PHYSICS 216 – HOMEWORK 4

Due in class Tuesday May 29, 2018.

Complementary reading: Baym Chapter 19; Sakurai and Napolitano Section 7.5

Problem 1 (Courtesy Edward Groth, Princeton University)

Consider two particles of identical mass m in motion in a two-dimensional rectangular box of sides $a < b$. In each of the four cases below, assume that the system of two particles is in the ground state. For each case, determine the ground state energy and normalized wavefunction. For each case, also determine the leading-order correction to the energy if there is an interaction between the particles of the form

$$V = a \cdot b \cdot V_0 \delta^{(2)}(\mathbf{r}_1 - \mathbf{r}_2)$$

(what sort of an interaction is this?). [Note: this is really a warm-up problem that doesn't necessarily involve second quantization.]
(a) Two distinguishable particles
(b) Two identical spin-0 particles
(c) Two identical spin-1/2 particles in the singlet state
(d) Two identical spin-1/2 particles in the triplet state. Looking at your results for the leading-order correction to the energy arising from the mutual interaction in this final case, how might you have predicted this before doing the calculation? Of all the results in this problem, this is the one that ties most directly into the area of second quantization for multi-particle systems.

Problem 2 (Baym Problem 19.1)

Construct explicit 4x4 matrices to represent the fermion creation and annihilation operators a_0, $a_0^\dagger$, $a_0 1$ and $a_1^\dagger$ for a two-level system. Check the following anticommutation relations: $\{a_0, a_0^\dagger\}$, $\{a_1, a_1^\dagger\}$, $\{a_0, a_1\}$ and $\{a_0, a_1^\dagger\}$.

Problem 3

In class, we defined the number operator for quanta in a box in two ways. One way was a simple sum over the occupancy in each plane wave state:

$$N = \sum_{\vec{p},\vec{\lambda}} a^{\dagger}_{\vec{p},\vec{\lambda}} a_{\vec{p},\vec{\lambda}}.$$

The other was via an integral over the density operator:

$$N = \int d^3r \sum_{\vec{\lambda}} \psi^{\dagger}_{\lambda}(\vec{r})\psi_{\lambda}(\vec{r}).$$

Show that these two expressions are equivalent.

Problem 4

Calculate the one-particle density matrix $G_s(x - x')$ for a $T = 0$ gas of fermions of density ρ in a one-dimensional box of length L. What is your interpretation of the meaning of this function?

Problem 5 (Baym Problem 19.4)

Two electrons are in plane wave states in a box. Using the formalism of second quantization, calculate the energy difference of parallel and antiparallel spin alignments to first order in the Coulomb interaction. This energy difference is often referred to as the "exchange energy."