PHYSICS 215B – HOMEWORK 3

Due Friday, February 7, at the end of the working day.

Complementary reading: Shankar, Chapter 5.

Problem 1

In class we developed the states of an infinite square well of width a centered about $x = 0$, labeled via the quantum number n. For example, for n odd, we found

$$\psi_n(x) \propto \alpha_n cos\Big(\frac{n\pi x}{a}\Big).$$

Find the values of α_n that normalize this wavefunction.

Problem 2

Consider a particle of mass m incident upon a potential step of height V_0, i.e., $V = 0$ for $x < 0$ and $V = V_0$ for $x \geq 0$.

a) Show that, for a particle with a kinetic energy of precisely $KE = V_0$ ($p_0^2 = 2mV_0$) , there is a 100% chance that the particle will be reflected by the potential barrier.

b) Suppose, instead, the particle's wavefunction is represented by a gaussian wavepacket whose position is known to a precision Δx. If the expectation value of the momentum of the particle is still $\sqrt{2mV_0}$, write down an integral that expresses the probability that the particle is reflected by the barrier. You may assume that $\hbar/2\Delta x < p_0$. You need not evaluate the integral.

Problem 3

Consider a particle of mass m in motion in the vicinity of a finite square well, with a potential function given by $V = 0$ for $-a \leq x \leq a$ and $V = V_0$ for $|x| > a$. Since the potential function is symmetric about $x = 0$, any solution $\psi(x)$ to the Schroedinger Equation must either be even ($\psi(-x) = \psi(x)$) or odd ($\psi(-x) = -\psi(x)$).

a) Consider an *even* solution $\psi(x)$ with energy $E < V_0$. Write down the function $\psi(x)$, up to normalization, in each of the three regions $x \leq -a$, $-a \leq x \leq a$, and $x > a$. Express the arguments of these functions in terms of x, E, and V_0.

b) Show that there exists at least one such value of E for any choice of V_0, i.e., that *any* finite square well has at least one even bound state.

c) For a particle of this mass, and a well of this size, what is the maximum V_0 for which there will be only one even bound state?

Problem 4

Consider a potential of the form

$$V(x) = \beta\delta(x)$$

where $\delta(x)$ denotes the Dirac delta function. For $\beta < 0$, this potential admits a single bound state. Find the energy of this state in terms of $|\beta|$ and the mass m of the trapped particle.

Problem 5

An object of mass m lies in the ground state of a one-dimensional infinite square well of dimension a. At $t = 0$ the extent of the square well is instantaneously doubled by extending one of the walls by a distance a, without disturbing the wavefunction of the object.

(a) What is the ratio of probablities of finding the object in the first excited and ground states of the stretched square well at $t = 0$?

(b) Instead of measuring the particle's properties at $t = 0$, you wait a time $t_1 = 2ma^2/h$, where h is Planck's constant. What is the same ratio of probabilities at t_1?

(c) Making the approximation that the object's wavefunction is dominated by these two states, estimate the probability that the object will bounce off the wall that wasn't moved in the time interval $0 < t < t_1$.

Problem 6

Shankar Exercise 5.3.3, Page 167.