PHYSICS 110B – HOMEWORK SET 6

Due Monday 5/17/04. Ten points per problem. Selected answers are provided.

Reading: Notes; supplements to be found on reserve (material TBA).

1.) For Fraunhofer diffraction from a single slit, we found in class that, at distance s from the center of the slit, the contribution to the overall amplitude of the electric field from an element ds is given by

$$dE = \gamma_0 \cos(kx - \omega t + ks\ \sin\theta) ds,$$

where θ is the angular displacement relative to the normal of the slit, as measured on a distant screen. Integrating over the slit width b (from $-b/2$ to $+b/2$), show that the total electric field observed on the distant screen at the angle θ is given by

$$\gamma_0 b \frac{\sin\beta}{\beta} \cos(kx - \omega t)$$

$$\beta = \frac{1}{2} kb \sin\theta.$$

Hint: Consider elements ds at s and $-s$ simultaneously, and use the identity

$$\cos(a) + \cos(b) = 2\cos[\frac{1}{2}(a+b)] \cos[\frac{1}{2}(a-b)].$$

2.) Anti-reflective Coatings. A thin film, of thickness d and index of refraction n_1, is deposited on glass, with index $n_2 > n_1$. Plane-wave radiation of wavelength λ is incident on the film with an angle of incidence θ, reflecting back into the air from both the front and back surface of the film. Show that the reflected intensity will be a minimum when

$$d = \frac{\lambda}{4n_1 \cos\theta_1}$$

with

$$\frac{\sin\theta_1}{\sin\theta} = \frac{1}{n_1}$$

(For $n_2 > n_1$, the change in phase at reflection will usually be the same at the front and back surface of the film. Assume this is the case).

A durable anti-reflective coating, with good transmission throughout the visible and into the near ultraviolet, is magnesium fluoride (MgF_2), for which $n = 1.38$ (glasses typically have $n \simeq 1.5$). What thickness of MgF_2 should you deposit on the surface of your TV set to minimize the reflection of sunlight incident at 30°? (The spectrum of sunlight peaks in the yellow/green, at a wavelength of $\lambda \simeq 550$ nm.)

3.) Can you show that

$$\frac{e^{Nx}-1}{e^x-1} = [1 + e^x + e^{2x} + ... + e^{(N-1)x}]?$$

Hint: You don't need to expand and regroup the exponentials. Just use a little algebra and sum-index manipulation.

4.) A typical diffraction grating might have 1000 slits, with a slit width of $b = 5\mu$m and a slit pitch (separation) of $a = 20\mu$m. Meanwhile, a typical visible light wavelength (greenish-blue) is $\lambda = 0.5\mu$m. The diffraction grating has principle maxima at $\alpha = n_p\pi$ and local maxima at $\alpha = n_l\pi/N$, for any positive integers n_p and n_l (as discussed in class, $\alpha = (1/2)ka\sin\theta$). For this typical grating, find the angles θ for the first principle maximum ($n_p = 1$) and the local maximum immediately following the first principle maximum $n_l = N + 1$. What is the ratio of intensities between these two maxima? What is the ratio of intensities between the first, second, and tenth principle maxima? Estimate the *resolving power* of the grating, i.e., if you shine another wavelength of light onto the grating simultaneous with the $\lambda = 0.5\mu$m light, by roughly how much does this new wavelength have to differ from $0.500\mu m$ so that their first principle maxima don't overlap?

5.) The centers of two slits of width b are separated by a distance of a. This double-slit aperture is illuminated by plane-wave radiation of wavelength λ, and its diffraction pattern is observed at a far-off screen as a function of the angle θ relative to the direction of the original plane wave. Show that the intensity observed on the screen is given by

$$I(\theta) = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\alpha,$$

where

$$\alpha = \frac{a\pi\sin\theta}{\lambda}$$

$$\beta = \frac{b\pi\sin\theta}{\lambda}$$

and I_0 is the intensity that is observed at $\theta = 0$ if one of the slits is covered. Plot $I(\theta)$ (in units of I_0) versus $\sin\theta$ for the case $a = 3b$. What does this diffraction pattern look like in the limits that $b/a \to 0$?

6.) Plane-wave radiation of wavelength λ is incident on a rectangular aperture of width a and height b, and the resulting diffraction pattern is observed on a screen a distance z away, where $z \gg a, b$. Consider the point (x, y) on the screen, where the horizontal (x) and vertical (y) distances are measured relative to the center of the aperture. Show that the observed intensity is

$$I(x,y) = I_0 \frac{\sin^2\beta_x}{\beta_x^2}\frac{\sin^2\beta_y}{\beta_y^2}$$

$$\beta_x = \frac{2\pi a x}{2\lambda z}$$

$$\beta_y = \frac{2\pi b y}{2\lambda z}.$$

7.) From the rotation frequency and light spectrum, you determine that the two stars in a binary system are separated by about 10^9 meters. With the Mount Palomar 200-inch diameter telescoped, the two stars in the binary system are just resolved at $\lambda = 500$ nm. What is the distance to the binary system?