PHYSICS 215B – HOMEWORK 2

Due Wednesday, January 29 at the end of the working day.

Complementary reading: Shankar, Chapters 4 and 9. You might also read Chapters 2 and 3, but that would just be for fun.

Problem 1

The following problem will help to reinforce the connection between the bra/ket notation and the underlying Fourier calculus. We start with an alternative development of the representaiton of the Dirac δ-function in terms of complex exponentials.
(a) As we have seen, one definition of the δ-function is

$$\delta(x-x') = \lim_{\epsilon\to 0} \frac{1}{\sqrt{\pi\epsilon^2}} \exp\Big(-\frac{(x-x')^2}{\epsilon^2}\Big).$$

By appropriate use of complex integration, verify that this expression is equivalent to

$$\delta(x-x') = \lim_{a\to 0} \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ik(x-x')-ak^2} dk.$$

(b) Let $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k) e^{ikx} dk$. Show that

$$\int_{-\infty}^{+\infty} |f(x)|^2 dx = \int_{-\infty}^{+\infty} |a(k)|^2 dk.$$

(c) Rewrite the expression you have just justified in terms of Dirac bras and kets.

Problem 2

Shankar, Excercise 4.2.1, Page 129. For part (5), the L_z measurement is made *after* the L_z^2 measurement.

Problem 3

Shankar, Excercise 4.2.2, Page 139.

Problem 4

Shankar, Excercise 4.2.3, Page 139.

Problem 5

An ensemble of atoms has a transition of energy E_0 excited at time $t = 0$. The atoms in the ensemble decay back to the ground state by emitting a photon. The mean lifetime of the transition back to the ground state is Γ. Show that the distribution of emitted photon energies is given by

$$P(E) \propto \frac{1}{1 + \frac{4\Gamma^2}{\hbar^2}(E - E_0)^2}.$$

What is the precise relation between the FWHM of $P(E)$ and the lifetime of the excited state?