PHYSICS 215B – HOMEWORK 5

Due at the end of the working day Wednesday, February 26.

Complementary reading: Shankar, Chapters 11 and 12.1-12.5.

Problem 1

Shankar Exercise 11.4.1, Page 300.

Problem 2

Let Θ denote the time reversal operator, with the following property, which derives from classical correspondence:

$$\Theta \vec{J} \Theta^{-1} = -\vec{J}$$

Recall also that Θ is anti-unitary, i.e., that $\langle \Theta\psi_a | \Theta\psi_b \rangle = \langle \psi_b | \psi_a \rangle$ and $\Theta c|\psi\rangle = c^* \Theta|\psi\rangle$, where c is any complex number.

Prove that $\Theta|j,m\rangle = e^{i\delta}|j,-m\rangle$ for some phase angle δ.

Problem 3

Show that $[L_x, L_y] = i\hbar L_z$, that $[L^2, L_x] = 0$, and that $[L_z, L_\pm] = \pm\hbar L_\pm$.

Problem 4

Shankar Exercises 12.3.3 and 12.3.4, Page 315.

Problem 5

Shankar Exercise 12.5.3, Page 329.

Problem 6

For j = 1, write down the 3x3 matrices J_+ and J_-. Combine these to find J_y, and show that $(J_y/\hbar)^{2n+1} = J_y/\hbar$; $(J_y/\hbar)^{2n} = (J_y/\hbar)^2$. Exponentiate to show that

$$d^{(1)}_{0,1} = -d^{(1)}_{1,0} = -d^{(1)}_{0,-1} = d^{(1)}_{-1,0} = (1/\sqrt{2})\sin(\theta)$$

$$d^{(1)}_{1,1} = d^{(1)}_{-1,-1} = 1/2(1+\cos(\theta))$$

$$d^{(1)}_{-1,1} = d^{(1)}_{1,-1} = 1/2(1-\cos(\theta))$$

$$d^{(1)}_{0,0} = \cos(\theta)$$

Problem 7

A W^- boson decays into an electron and a neutrino. We don't need to know what these things are, other than to say that the W^- boson has $J = 1$ while the electron and neutrino each have $J = 1/2$. Also, when the W decays into the electron and the neutrino, the neutrino's angular momentum is directed along its direction of motion, while the electron's angular momentum is directed against its direction of motion.

Assuming that the W^- is polarized in the +z direction (i.e., that its angular momentum is directed along the z axis), use the result of the preceding problem to predict $P(\theta)$, the electron angular distribution – the relative probability of finding the electron flying off at an angle θ relative to the z axis in the rest frame of the W^-. Don't forget that probabilities are given by the square of the wavefunction, and not by the wavefunction itself.