PHYSICS 215B – HOMEWORK 7

Due at the end of the day Wednesday March 19.

Complementary reading: Shankar, Chapter 14 and 15.

Problem 1

In class, we introduced the *minimal interaction potential*

$$V = -\frac{q}{c}\vec{v} \cdot \vec{A} + q\phi.$$

Using the Euler-Lagrange equations, show that this is equivalent to the Lorentz Force Law.

Problem 2

Shankar, Exercise 14.3.2, page 384. Just do the case of the positive eigenvalue.

Problem 3

In the presence of a magnetic field $\mathbf{B}$, the dynamics of an otherwise-free spin-1/2 electron is dictated by the Hamiltonian

$$H = -\mu_B \sigma \cdot \mathbf{B}$$

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli spin matrices. Assume that for all times $t < 0$ the magnetic field is given by $\mathbf{B} = (0, 0, B_z)$ and the spin of the electron under consideration is oriented in the direction of the magnetic field.

a) Use the time-depedent Schrödinger Eqaution to demonstrate that the $t < 0$ wave function for the electron's spin state, in terms of the basis $\binom{1}{0}, \binom{0}{1}$ of the eigenstates of the $t < 0$ Hamiltonian, can be written as

$$\psi(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\omega t}$$

with $\omega = -\mu_B B_z/\hbar$.

At $t = 0$ an additional field component is introduced, leading to an overall magnetic field vector of $\mathbf{B} = (B_x, 0, B_z)$, and to a precession of the spin vector with an angular frequency Ω.

(b) Write down the explicit form of the Hamiltonian for $t > 0$.

(c) Determine the precession frequency Ω in terms of B_x, B_z, μ_B and fundamental constants.

(d) For certain times, the probability of finding the electron with its spin oriented in the $-\hat{z}$ direction will be maximal. In terms of the same quantities as for (c), what is this maximal probablity?

Problem 4

Shankar, Exercise 15.1.2, page 407.

Problem 5

Shankar, Exercise 15.2.1, page 412; part (1) only.

Problem 6

Shankar, Exercise 15.2.2, page 413. Get the relative signs right, but don't worry about the absolute signs (don't worry about sign conventions).

Problem 7

Shankar, Exercise 15.2.5, page 415.