PHYSICS 215B – HOMEWORK 4

Due Friday, February 14 at the end of the working day.

Complementary reading: Shankar, Chapter 7. You could also read Chapter 6, but for entertainment purposes only.

Problem 1

Shankar Exercise 7.3.7 Page 202.

Problem 2

Shankar Exercise 7.3.4 Page 196. Might he mean (7.3.23)-(7.3.26) instead?

Problem 3

Two distinguishable objects of mass m, moving in a common potential of the form $U = \frac{1}{2}Cx^2$, are coupled through a potential $U_c(x_1, x_2) = \frac{1}{2}k(x_1 - x_2)^2$. Here x_1 and x_2 are the coordinates of particles 1 and 2, respectively, and C and k are greater than 0.

(a) Write down the full time-independent Shroedinger Equation for the system in terms of the individual coordinates x_1 and x_2.

(b) Re-writing the Hamiltonian in terms of the center of mass and relative variables $R = \frac{1}{2}(x_1 + x_2)$ and $r = (x_1 - x_2)$, show that the Shroedinger equation can be separated into terms depending separately on R and r.

(c) Assuming $k = C/2$, list the first five energy levels of the system in terms of C and m.

Hint: it may help to know that

$$\frac{d^2}{dx_1^2} + \frac{d^2}{dx_2^2} = 2[\frac{d^2}{d(x_1 + x_2)^2} + \frac{d^2}{d(x_1 - x_2)^2}]$$

Problem 4

Again using (7.3.23)-(7.3.26), show that

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}X - i\sqrt{\frac{1}{2m\omega\hbar}}P$$

acts as the raising operator, i.e., that

$$a^\dagger|n\rangle = C_n^\dagger|n+1\rangle$$

for some constant $C_n^\dagger$.

Problem 5

The potential energy of an ideal pendulum bob of mass m, suspended by a massless string of length l, is given by $V(\theta) = mgl * (1 - \cos\theta)$, where θ is the angle of the bob and string relative to the equilibrium (vertical) orientation.

In terms of the linear arc-length displacement s of the bob from its equilibrium position, write down the one-dimensional Schrödinger equation governing the motion of the bob. Expanding the expression for potential energy, rewrite this equation as the Schroedinger equation for a perturbed harmonic oscillator (keep only the leading term in the perturbation, which is one term beyond the term that produces the harmonic-oscillator potential term).

Under the assumption that the perturbation has no effect (i.e., that the pendulum behaves as a perfect harmonic oscillator), what is the ground-state energy of the pendulum?

We can get a leading-order correction to the ground-state energy by calculating the expectation value for the perturbed Hamiltonian for the unperturbed ground state wavefuction. In other words, take the expectation value of this new (first-order) Hamiltonian for the ground-state of the unperturbed Hamiltonian. What is the difference between this and the ground-state energy of the unperturbed system? Don't be afraid to use your raising and lowering operators for this step.

Problem 6

Shankar Exercise 7.4.5 Page 212. Parts (1) and (2) only.