# PHYSICS 110A WINTER 2003

## FINAL EXAM

## PUT YOUR NAME ON THE EXAM RIGHT AWAY!

PROBLEM 1 [30 POINTS]

Three infinite straight conducting wires are arranged to be parallel with the $z$ axis. The first lies with $(x, y) = (0, 0)$, the second with $(x, y) = (b, 0)$ and the third with $(x, y) = (b/2, \sqrt{3}b/2)$; i.e., the wires form an equilateral triangle of side $b$ in their $x$-$y$ projection. The first two wires each carry a current of magnitude $I$ in the $+\hat{z}$ direction, while the third carries a current of magnitude $I$ in the $-\hat{z}$ direction. Calculate the magnitude and direction of the force per meter on the third wire due to the magnetic fields of the first two wires.

PROBLEM 2 [35 POINTS]

Two concentric hollow spherical shells with radius $a$ and $b$ carry charges of $+Q$ and $-Q$ respectively, with $b > a$ and with the inner (radius $a$) sphere possessing the positive charge.

a) Assuming that you start with the charges dispersed at infinity, how much energy does it take you to assemble this distribution?

The same charge distribution is assembled around a dielectric of susceptibility $\chi_e$ (i.e., after the distribution is assembled, the region between the two concentric spheres is filled with dielectric).

b) How much energy does it take to assemble the charge distribution in this case?

PROBLEM 3 [30 POINTS]

A thin wire lying along the $z$ axis carries a current $I$ in the $+z$ direction. This wire is surrounded by a solid conducting cylinder of radius $a$, carrying a uniform current density. If the total current in the cylinder is also $I$, but in the $-z$ direction, what is the value of the magnetic field (magnitude and direction) at all points $(s, \phi, z)$ in space? You may ignore the premeability of the conductor (i.e., you may assume that $\mu = \mu_0$ for the conductor).

PROBLEM 4 [35 POINTS]

An infinite conducting slab of thickness $d$ and permeability $\mu$ lies in the $x$-$y$ plane, evenly straddling the plane defined by $z = 0$. The conductor carries a uniform current density of $\vec{J} = J_0 \hat{y}$ throughout it. In what follows, your answers should be expressed in terms of the permeability $\mu$, and not the magnetic susceptibility $\chi_m$.

a) Find the magnetic field (magnitude and direction) at all points inside the conductor.

b) Identify the magnitude and direction of any (effective) bound currents flowing on the surface of the conductor.

c) Identify the distribution and direction of any (effective) bound currents flowing in the bulk (volume) of the conductor.

PROBLEM 5 [35 POINTS]

A spherical shell of radius $\rho$ has a potential given by $V(\rho, \theta, \phi) = V_0(3\cos^2\theta - 3\cos\theta - 1)$.

a) Find the potential at all points $(r, \theta, \phi)$ outside the shell.

b) Approximate the value of the electric field (magnitude and direction) for all points $(r, \theta, \phi)$ such that $r \gg \rho$.

PROBLEM 6 [35 POINTS]

Two thin wire loops of radius $a$, each carrying current $I$, lie in the $x$-$y$ plane, co-axial with the $z$ axis. One lies at $z = b$ and the other at $z = -b$. When looked at from above (from a point of high $z$), the current in the loop at $z = b$ circulates counterclockwise, while the current in the loop at $z = -b$ circulates clockwise.

a) In the limit that $r \gg a$ and $r \gg b$, find the value of the magnetic vector potential $\vec{A}(x, y, 0)$ for any point in the $x$-$y$ plane (work in the coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$).

b) From this, calculate the value $B_z$ of the z-component of the magnetic field at any point on the $x$-$y$ plane far away from the two loops.

c) Consider a point $\vec{r} = (x, 0, 0)$ for $x \gg a, b$. Approximate the value (magnitude and direction) of the vector potential $\vec{A}$ at a small distance $z$ above this point, i.e., at the point $(x, 0, z)$ for $z \ll x$.

d) What is the value of the magnetic field, both magnitude and direction, at the point $(x, 0, 0)$?