PHYSICS 215B – HOMEWORK 1

Due Wednesday, January 22 2014, at the end of the working day.

Complementary reading: Shankar, Chapter 1.

Problem 1

Shankar, Exercise 1.3.1, page 15

Problem 2

The Legendre polynomials are defined by

$$P_l(x) = \frac{1}{2^l l}(\frac{d}{dx})^l (x^2-1)^l.$$

Note that the set $\{1, x, x^2, x^3, ...\}$ forms a basis of the space of square-integrable functions on the region $|x| \leq 1$; however, this basis is not orthonormal. Use the Gram-Schmidt procedure to form the first three members of an orthonormal basis. Show that these are proportional to the corresponding Legendre polynomials.

Problem 3

Shankar, Excercise 1.7.1, part (3) only, Page 30. Just to get a little practice with some slightly formal thinking. Please do not make use of results from prior sections of the problem, but work explicitly in terms of matrix elements and summed indices.

Problem 4

Shankar, Excercise 1.8.5, Page 42

Problem 5

Shankar, Exercise 1.8.10, Page 46.

Problem 6

Show that

$$\delta(f(x)) = \sum_n \frac{\delta(x-x_n)}{|df/dx_n|},$$

where x_n are the zeros of $f(x)$, and df/dx_n is the value of df/dx when evaluated at $x = x_n$ (assume that $df/dx_n \neq 0$ for all n). Use this result to obtain simplified expressions for $\delta(ax)$ and $\delta(x^2 - a^2)$.

Problem 7

Consider a system of two states

$$\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix}$$

governed by the Hamiltonian

$$\begin{pmatrix}E & \Delta\\ \Delta & E\end{pmatrix}.$$

Find the propagator U for this system; i.e., find the matrix U for which we can write

$$\begin{pmatrix}\psi_1(t)\\ \psi_2(t)\end{pmatrix} = U\begin{pmatrix}\psi_1^0\\ \psi_2^0\end{pmatrix}.$$

Assuming $(\psi_1^0, \psi_2^0) = (1, 0)$, find an expression that represents the probability of finding the system in the state $(1, 0)$ as a function of time.