PHYSICS 216 WINTER 2018 – HOMEWORK 3

Due in class Tuesday May 15, 2018.

Complementary reading: Shankar Chapter 10 and 14, Baym Chapters 14, 18 and 20 (p460-468)

Problem 1

Shankar, Exercise 14.3.2 (1) and (2), page 384, but just do the case of the positive eigenvalue. This is the problem that starts "Show that the eigenvectors of $\sigma \cdot \hat{\mathbf{n}}$ are given by..."

Problem 2

In the presence of a magnetic field $\mathbf{B}$, the dynamics of an otherwise-free spin-1/2 electron is dictated by the Hamiltonian

$$H = -\mu_B \sigma \cdot \mathbf{B}$$

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli spin matrices. Assume that for all times $t < 0$ the magnetic field is given by $\mathbf{B} = (0, 0, B_z)$ and the spin of the electron under consideration is oriented in the direction of the magnetic field.

a) Use the time-dependent Schrödinger Eqaution to demonstrate that the $t < 0$ wave function for the electron's spin state, in terms of the basis $\binom{1}{0}, \binom{0}{1}$ of the eigenstates of the $t < 0$ Hamiltonian, can be written as

$$\psi(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\omega t}$$

with $\omega = -\mu_B B_z/\hbar$.

At $t = 0$ an additional field component is introduced, leading to an overall magnetic field vector of $\mathbf{B} = (B_x, 0, B_z)$, and to a precession of the spin vector with an angular frequency Ω.

(b) Write down the explicit form of the Hamiltonian for $t > 0$.

(c) Determine the precession frequency Ω in terms of B_x, B_z, μ_B and fundamental constants.

(d) For certain times, the probability of finding the electron with its spin oriented in the $-\hat{z}$ direction will be maximal. In terms of the same quantities as for (c), what is this maximal probablity?

Problem 3

Consider a universe in which the spin-orbit coupling is the only perturbation to the coulomb potential for a hydrogenic atom in a field-free region of space.

In class, we presented the $2S$ and $2P$ spin-orbit energy shifts. Scaling from these, determine the $3S$, $3P$ and $3D$ spin-orbit energy shifts in terms of the fine structure scale factor $\lambda = 1.45 \times 10^{-3}$ eV. Hint: You'll need to calculate the expecation values of the r^{-3} operator (why)? For this, pp 356-357 ('The Wave Functions') in Shankar is very helpful.

Hydrogen atoms in a plasma are excited into their second excited (n=3) state, whence they decay back into their ground state with the emission of a photon. Calculate the energy and degenaracy of these transitions in terms of the electron charge, the Bohr radius a_0, and the fine structure scale factor λ.

Which of these transitions are 'second order', i.e., violate selection rules for electromagnetic decays?

Problem 4

Consider the same system as that of the previous problem. A uniform magnetic field of strength B is now switched on in the region of the plasma. Again calculate the energy and degeneracy of the transition(s) back to the ground state, the former in terms of the 'Bohr magneton' $\mu_B = q/2m_e c$ in addition to the above-mentioned constants and the magentic field strength B.

For this case, consider only the 'first order' decays, i.e., decays that satisfy selection rules for electromagnetic dipole decays.

Problem 5

Two *identical* spin-0 objects of mass m, moving in a common potential of the form $U = \frac{1}{2}Cx^2$, are coupled through a potential $U_c(x_1, x_2) = \frac{1}{2}k(x_1 - x_2)^2$. Here x_1 and x_2 are the coordinates of particles 1 and 2, respectively, and C and k are greater than 0.

(a) Write down the full time-independent Schrödinger Equation for the system in terms of the individual coordinates x_1 and x_2.

(b) Re-writing the Hamiltonian in terms of the center of mass and relative variables $R = \frac{1}{2}(x_1 + x_2)$ and $r = (x_1 - x_2)$, show that the Shroedinger equation can be separated into terms depending separately on R and r.

(c) Assuming $k = C/2$, list the first five energy levels of the system in terms of C and m. Remember that the two particles are identical bosons.

Hints: it may help to know that

$$\frac{d^2}{dx_1^2} + \frac{d^2}{dx_2^2} = 2[\frac{d^2}{d(x_1 + x_2)^2} + \frac{d^2}{d(x_1 - x_2)^2}]$$

and that the harmonic oscillator wavefunctions alternate between even and odd, with the ground-state wavefunction even.