PHYSICS 110A WINTER 2002

FINAL EXAM

PUT YOUR NAME ON THE EXAM RIGHT AWAY!

PROBLEM 1 [35 POINTS]

A infinite cylinder of radius R and surface charge density σ A/m^2 rotates about its axis with angular velocity ω.

a) What are the values of the electric and magnetic field (magnitude direction) on the axis of the cylinder?

Instead, consider a cylinder with all the same properties, but with a finite length L.

b) Approximate the values of the electric and magnetic fields (magnitude and direction) along the axis of the cylinder, at points z far above the cylinder ($z \gg L$).

PROBLEM 2 [30 POINTS]

A sleep-deprived student in the Physics 134 lab is confronted with the following magnetostatic vector potential:

$$\vec{A}(s,\phi,z) = 2\beta s z \hat{s} + k\hat{\phi} + \beta s^2 \hat{z}.$$

This student needs to find the current distribution $\vec{J}(s,\phi,z)$ which produces this field.

NOTE: You can get part b) even if you're totally baffled by part a)!!

a) If she were to use the expression (5.62) from the book to find the current distribution (even heeding the footnote at the bottom of the page), she would get the wrong answer. Why? Make sure your reasoning is very clear, and that your claims are fully supported by whatever calculations may be necessary to do so!

b) What *is* the current distribution $\vec{J}(s,\phi,z)$ which produces this field?

PROBLEM 3 [30 POINTS]

An effectively infinite cylindrical shell of radius s_1 carries a solenoidal current density of K A/m in the $+\hat{\phi}$ direction. A similarly infinite concentric cylindrical shell of larger radius s_2 carries an opposing solenoidal current density of $2K$ A/m in the $-\hat{\phi}$ direction. The regions inside the inner cylinder, between the inner and outer cylinder, and outside the outer cylinder are all empty (vacuum).

a) What is the magnetic field (magnitude and direction) at every point in space?

Now, the region between the inner and outer cylinders is filled with a material possessing magnetic susceptibility χ_m.

b) Now what is the value of the magnetic field (magnitude and direction) at every point in space?

c) Identify the location, magnitude and direction of all bound (effective) currents in the system.

PROBLEM 4 [35 POINTS]

A thin disk of radius a and possessing a charge density of σ C/m^2 rotates at an angular velocity of ω rad/s about the z axis.

a) For distances $s \gg a$ in the plane of the rotating disk, the magnitude of the magnetic vector potential (in the gauge $\vec{\nabla} \cdot \vec{A} = 0$) can be written as

$$A = \frac{B}{s^k}.$$

What are the values of B and k?

b) What is the direction of $\vec{A}$? Be very clear and specific in your description of the direction.

A region of space possesses a magnetic field of the form

$$\vec{B} = \beta y \hat{x} + \beta z \hat{y} + \beta x \hat{z}.$$

c) If the spinning disk is placed in this region, what will be the magnitude and direction of the magnetic force on the disk? Please make sure your reasoning is clear.

PROBLEM 5 [35 POINTS]

A rectangular metal pipe, running parallel to the z-axis (from $-\infty$ to $+\infty$) has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at a potential specified by

$$V(b, y) = \beta \sin \frac{4\pi y}{a}.$$

a) What is the potential at every point (x, y, z) within the pipe?

b) What is the surface charge density on the bottom, i.e., on the side with $y = 0$?

PROBLEM 6 [35 POINTS]

In what follows, your answers need not be precise, but only true to good approximation. However, make sure that you express your answers solely in terms of the provided constants $r, d, \epsilon_r, \mu, E_0$ and B_0, and well-known physical constants.

A very thin disk of radius r and thickness d ($d \ll r$) is manufactured from a dielectric with relative permittivity ϵ_r. This disk is placed in an electric field of magnitude E_0, oriented perpendicular to its flat ends.

a) What is the magnitude and direction of the electric field within the disk?

b) What is the electric dipole moment of the disk?

A very tall cylinder of radius r and height d ($d \gg r$) is manufactured from a metal with magnetostatic permeability of μ. The cylinder is placed in a magnetic field of magnitude B_0, oriented parallel to the axis of the cylinder.

c) What is the magnitude and direction of the magnetic field within the cylinder?

d) What is the magnetic dipole moment of the cylinder?