PHYSICS 215B – HOMEWORK 1

Due Wednesday, January 22 2014, at the end of the working day.

Complementary reading: Shankar, Chapter 1.

**Problem 1**

Shankar, Exercise 1.3.1, page 15

**Problem 2**

The Legendre polynomials are defined by

\[ P_l(x) = \frac{1}{2^l l!} \left( \frac{d}{dx} \right)^l (x^2 - 1)^l. \]

Note that the set \{1, x, x^2, x^3, \ldots\} forms a basis of the space of square-integrable functions on the region \(|x| \leq 1\); however, this basis is not orthonormal. Use the Gram-Schmidt procedure to form the first three members of an orthonormal basis. Show that these are proportional to the corresponding Legendre polynomials.

**Problem 3**

Shankar, Exercise 1.7.1, part (3) only, Page 30. Just to get a little practice with some slightly formal thinking. Please do not make use of results from prior sections of the problem, but work explicitly in terms of matrix elements and summed indices.

**Problem 4**

Shankar, Exercise 1.8.5, Page 42

**Problem 5**

Shankar, Exercise 1.8.10, Page 46.

**Problem 6**

Show that

\[ \delta(f(x)) = \sum_n \frac{\delta(x - x_n)}{|df/dx_n|}, \]

where \(x_n\) are the zeros of \(f(x)\), and \(df/dx_n\) is the value of \(df/dx\) when evaluated at \(x = x_n\) (assume that \(df/dx_n \neq 0\) for all \(n\)). Use this result to obtain simplified expressions for \(\delta(ax)\) and \(\delta(x^2 - a^2)\).
Problem 7

Consider a system of two states

\[
\begin{pmatrix}
\psi_1(t) \\
\psi_2(t)
\end{pmatrix}
\]

governed by the Hamiltonian

\[
\begin{pmatrix}
E & \Delta \\
\Delta & E
\end{pmatrix}.
\]

Find the propagator \( U \) for this system; i.e., find the matrix \( U \) for which we can write

\[
\begin{pmatrix}
\psi_1(t) \\
\psi_2(t)
\end{pmatrix} = U \begin{pmatrix}
\psi_1^0 \\
\psi_2^0
\end{pmatrix}.
\]

Assuming \((\psi_1^0, \psi_2^0) = (1, 0)\), find an expression that represents the probability of finding the system in the state \((1, 0)\) as a function of time.