PHYSICS 110A – HOMEWORK SET 5

Due Monday 3/8/10. Ten points per problem. Answers provided where appropriate.

Reading: Griffiths, Chapter 5.

1.) 5.10

$$\frac{\mu_0 I^2 a^2}{2\pi s(s+a)}$$

$$\frac{\mu_0 I^2}{2\pi}[\frac{a}{s} - \frac{2}{\sqrt{3}}\ln(1+\frac{\sqrt{3}a}{2s})]$$

2.) 5.13. Answer to b), for $s < a$ and $s > a$, respectively:

$$\frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi}$$

$$\frac{\mu_0 I}{2\pi s}\hat{\phi}$$

3.) 5.14; $\vec{B} = -\mu_0 J z \hat{y}$ for $-a < z < a$; $\vec{B} = -\mu_0 J a \hat{y}$ for $z > a$;

4.) 5.22

5.) 5.23

$$\frac{k}{\mu_0 s^2}\hat{\phi}$$

6.) 5.29

$$\frac{\mu_0 \omega Q}{4\pi R}[(1 - \frac{3r^2}{5R^2})\cos\theta\hat{r} - (1 - \frac{6r^2}{5R^2})\sin\theta\hat{\theta}]$$

7.) 5.31

8.) 5.34; answer to b)

$$\frac{\mu_0}{4\pi}\frac{I\pi R^2}{r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

9.) 5.36

10.) A magnet with a uniform magnetic field (known as a *dipole magnet*) will bend a beam of charged particles along the arc of a circle. Instead, consider the effect of a *quadrupole magnet*, for which the y component of the magnetic field has the dependence

$$B_y = Gx.$$

Consider a beam of positively charged particles with a circular cross section, i.e., a beam that would punch a circular hole through a piece of paper placed in its path. Assume that this beam passes through a quadrupole magnet travelling in the $+\hat{z}$ direction, centered on $x = y = 0$. (The quadrupole magnet is short – the beam only passes through it for a small distance in z, beyond which it travels in a field-free region).

a) Argue that the quadrupole magnet will act as a *focussing lens* for particles with $x \neq 0$. i.e., that particles with $x \neq 0$ will be redirected by the field of the quadrupole magnet to converge at $x = 0$ at a focal point in z which is in the field-free region somewhere downstream of the magnet. Approximate the focal length of this magnet in terms of the quadrupole strength G, the beam momentum p, the beam particle charge Q, and the z-extent d of the magnet, in the limit that d is small. (Answer: $f = p/dQG$)

b) In the vacuum gap of the quadrupole magnet, through which the beam passes, there are no sources of magnetic field. Considering the appropriate field-source relation for the magnetic field, argue that the quadrupole magnet will act as a *defocussing lens* for particles with $y \neq 0$, with the same focal length as that of the focussing effect for particles displaced in x.

11.) 5.39; answer to b) is $V = vBt$.

12.) 5.46

13.) 5.61