PHYSICS 133
HOMEWORK I

This homework is due at the beginning of the third class period.

Problem 1

The probability of decay of an unstable particle between times t and $t + dt$ is given by the exponential distribution

$$P(t)dt = \frac{1}{\tau} \exp\left(-t/\tau\right)\, dt.$$

Show the following:

(a) The probability is correctly normalized (integrated over all time, the probability that the particle will decay is exactly 1);

(b) The mean value of the decay time is the mean lifetime τ;

(c) The root-mean-square deviation of decay times about the mean decay time τ is again just τ. [Hint: you need to find the distribution-weighted-mean of the value $(t - \tau)^2$ along the way...]

Problem 2

The following are the results of four measurements of the height of a person (in cm): 165.6 ± 0.3, 165.1 ± 0.4, 167.2 ± 1.6, and 166.3 ± 0.9.

(a) Calculate the straight average, ignoring the errors.

(b) Calculate the best estimate of the person's height and the uncertainty, taking into account the errors properly. Does the difference go in the direction you would expect? Why?

Problem 3: The famous $\sqrt{12}$ factor!

A particle tracking detector can locate a particle in one dimension to within a spatial interval d. In other words, the detector gives a signal when the particle passed somewhere along the strip in the interval between $x = 0$ and $x =$ d. With the reasonable assumption that particles are incident uniformly across the detector (i.e., that the probability distribution between x = 0 and x = d is a constant), show that the standard (root mean square) deviation of the resulting position measurement is given by $\sigma = d/\sqrt{12}$.

Problem 4

We can represent a sinusoidal current of the form

$$I(t) = I_0 \cos(\omega t)$$

as the real part of a complex current

$$I(t) = \Re(I_0 e^{j\omega t}).$$

Making use of this, show that we can represent the relation between the current through and voltage across a capacitor of capacitance C Farads in terms of the complex impedance

$$Z_C = \frac{1}{j\omega C}.$$

Rewrite Z_C in the form $Z_0 e^{i\phi}$ where Z_0 is the magnitude of the capacitor's complex impedance and ϕ is its phase. Section 6.4 of the Lab Manual can be a helpful reference for this problem.

Problem 5

Find expressions for the magnitude of the complex impedance of a sinusoidally-excited circuit with a resistor of value R Ohms and a capacitor of value C Farads

a) when R and C are in series

b) when R and C are in parallel.

Also find the phase of the complex impedance for the series circuit. In all cases, you answer will be a function of the angular frequency ω of the induced oscillation. Again, Section 6.4 can be a good reference.