PHYSICS 216 WINTER 2018 – HOMEWORK 1

Due in class Tuesday April 16, 2018.

Complementary reading: Sakurai and Napolitano 5.1-2 and 5.4, Shankar Chapter 17, Baym Chapter 11

Note: For this and all subsequent homework assignments, problems from Shankar can be found at
http://scipp.ucsc.edu/∼schumm/ph216/Shankar_homework.pdf

The main course website is
http://scipp.ucsc.edu/∼schumm/ph216/

## Problem 1

As we all know, the potential energy of an ideal pendulum bob of mass $m$, suspended by a massless string of length $l$, is given by $V(\theta) = mgl*(1-\cos\theta)$, where $\theta$ is the angle of the bob and string relative to the equilibrium (vertical) orientation.

In terms of the linear arc-length displacement $s$ of the bob from its equilibrium position, write down the one-dimensional Schroedinger equation governing the motion of the bob. Expanding the expression for potential energy, rewrite this equation as the Schroedinger equation for a perturbed harmonic oscillator (keep only the leading term in the perturbation).

Under the assumption that the perturbation has no effect (i.e., that the pendulum behaves as a perfect harmonic oscillator), what is the ground- state energy of the pendulum? What is the difference between this and the true ground-state energy of the system, to leading order in perturbation theory?

## Problem 2

Consider a hydrogen atom compoased of a hypothetical spinless electron and proton, so that fine and hyperfine effects are not in play. As we know, there is a four-fold degenerate transition between the ground and first orbitally-excited state, which has a transition energy of

$$E_{L_\alpha} = \frac{3}{8}(m_e c^2)\alpha.$$

However, if this atom is placed in a constant, spatially uniform electric field, the degeneracy is broken. Calculate the resulting energy of the four transitions to leading order in the electric field strength $\varepsilon$. Express the resulting eigenstates of the perturbed ground and first-orbitally-excited states in terms of the unperturbed hydrogenic egienstates $\psi_{nlm}$.

## Problem 3

An electron is contrstained to move in *two* dimensions under a potential

$$V(r) = -\frac{ke^2}{r},$$

where $r$ is the distance from the center of motion (origin) and $k$ is the electrostastic force constant. Using the variation principle with the three-dimensional Hydrogen ground-state wavefunction

$$\psi(r, \theta, \phi) \propto e^{-r/a_0}$$

as your trial-function inspiration, estimate the ground-state energy of this 'two-dimensional hydrogen atom'. Express your answer in electron-Volts, making use of the fact that

$$\frac{m_e k^2 e^4}{2\hbar^2} = 13.6\text{eV}.$$

Can you explain the difference you see between your answer and that for the three-dimensional Hydrogen atom? In doing so, it may help to note that the Bohr radius is given by

$$a_0 = \frac{\hbar^2}{ke^2 m_e}.$$

## Problem 4

Consider a particle in motion in an attractive central potential $V(r)$ that increases monotonically from $r = 0$ to $r = \infty$ such that $V(\infty) = 0$. Show that the quantum condition for bound $S$-wave ($l = 0$) states is given by

$$2\int_0^a \sqrt{2\mu(E - V)}dr = (N + \frac{3}{4})h,$$

where $a$ is the classical turning point.

HINT: This is a 3D problem; you'll want to work in spherical coordinates.

## Problem 5

Shankar 16.2.5

**Problem 6**

Shankar 16.2.7