PHYSICS 110B SPRING 2003

MIDTERM II

PUT YOUR NAME ON THE EXAM RIGHT AWAY!

PROBLEM 1 [25 POINTS]

A plane electromagnetic wave with wavelength λ propagates in the $+z$ direction. The wave is polarized in the $+y$ direction, and the amplitude of the electric field oscillation is E_0.

a) Write down, in either the real or complex representation, the explicit vector expressions $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$ for the electric and magnetic fields associated with this plane wave. Assume that the electric field oscillation is at its maximum at $(x, y, z, t) = (0, 0, 0, 0)$. Express your answer, to the fullest extent possible, in terms of the given parameters E_0 and λ.

Instead, this same plane wave, again with a $+y$ polarization, propagates in the x-z plane, with a direction of $45°$ towards the $+x$ axis relative to the $+z$ axis.

b) What is the wavevector $\vec{k}$ of this wave? Express $\vec{k}$ explicitly in terms of its x,y, and z components, and the wavelength λ.

c) Again write down, in either the real or complex representation, the full vector expressions $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$ for the electric and magnetic fields associated with this plane wave. Assume that the electric field oscillation is at its maximum at $(x, y, z, t) = (0, 0, 0, 0)$.

d) Calculate the time-averaged power passing through a 1 m^2 area oriented parallel to the x-y plane.

PROBLEM 2 [25 POINTS]

A long straight wire contains a net electrical charge of λ C/m. This wire runs down the axis of an infinite solenoid of radius s_0, which is wound with n turns per meter. Calculate the total momentum and angular momentum (magnitude and direction, in both cases) carried in the fields in a length z_0 of this configuration, assuming that a current I courses through the turns of the solenoid. Angular momentum is only defined up to a chosen axis; let the axis of choice be the obvious one; i.e., the axis of the solenoid along which the wire lies.

PROBLEM 3 [25 POINTS]

Consider a rectangular waveguide of dimensions a and $b = \frac{1}{2}a$. The coupling into the waveguide is somehow arranged so that only the TE_{32} mode can be excited within the guide.

a) If the applied frequency ω is exactly twice the minimum necessary to excite this finicky waveguide, what is the wavenumber k of this excitation (as in the text, k is the wavenumber associated with oscillations in the direction of propagation)? Your answer should depend only upon a, the dimension of the waveguide.

For electromagnetic waves in vacuum, we know that the electric and magnetic fields oscillate in a direction perpendicular to the direction of propagation of the wave, and so all the energy contained in the wave is accounted for by these transverse oscillations. We also know, though, that for guided waves, there can be 'longitudinal' oscillations that are parallel to the direction of motion. So for guided waves, there can be energy contained in longitudinal oscillations.

a) For the TE_{32} mode of the above waveguide, excited at $\omega = 2\omega_{min}$, find the fraction of the energy contained the longitudinal oscillations, separately for the electric and magnetic fields.

PROBLEM 4 [25 POINTS]

As discussed in section 9.3.2 of Griffiths, a light wave traveling through vacuum that encounters an insulator at normal incidence will have a fraction

$$R = (\frac{1-n}{1+n})^2$$

of its intensity reflected back into the vacuum, *provided that* $\mu \simeq \mu_0$ *for the medium.*

For this problem, assume instead that $\epsilon = \epsilon_0$ for the medium, and that its index of refraction is instead generated by its magnetic permeability $\mu = \mu_r \mu_0$. Calculate the reflection coefficient R for the case that $\mu_r = 2$.