

## PHYSICS 110B – HOMEWORK SET 2

Due Monday 4/12/04. Ten points per problem. Selected answers are provided.

Reading: Notes; supplements to be found on reserve in Purcell, Chapter 8, and Bleaney and Bleaney, Chapter 9 and Section 12.5.

1.) Consider an RLC circuit driven by a voltage  $V = V_0 \cos(\omega t)$ . By explicitly solving the second-order differential equation developed in class, show that the voltage across the capacitor is given by

$$V_c = \frac{1}{\omega C} \frac{V_0}{\sqrt{R^2 + (1/\omega C - L\omega)^2}} \sin(\omega t + \phi)$$

where

$$\tan \phi = \frac{1/\omega C - L\omega}{R}.$$

Calculate the impedance of this circuit.

2.) We can represent the sinusoid  $S(\omega, A, \phi) = A \cos(\omega t + \phi)$  via the complex exponential  $Ae^{i\phi}$ , by which we mean that  $S(\omega, A, \phi) = \text{Re}[Ae^{i\phi}e^{i\omega t}]$ . Show that the sum of representations

$$A_s e^{i\phi_s} = A_1 e^{i\phi_1} + A_2 e^{i\phi_2}$$

represents the sum  $S(\omega, A_1, \phi_1) + S(\omega, A_2, \phi_2)$  of the represented sinusoids. Express  $A_s$  and  $\phi_s$  in terms of  $A_1$ ,  $A_2$ ,  $\phi_1$  and  $\phi_2$ .

3.) Show that the effective impedance  $Z_e$  of two circuit elements in parallel is given by

$$\frac{1}{Z_e} = \frac{1}{Z_1} + \frac{1}{Z_2},$$

where  $Z_1$  and  $Z_2$  are the impedances of the two circuit elements.

4.) In introductory physics, we learn that the power consumed by a DC circuit is given by  $P = IV$ , where  $V$  is the voltage across the circuit and  $I$  the current through it. For AC circuits, it should still be the case that the *instantaneous* power through the circuit at time  $t$  is given by  $P(t) = I(t)V(t)$ . Let's explore this for the series RLC circuit analyzed in class.

Consider first the general case of an AC circuit, for which a current  $I(t) = I_0 \cos(\omega t + \phi)$  is driven by a voltage  $V(t) = V_0 \cos(\omega t)$ . Show that, if indeed  $P(t) = I(t)V(t)$ , then the average power consumed by this circuit is given by

$$\bar{P} = \frac{1}{2} I_0 V_0 \cos \phi.$$

If this is correct, what would be the average power consumed by a series RLC circuit as a function of the driving frequency  $\omega$ ?

Taking a look at the series RLC circuit diagram, we see that the only mechanism by which the circuit can consume power is via power dissipation in the resistor, for which  $P(t) = I^2(t)R$ . Calculate the mean power dissipated by the resistor in our series RLC circuit, compare it to the mean power consumed just above, and comment.

5.) In class, we considered a *series* RLC circuit driven by a sinusoidal voltage  $V = V_0 \cos(\omega t)$ . For this problem, we'll consider a *parallel* driven RLC circuit, i.e., an RLC circuit for which the resistor, inductor, and capacitor are all connected in parallel with each other and with the voltage source. Show that the magnitude of the impedance of such a circuit is given by

$$Z^2 = \frac{R^2}{1 + R^2(\omega C - 1/\omega L)^2}$$

and that the current leads the voltage by a phase angle  $\phi$ , where

$$\tan\phi = R\omega C - \frac{R}{\omega L}.$$

6.) Rather than overall amplitude, AC voltage sources are usually rated in terms of the root-mean-square (rms) voltage they deliver. (The 'rms' value of an oscillatory function is just the time-averaged value of the square of the function over one period of its oscillation).

The 60-Hz oscillation of U.S. wall-plug voltage is rated in this way. Express  $V(t)$  for U.S. wall-plug voltage in the form  $V(t) = V_0 \cos(\omega t)$ , given that it is rated at 120 V rms. (Answer:  $V(t) \simeq 170 \cos(377t)$ .)

Show that the average power dissipated by a driven AC circuit can be written in the form  $\bar{P} = V_{rms} I_{rms} \cos\phi$ , where  $V_{rms}$  and  $I_{rms}$  are the rms values of the driving voltage and current through the circuit, and  $\phi$  is the phase angle between the voltage and current.

7.) A resistor  $R$  and capacitor  $C_2$  are connected together in parallel. This parallel combination of  $R$  and  $C_2$  is then connected in series with a capacitor  $C_1$ . This circuit is then plugged into a 120V wall socket in your house. If  $R = 10,000\Omega$ ,  $C_1 = 0.5\mu\text{F}$ , and  $C_2 = 0.2\mu\text{F}$ , what is the average power consumed by the circuit? (Answer: 0.64 Watt).

8.) Show that, for a series RLC circuit driven by a voltage  $V_0 \cos(\omega t)$ , the value  $\omega_0$  which drives the circuit into resonance (maximum current) also provides for a current that is exactly in phase with the driving voltage. Can you argue on physical grounds that this makes sense? What is  $\omega_0$  in terms of  $R$ ,  $L$ ,  $C$ , and  $V_0$ ?

Now, consider two identical RLC circuits, coupled via a mutual inductance  $M = L$ , with one driven by the potential  $V_0 \cos(\omega t)$ , and the other undriven (a perfectly coupled unit-gain transformer). Using the criterion that the circuit is in resonance when the current in the driven circuit is exactly in phase with the driving voltage, show that the resonant frequency becomes

$$\omega_0 = \sqrt{\frac{1}{2LC - R^2C^2}}.$$