

**PHYSICS 110A WINTER 2002**

**MIDTERM II**

**PUT YOUR NAME ON THE EXAM RIGHT AWAY!**

**PROBLEM 1 [25 POINTS]**

A charge of magnitude  $+Q$  rests at the point  $(0,0,d)$ , while a charge of magnitude  $-Q$  rests at the point  $(0,0,d+a)$ . The distance  $a$  is small relative to  $d$ . A infinite conducting plane at  $z=0$  is held at ground ( $V=0$ ).

a) What is the electrostatic potential midway between the two charges?

b) Using the multipole expansion, for  $z$  much greater than  $d$  we can approximate the potential  $V(0,0,z)$  as

$$V(0,0,z) = \frac{A}{z^k}$$

What are the values of  $A$  and  $k$ ?

PROBLEM 2 [25 POINTS]

A spherical shell of radius  $\rho$  has a potential given by  $V(\rho, \theta, \phi) = \beta \cos \theta$ . The spherical shell lies at the center of a spherical cavity of radius  $R$ . Surrounding the cavity is a conductor held at ground (see diagram). Find the function  $V(r, \theta, \phi)$  which gives the potential at any point  $(r, \theta, \phi)$  within the cavity.

PROBLEM 3 [25 PTS]

An effectively infinite conducting cylinder with radius  $\rho$  and surface charge density  $\sigma$  is frozen into a block of ice ( $\epsilon_r = 100$ ).

a) Find the magnitude and direction of the electric field at every point in space outside the cylinder.

b) How much stronger or weaker is this field relative to the case for which the cylinder is surrounded by vacuum?

PROBLEM 4 [25 PTS]

In a region of space, the constant electric field produced by a far-off distribution of free charge has a magnitude  $E_0$ , and is directed in the  $+\hat{z}$  direction.

An effectively infinite planar slab of dielectric of thickness  $d$  is brought into this region of space, and oriented perpendicular to the  $z$  axis, i.e., parallel to the  $x - y$  plane. The external field due to the far-away free charge distribution induces a polarization of magnitude  $P$  inside the slab. The direction of  $\vec{P}$  is identical to that of the external field.

a) If the far-off distribution of free charge is unchanged as the slab is brought into position, what is the value of the electric field inside the slab? DON'T FORGET: the electric field has both a magnitude and direction.

b) What would the electric field be if, instead of being oriented in the  $+\hat{z}$  direction, the field was oriented at an angle of  $45^\circ$  between the  $+\hat{x}$  and  $+\hat{z}$  directions? Express your answer by writing down separate expressions for  $E_x$ ,  $E_y$  and  $E_z$ . Again, the free charge distribution is unchanged by the introduction of the slab.

Facilitating fact: the field due to an infinite sheet of charge with area density  $\sigma$  has magnitude  $\sigma/2\epsilon_0$ .