PROBLEM 1 [25 POINTS]

A charge $+Q$ sits at the point $(0, 0, 2d)$, while a charge $-Q$ sits at $(0, 0, -2d)$.

a) What is the electrostatic potential at the point $(r/\sqrt{2}, 0, r/\sqrt{2})$, in the limit that $r \gg d$?

b) What is the magnitude of the force that a charge q would feel at that point?

PROBLEM 2 [20 POINTS]

A charge $q > 0$ moves towards the center of a grounded conducting sphere with a speed v. At $t = 0$, the charge is a distance a from the center of the sphere, where $a > R$, and R is the radius of the sphere. What current (amount of charge per second) flows out of ground onto the sphere at $t = 0$? Make sure to specify whether the current is positive (positive charge flowing onto sphere) or negative.

PROBLEM 3 [25 POINTS]

The following problem illustrates several essential points about the electrostatics of insulators.

An infinitely long insulating cylinder of radius a, with a 'frozen in' polarization of

$$\vec{P} = ks\hat{s},$$

sits at rest deep in space, far from any other sources of electric field.

a) What is the electric field at every point in space, inside and outside the cylinder, in the vicinity of the cylinder?

b) Calculate the insulator's electrical susceptibility χ_e.

PROBLEM 4 [30 POINTS]

A rectangular pipe, running parallel to the z axis (from $-\infty$ to $+\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$ (see diagram). The fourth side, at $x = b$, is maintained at the potential

$$V(y) = V_0 \sin(2\pi y/a).$$

Find the potential $V(x, y, z)$ for all points (x, y, z) inside the pipe. It may be convenient to recall that

$$\sinh(x) = \frac{e^x - e^{-x}}{2}.$$