PHYSICS 216 – HOMEWORK 5

Due in my office by the end of the day Monday June 11, 2018.

Complementary reading: Sakurai and Napolitano, Sections 6.1-6.4 and 6.6-6.8; Shankar Chapter 19; Baym Chapter 9

Problem 1

Show that the Green's function

$$G(\vec{r}) = -\frac{m}{2\pi\hbar^2}\frac{e^{ikr}}{r}$$

satisfies SE for point-source scattering

$$(\frac{\hbar^2}{2m}\nabla^2 + E)G(\vec{r}) = \delta(\vec{r}).$$

Problem 2

SCATTERING FROM A YUKAWA POTENTIAL

An object of mass m and wave-number k scatters elastically from a central potential of the form

$$V(r) = \frac{V_0}{r}e^{-\lambda r}.$$

Calculate the magnitude q of the momentum transferred to the particle by the scattering potential as the particle undergoes a scattering through an angle θ.

To leading order in the Born expansion, calculate the cms differential scattering cross section

$$\frac{d\sigma}{d\Omega}(\theta) = |f(\theta)|^2$$

for the elastic scattering of the object from the potential.

Problem 3

Shankar, Problem 19.5.3. This problem begins: (1) Show that $\sigma_0 \to 4\pi r^2$... Note that a "hard sphere" is one for which the potential is infinite everywhere inside some radius r_0, and 0 elsewhere.

Problem 4 (Courtesy of Howie Haber)

Consider the case of low-energy scattering from a spherical delta-function shell,

$$V(\vec{r}) = V_0 \delta(r - a).$$

Under the assumption that $ka \ll 1$ (so that only s-wave scattering is important), calculate the scattering amplitude $f(\theta)$, the differential cross section $d\sigma/d\theta$, and the total cross section.

HINT: Solve the time-independent Schrödinger squation exactly, in the case of $l = 0$, for the radial wavefunction $R(r) \equiv u(r)/r$. Consider the cases of $r < a$ and $r > a$ separately. By integrating the Schrödinger equation from $r = a - \epsilon$ to $r = a + \epsilon$, for infinitessimal ϵ, show that

$$\frac{du}{dr}|_{a+\epsilon} - \frac{du}{dr}|_{a-\epsilon} = \frac{2mV_0}{\hbar^2} u(a).$$

Inserting your espressions for $u(r)$ into the equation above, determine the s-wave phase shift. In particular, find an expression for $\tan\delta_0$ in terms of V_0 and the wave number k. Evaluate the phase shift in the limit $ka \ll 1$ to simplify the expression, which should then allow you to complete the problem.

Problem 5

A colliding beam experiment of counter-circulating equal-energy beams involves red and green particles which, except for their color, are identical in every other way. Each beam pulse consists of N particles evenly distributed in a cylinder of radius a and length L. Beam pulses pass through one another at a fixed spatial location at a frequency of f times per second. The amplitude for the scattering of two of the particles off one another is dominated by the first two terms of the partial wave expansion, with

$$\frac{1}{k} T_0 = \sqrt{\sigma_0}$$

$$\frac{1}{k}T_1 = e^{i\delta}\frac{1}{3}\sqrt{\sigma_0}$$

where k is the magnitude of the relative wave number of the plane waves describing each of the counter-circulating beams. A small square detector of area A is mounted R meters away from the collision point, at a angle θ_d from the beam direction.

Note: I just realized that Shankar does use the nomenclature T_l; my T_l is Shankar's $e^{i\delta_l}\sin\delta_l$.

a) If the particles are bosons, calculate the rate of particles scattered into the detector (number of particles per second) under the assumption that one beam is entirely red and the other entirely green.

b) Calculate the rate of bosons scattered into the detector under the assumption that each beam is exactly half red and half green.

c) If the particles are spin-half fermions rather than bosons, calculate the rate of particles scattered into the detector under the assumption that one beam is entirely red and the other entirely green. Assume the beams are unpolarized.

d) If the particles are spin-half fermions, calculate the rate of particles scattered into the detector under the assumption that each beam is exactly half red and half green. Again assume the beams are unpolarized.

e) Calculate the total cross section for each of the following four cases: distinguishable bosons, indistinguishable bosons, distinguishable fermioins, unpolarized indistinguishable fermions.