PHYSICS 216 WINTER 2018 – MIDTERM 1

Due in class Tuesday May 15, 2018. Open book, open notes, but everything else "closed". You are to work on your own of course.

Please take the exam in one continuous 2 and 1/2 hour block. You may stop the clock for bathroom breaks...

Problem 1 [25 pts]

An object of mass m moves under the influence of a potential of the form

$$V(x) = \frac{1}{2}m\omega^2 x^2 + \frac{\beta m^2 \omega^3}{\hbar} x^4,$$

where β is much less than one. To leading order in PT, calculate the fractional shift in ground-state energy caused by the x^4 term in $V(x)$.

Problem 2 [25 pts]

An object of mass m and energy $E = p^2/2m$ moves in one dimension in the direction of positive x. At the point $x = -a$, the particle's path is divided into two, with an equal probability that the particle takes either path. The paths recombine at $x = +a$. For one path, the potential is 0 the entire way. For the other path, the potential rises linearly from 0 at $x = -a$ to V_0 at $x = 0$ and then falls linearly back to 0 at $x = +a$. The potential is also 0 for $x < -a$ and $x > +a$. Given E, p and m, and the value $V_0 = 0.1E$, for what minimum value of a will the particle have 0 probability of making it to values of x greater than $+a$?

Problem 3 [25 pts]

An object of mass m moves in an infinite square well extending between $x = 0$ and $x = a$. In the midst of this square well is an additional potential of the form

$$V^{(1)}(x) = \frac{\pi^2\hbar^2}{4ma}\delta(x - \frac{a}{2}).$$

Using the variational principle, estimate the ground state energy of this system. To make the calculation more tractable, you may work with an unnormalized trial wavefunction, although you'll want to make use of the fact that the infinite square well eigenstates carry a normalization factor $\sqrt{\frac{2}{a}}$.

Problem 4 [25 pts]

At $t = -\infty$, an object of mass m is in motion in the ground state of a harmonic osciallator with a Hamiltonian given by

$$H^{(0)} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

The object is perturbed by a potential of the form

$$H^{(1)} = \beta\hbar\omega e^{2\omega t}(a + a^\dagger),$$

with β much less than one. At leading order, what is the probability of finding the object in the first excited state of the oscillator at $t = 0$?