PHYSICS 101B – HOMEWORK SET 4

Reading: Tipler and Llewellyn, Sections 8.4–8.5 and 10.1–10.4.

Due Friday 3/2/07.

1.) Consider a beam of particles travelling through a medium with an interaction probablity of γ per meter.

a) Show that the the change ΔN in the number of unscattered particles suffered during the traversal of a small length Δx surrounding the position x is given by

$$\frac{\Delta N}{\Delta x} = -\gamma N(x)$$

where N(x) is the number of surviving particles at the position x.

b) Letting $\Delta x \to 0$, show that you get a differential equation for $N(x)$ whose solution is given by $N(x) = N_0 e^{-\gamma x}$ where N_0 is the original number of particles in the beam. (Hint:$\int dN(x)/N(x) = \ln N(x)$.)

c) Show thus that the mean-free-path, or mean survival distance, for any particle in the beam is just $\lambda = 1/\gamma$. Note that the survival probability distribution implied by b) is not normalized, so you'll need to divide your weighted mean by the unweighted integral $\int N_0 e^{-\gamma x} dx$ in order to get the correct answer.

2.) Starting with Ohm's law, show that the current density J for a wire of length L, cross-sectional area A, and resistivity ρ, is given by

$$J = \frac{V}{\rho L}$$

where V is the applied potential difference between the two ends of the wire. Show that this is equivalent to the expression $J = \sigma E$ where $\sigma = 1/\rho$ is the conductivity, and E the (uniform) electric field inside the wire.

3.) Problem 10.14 (8.17×10^6, 1.00×10^7, 1.42×10^7 $(\Omega - m)^{-1}$))

4.) Problem 10.17, except do it for gold, for which I know the density to be 19.4 g/cm^3. ($6.41 \times 10^4 K$)

5.) Problem 10.18. The answer 0.1% is very approximate.

In the following problems, you will approximate the Fermi-Dirac distribution function $f_{FD}(E;T)$ in the following way. At $T = 0$, the distribution is the same as the true distribution: equal to 1 for $E < E_F$ and 0 for $E > E_F$. Now, assume that for $T \neq 0$, the distribution is 1 for $E < E_F - kT$, 0 for $E_F - kT < E < E_F$, 1 for $E_F < E < E_F + kT$, and finally 0 for $E > E_F + kT$. Thus, the effect of going to higher temperature is just to move the electrons with energy within kT *below* the Fermi energy to be within energy kT *above* the Fermi energy.

6.) Plot the true distribution $f_{FD}(E)$ and this approximate distribution on the same plot. Make the plot for the specific case of gold at room temperature (T = 300K). Make your plot over the range $\pm 3kT$ about E_F. Label your axes in terms of electron volts (eV).

7.) Using our approximate $f_{FD}(E)$, find the total energy contained in the motion of the free electrons of piece of metallic gold of volume V as a function of temperature. Don't forget that

$$n(E) = g(E) f_{FD}(E)$$

where $g(E)$ is the appropriate density of states for a 3-d infinite square well of volume V – see equation 8-66 in the text. You will find the following expansion useful:

$$(E_F \pm kT)^{5/2} \simeq E_F^{5/2}[1 \pm \frac{5}{2}\frac{kT}{E_F} + \frac{15}{8}\frac{(kT)^2}{E_F^2}]$$

Answer:

$$U \simeq \frac{8\pi(2m)^{3/2}}{5h^3} V {E_F}^{5/2} + \frac{6\pi(2m)^{3/2}}{h^3}\sqrt{E_F} V (kT)^2$$

(Can you derive that expansion?)

8.) Use this to find the contribution of the free electrons to the molar heat capacity of gold at room temperature. Compare this to the result 10.45 (pg. 461 of text; use $\alpha = \pi^2/4$) from the more rigorous calculation due to Sommerfeld. Answer:

$$C_V \simeq \frac{A}{\rho}\frac{12\pi(2m)^{3/2}}{h^3} k^2 \sqrt{E_F} T$$

where A is the atomic mass and $\rho = 19.4 g/cm^2$ is the density of gold.

9.) This result is substantially different than that for a gas of distinguishable particles. Why?