PHYSICS 215B – HOMEWORK 6

Due Wednesday March 12 at the end of the day.

Complementary reading: Shankar, Chapters 12.6 and 13.

Problem 1

Consider a plane wave representing a state of total momenet $\vec{p} = \hbar\vec{k}$, with

$$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}.$$

a) Write down the wavefunction of this state.

b) Working in cartesian coordinates, demonstrate that the Laplacian is the appropriate second derivate to use for the three-dimensional Schrödinger equation.

Problem 2

Shankar Problem 12.5.11 page 338

Problem 3

Shankar Problem 12.6.8 page 349

Problem 4

Shankar Problem 13.1.1 page 357

Problem 5

Shankar Problem 13.1.3 page 357

Problem 6

A Hydrogen atom is in an eigenstate of the z projection of its orbital angular momentum, with $m_l = +1$. The atom is also in a pure eigenstate of radial excitation, i.e., it has a well-defined radial quantum number n. The atom, however, is not in an eigenstate of total orbital angular momentum; instead, its wavefunction is an even mixture of two different eigenfunctions of total orbital angular momentum.

a) What is the lowest value of n that this atom's wavefunction can have?

b) For this smallest allowable value of n, determine the expectation value of the magnitude of the total orbital angular momentum for this wavefunction.

c) Under the same assumption as for a) and b), calculate the probability of finding the atom's electron within a cone of angle $\frac{\pi}{3}$ of the polar axis, for any value of r or ϕ.

Problem 7

[Thanks to Howie Haber for this problem]

One might conculde from the lack of angular momentum in the ground state of the hydrogen atom that the electron is stationary. To show this is not the case, calculate the probability that the electron's momentum, if measured, would be found to lie in a momentum element d^3p centered at momentum $\vec{p}$. What are the electron's mean kinetic and potential energies?

Some possible hints: It may help (which can be done without loss of generatlity) to consider $\vec{p}$ to be in the $\hat{z}$ direction. In addition the following integral (which you might be able to evaluate on your own), and its derivates with respect to the parameter a, may be of help:

$$\int_0^\infty e^{-ay}\sin(my)dy = \frac{m}{(a^2+m^2)^2}$$