

Photon Attenuation

Three modes, all catastrophic

(a) Photoelectric effect

Photon absorbed by orbiting e^- , recoil energy taken up by nucleus. More recoil energy \Rightarrow less likely to occur (Nuclear atomic form factor) \Rightarrow "edges" at exact ionization energies

Both the PE cross section and K shell energies are quickly rising func. of Z (Z^5 and Z^2 , respectively), but even for the heaviest elements, PE effect does not dominate above 500 keV or so.

(b) Compton scattering



Calculable in QED, under generally valid assumption that the atomic electrons are free ($E_{\text{trans}} \gg E_{\text{binding}}$)

$$\left(\frac{h\nu}{m_0 c^2}\right)^2 = \left[\frac{h\nu'}{m_0 c^2}\right]^2$$

$$\gamma = \frac{h\nu}{m_0 c^2}$$

$$s = \frac{T}{h\nu}$$

$$\frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_0 c^2 \gamma^2} \left[2 + \frac{s^2}{\gamma^2 (1-s)^2} + \frac{s}{1-s} \left(s - \frac{2}{\gamma} \right) \right]$$

For very low T , cuts off since momentum transfer too small to kick electrons out of atom (atom neutral). Otherwise σ_T falls slowly as particle becomes relativistic (γ^2 in denom).
~~Then, Compton picks up as PE~~

Contributions to Photon Cross Section in Carbon and Lead

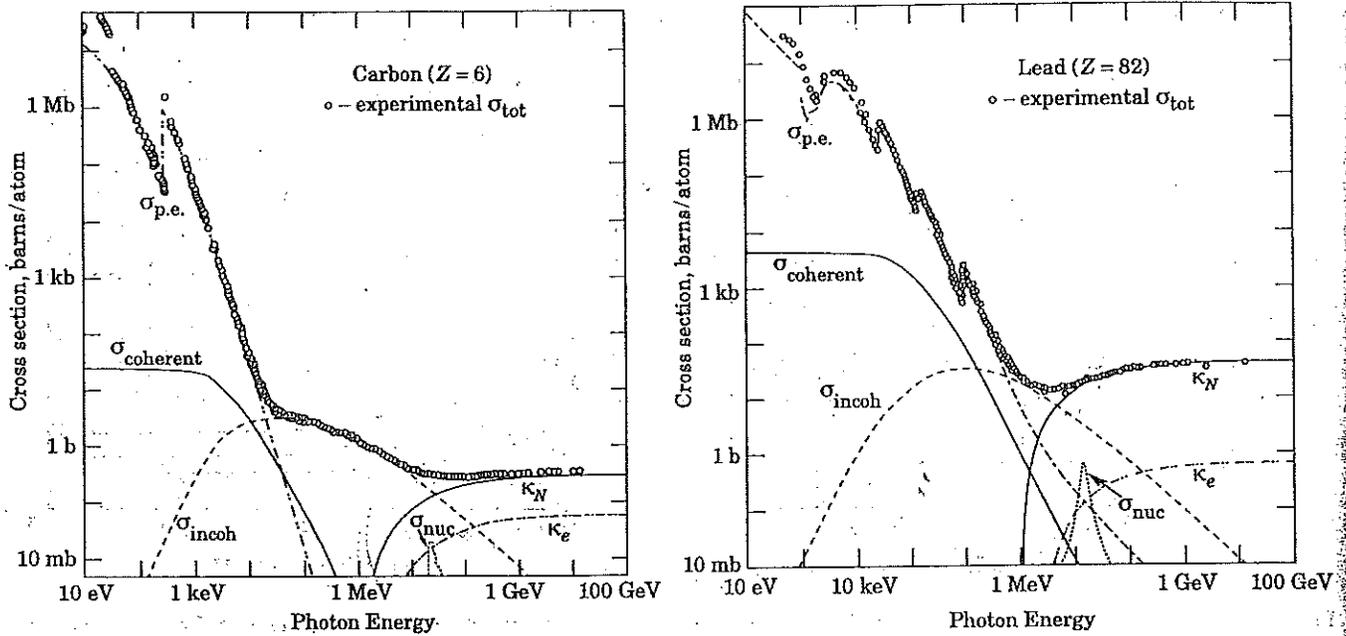


Figure 11.3: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes.

- $\sigma_{p.e.}$ = Atomic photo-effect (electron ejection, photon absorption)
- $\sigma_{coherent}$ = Coherent scattering (Rayleigh scattering—atom neither ionized nor excited)
- $\sigma_{incoherent}$ = Incoherent scattering (Compton scattering off an electron)
- κ_n = Pair production, nuclear field
- κ_e = Pair production, electron field
- σ_{nuc} = Photonuclear absorption (nuclear absorption, usually followed by emission of a neutron or other particle)

From Hubbell, Gimm, and Øverbø, *J. Phys. Chem. Ref. Data* 9, 1023 (80). The photon total cross section is assumed approximately flat for at least two decades beyond the energy range shown. Figures courtesy J.H. Hubbell.

Fractional Energy Loss for Electrons and Positrons in Lead

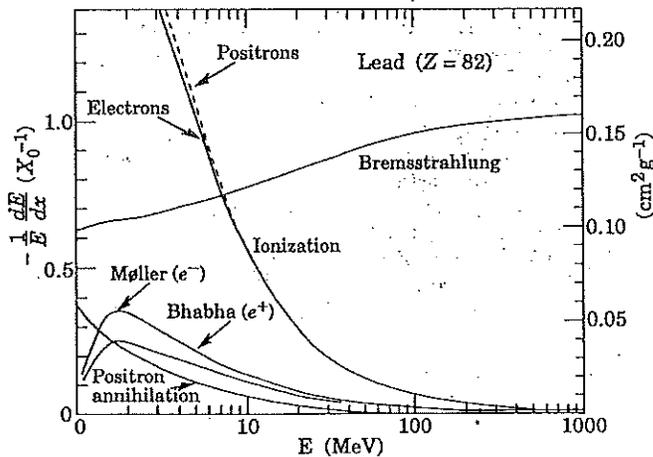
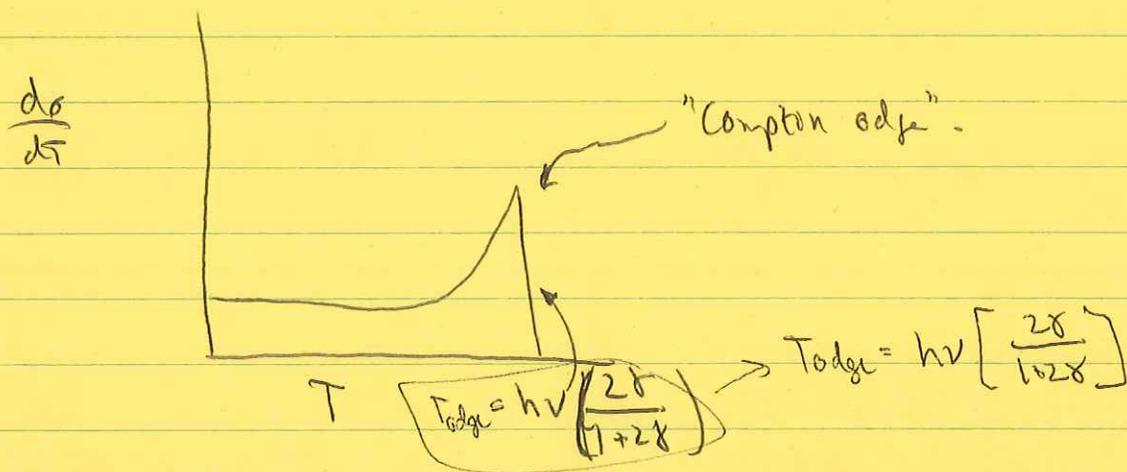


Figure 11.4: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Moller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\text{Pb}) = 5.82 \text{ g/cm}^2$, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials, namely $X_0(\text{Pb}) = 6.4 \text{ g/cm}^2$. The development of electron-photon cascades is approximately independent of absorber when the results are expressed in terms of inverse radiation lengths (i.e., scale on left of plot).

As a func of T , this looks roughly like



which is the energy deposited observed for a thin detector
 For ^{137}Cs . (For Compton-dominant energy photons), ^{137}Cs are
 in which the scattered photon always escapes.

Compton total cross-section: For very low T , cuts off
 since momentum transfer too small to distinguish electrons
 from the rest of the atom (so atom looks neutral). Otherwise,
 σ_c fall as as incident photon energy increases ($1/x^2$).

Thus, Compton picks up as PE falls off (100 keV - 2 MeV),
 but then dies away again, to be overtaken by pair production
 (2-20 MeV).

Especially, for heavy materials, there tends to be a tough
 where Compton is falling + pair production is rising \Rightarrow ~ few
 MeV photons can be relatively penetrating!!

PDG III.22 + explain

(carbon, low photon
 atom. h ν)

R7D 2b

Note: L_{rad} is electron radiation length

A

<u>Material</u>	<u>$L_{rad}(cm)$</u>	<u>$L_{rad}(gm/cm^2)$</u>	<u>$\lambda_I(cm)$</u>	<u>$\lambda_I(g/cm^2)$</u>
Be	35.3	65.2	40.7	75.2
Al	8.9	24.0	39.4	106.4
Si	9.4	21.8	45.5	106.0
Fe	1.8	13.8	16.8	131.9
W	0.35	6.8	9.6	185
Pb	0.56	6.4	17.1	194
U	0.32	6.0	10.5	199

shielding

↓
↓
↓
↓
↓

↓

10

4

100

- MCS
- \hat{C} radiation
- North Mod.
- Secondary emission?

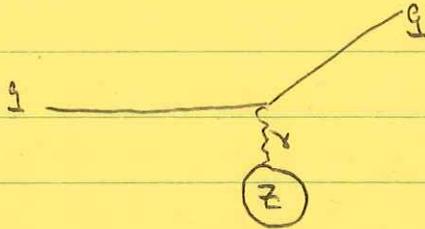
Mass:

Include en showers of EM calorimetry $\Rightarrow E_c \Rightarrow$ # prop to E_{in} ; det. in \hat{C} rad, and interaction. Total absorption cal. vs. sampling. micro radius, shower size. Tower sharing? Typical materials & resolutions. Constant term. EGS4.

* Problem: show uniform dist has $\sigma = \sqrt{N}$

Multiple Coulomb Scattering

Consider a charged particle travelling through a medium. In addition to ionization, which for the most part does not deflect the particles (e^- 's being an exception), the particle also suffers coherent nuclear scatters:



These do lead to minute deflection of the particles. Typically, in a macroscopic material thickness, the particle will undergo numerous deflections, leading to a macroscopic deflection angular & transverse deflection which must be treated statistically.

Since there are many scatters, the central limit theorem suggest a gaussian treatment is adequate. However, for large scatters, ~~for the energy loss distribution, the Rutherford cross section~~

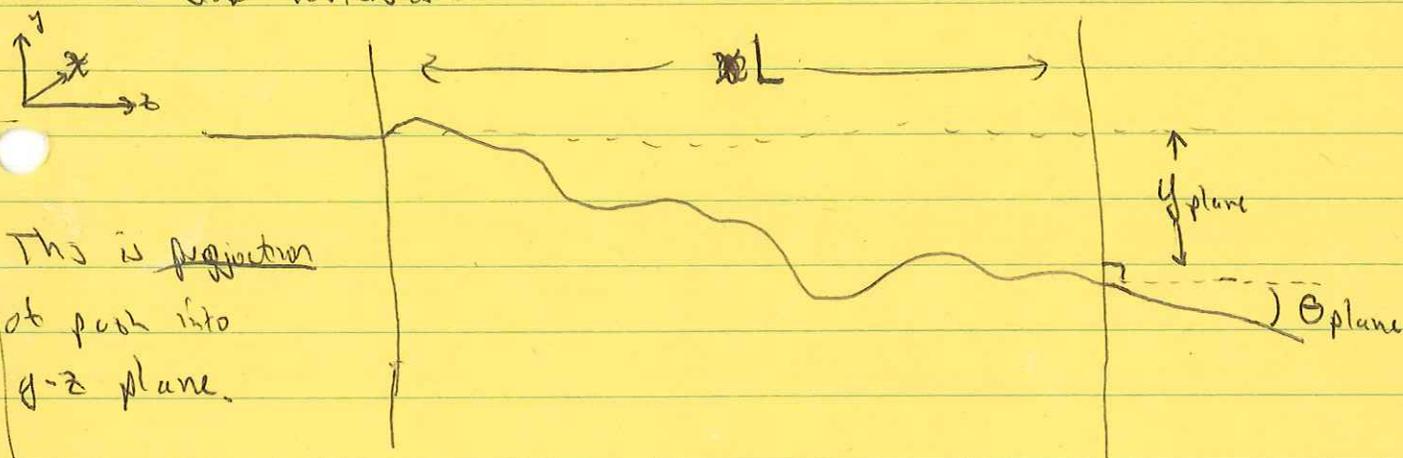
$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4(\theta/2)} \quad (\text{Rutherford section})$$

\Rightarrow non-gaussian tails!

has an infinite variance, and Gaussian scattering is not obtained even in the limit $N \rightarrow \infty$. Thus, one typically uses the Gaussian approximation for detector design, but a more rigorous "Molière" treatment for accurate analysis

of physics done: e.g., MCS by the material in precision particle trackers. In general, these models are complex, and packaged in standard MC's. Thus, for our purposes, will confine ourselves to the Gaussian treatment, which is ~~appropriate~~ always appropriate for a qualitative understanding, and sometimes quantitative results (track-fitting errors!)

The particle's deflection introduces a transverse walk in its path through the material. We define ~~two~~ x and y variables:



Note that CS is similar to pair production, ^{and bremsstrahlung} in that they all three are ~~both~~ coherent nuclear processes, and thus ~~so~~ should have the same material property dependence. Thus, MCS is yet another process characterized by LRAD!

What typically is done to provide a Gaussian approximation to get σ_{MCS}

$$\text{Al} \Rightarrow L_{\text{RAD}} = 24 \text{ g/cm}^2$$

$$\text{Au} \Rightarrow L_{\text{RAD}} = 6.5 \text{ g/cm}^2 \quad [\text{Au has } 0.34 \text{ } 0.6? \text{ } L_{\text{RAD}}]$$

Lect begin by considering the variable Θ_{plane} . What is typically done in order to provide a gaussian approximation to MCS is to set

$$\sigma_{\Theta_{\text{plane}}}^2 = \langle \Theta_{\text{plane}}^2 \rangle_{98}$$

where $\langle \Theta_{\text{plane}}^2 \rangle_{98}$ is the ^{mean square} RMS of the central 98% of the MOLIÈRE distribution. A ^{basic formula} formula ~~good to~~ ~~state~~ ~~no~~ vary soon anymore: is

$$\sigma_{\Theta}^{\text{plane}} = \frac{14.5 \text{ MeV}}{\beta p c} \approx \sqrt{\frac{k}{L_{\text{RAD}}}}$$

where the \sqrt{k} dependence is just due to the random walk characteristic of the transverse path. Note that

- (1) σ increases as the $\sqrt{\text{path length}}$
- (2) σ decreases linearly w/ momentum

A slightly more complicated formula, ^{to better than 10% for} good for $10^{-3} < L/L_{\text{RAD}} < 100$, is now recommended: (106)

$$\sigma_{\Theta}^{\text{plane}} = \frac{13.6 \text{ MeV}}{\beta c p} \approx \sqrt{\frac{k}{L_{\text{RAD}}}} \left[1 + 0.038 \ln \left(\frac{L}{L_{\text{RAD}}} \right) \right]$$

↑
multiple charge

⊗ Prob: Show dist $\exp(-z_1 \sigma_y) \exp(-z_2 \sqrt{1-s^2})$
 $\exp\left(-\frac{\theta^2}{\sigma_\theta^2}\right) \exp\left(-\frac{y^2}{\sigma_y^2} (1-s^2)\right) \exp\left(-s^2 \frac{\theta y}{\sigma_\theta \sigma_y}\right)$
 has right σ + covariance

Similarly, the rms displacement $\sigma_{y\theta}$ is found to be

$$\sigma_{y\theta} = \frac{L}{\sqrt{3}} \sigma_\theta^{\text{plane}} \quad (\text{here, } L \text{ is in cm, not } g/cm^2)$$

This is clearly correlated with σ_θ for a given path, y and θ^{plane} are clearly correlated ($y > 0$ implies $\theta^{\text{most likely}} > 0$). This is expressed in terms of the correlation coefficient

$$S_{y\theta} = \frac{\langle y \theta \rangle}{\sigma_y \sigma_\theta^{\text{plane}}} = \frac{\sqrt{2}}{2}$$

⊗ Prob: show $\frac{1}{2\sigma_\theta \sigma_y} \exp\left\{-\frac{1}{2\sigma_\theta \sigma_y}\right\}$
 $\left[\frac{\theta^2}{\sigma_\theta^2} - \frac{2s\theta y}{\sigma_\theta \sigma_y} + \frac{y^2}{\sigma_y^2}\right]$ has proper
 $\langle \theta \rangle, \langle y \rangle, \text{ and } \langle \theta y \rangle$

In practice, this means that if y, θ, z_1, z_2 are two independent Gaussian distributed variables, then

$$\theta = z_2 \sigma_\theta^{\text{plane}}$$

$$y = z_1 L \sigma_\theta^{\text{plane}} \sqrt{1-s^2} / \sqrt{3} + z_2 S_{y\theta} L \sigma_\theta^{\text{plane}} / \sqrt{3}$$

are appropriately distributed variables.

Finally, the two separate planar projections are independent, yielding

$$\sigma_\theta^{\text{space}} = \sqrt{2} \sigma_\theta^{\text{plane}}$$

$$\sigma_y = \sqrt{2} \sigma_{y\theta}$$

} r, θ are 3-d deviation from straight line path

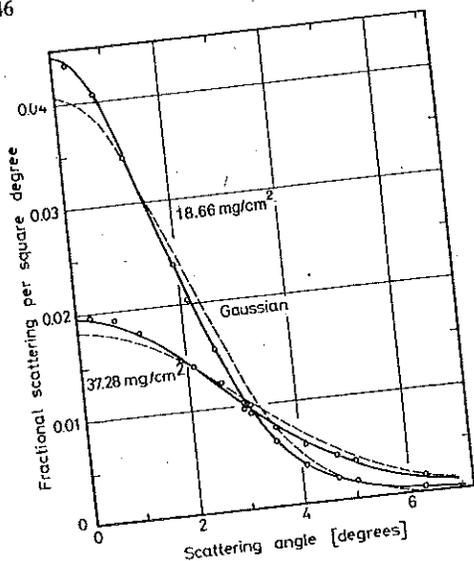


Fig. 2.15. Angular distribution of 15.7 MeV electrons scattered from a thin Au foil (from Hanson et al. [2.23]). The experimental values are compared with the Gaussian approximation to multiple scattering

δx : thickness of scatterer [cm]
 ρ : density of scatterer [g/cm^3]
 p : momentum of incident particle [MeV/c]
 $\beta = v/c$ of incident particle
 z : charge of particle in units of e

$$Q = \begin{cases} \sqrt{Z(Z+1)} & \text{for electron and positrons} \\ Z & \text{for other particles} \end{cases}$$

$$\bar{q} = \begin{cases} (Z+1)Z^{1/3} & \text{for electrons and positrons} \\ Z^{4/3} & \text{for other particles.} \end{cases}$$

For most calculations, it is usually not necessary to go beyond the first three terms. Figure 2.15 shows an example of this distribution for 15 MeV electrons passing through a thin gold foil. At small angles, this space angle distribution (with respect to solid angle!) is close to that of a Gaussian, but as angle increases, corrective terms come into play to form a long broad tail. The deflections at larger angles are generally due to one single, large angle Coulomb scattering in the material rather than to the cumulative effect of many small angle scatterings. The broad tail, therefore, should roughly follow that of the Rutherford $1/\sin^4(\theta/2)$ form for single scattering rather than that of a Gaussian. The transition between the small and larger angle regions is governed by plural scattering. This is given by Moliere as a correction to the small angle distribution.

2.5.1 Multiple Scattering in the Gaussian Approximation

If we ignore the small probability of large-angle single scattering, a good idea of the effect of multiple scattering in a given material can be obtained by considering the distribution resulting from the small angle ($< 10^\circ$) single scatterings *only*. In such a case, as we have seen, the probability distribution is approximately Gaussian in form,

$$P(\theta) \approx \frac{2\theta}{\langle \theta^2 \rangle} \exp\left(\frac{-\theta^2}{\langle \theta^2 \rangle}\right) d\theta \quad (2.88)$$

2.5 Multiple Coulomb

where we have use represents the mean from $\theta = 0$ to ∞ . should be equal to tion. By comparin; should be approx distribution's long

A better estim Lynch and Dahl [

$$\langle \theta^2 \rangle = 2 \frac{\chi_c^2}{1 + F^2}$$

where

$$v = 0.5 \frac{\Omega}{(1 - F)}$$

$$\Omega = \chi_c^2 / \chi_a^2$$

$$\chi_c^2 = 0.157 z \left(\right)$$

$$\chi_a^2 = 2.007 \times 1$$

The variable p is of the particle, z and α , the fine Moliere distribut since, as we have value for $\langle \theta^2 \rangle$. S scatters. For F formula yields r

As an illustr distributions sh

$$\langle \theta^2 \rangle = \begin{cases} 0.00 \\ 0.00 \end{cases}$$

By comparing that $\sigma = 1.94^\circ$; we can see a ge

A sometim dicular plane co also approxim

$$P(\theta_x) d\theta_x =$$

Supplement : Error Propagation

○ A word on error propagation:

Correlated Errors

Consider two ~~number~~ estimators $x_1 \pm \sigma$ and $x_2 \pm \sigma$

$$\bar{x} = \frac{x_1 + x_2}{2} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{2}}$$

Is average better on mean.

But what if x_1, x_2 are known with differing

precision?

$$x_1 \rightarrow x_1 \pm \sigma_1$$

$$x_2 \rightarrow x_2 \pm \sigma_2$$

Let

$$S_{1(2)} = \sigma_{1(2)}^2$$

$$W_{1(2)} = \frac{1}{S_{1(2)}} = \frac{1}{\sigma_{1(2)}^2}$$

save

Then,
$$\bar{x} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$$

"Error-weighted average"

$$\bar{x} = \frac{\frac{x_1}{\sigma_1^2} + \frac{x_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\sigma_{\bar{x}} = \left(\frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} \right)^{1/2} = \sqrt{\frac{1}{w_1 + w_2}}$$

↑ from considering convolution of 2 gaussian dist's.

(54)

For example, $\sigma_1 = \sigma_2 = \sigma$

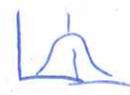
$$\bar{x} = \frac{\frac{x_1}{\sigma^2} + \frac{x_2}{\sigma^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma^2}} = \frac{\frac{1}{\sigma^2} (x_1 + x_2)}{\frac{1}{\sigma^2} (1+1)} = \frac{x_1 + x_2}{2} \quad \checkmark$$

$$\sigma_{\bar{x}} = \left(\frac{1}{\frac{1}{\sigma^2} + \frac{1}{\sigma^2}} \right)^{1/2} = \left(\frac{\sigma^2}{2} \right)^{1/2} = \frac{\sigma}{\sqrt{2}} \quad \checkmark$$

Note also that if $\sigma_1^2 < \sigma_2^2$ then mean is closer to x_1 than x_2 \checkmark

$$\sigma_{ij} = \int \int (x_i - \bar{x}_i)(x_j - \bar{x}_j) P(x_1, \dots, x_n) dx_1 \dots dx_n$$

But now, what if instead of measuring a single observable x , we are measuring multiple observables $\vec{x} = (a, b, c, d, \dots)$

[Example: fitting a resonance  for $a = \text{mass}$ $b = \text{width}$]

Then, we measure a vector \vec{x} of observables, with an error matrix

$$\sigma_{ij}^{(1)} = \int \int (x_i - \bar{x}_i)(x_j - \bar{x}_j) P(x_1, \dots, x_n) dx_1 \dots dx_n$$

$$S = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \dots \\ \sigma_{21} & \sigma_{22} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$S = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & \dots \\ \sigma_{12} & \sigma_{22}^2 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$\sigma_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$
 $\sigma_1 = \sqrt{\sigma_{11}}$ $\sigma_2 = \sqrt{\sigma_{22}}$
 $-1 \leq S_{12} \leq +1$

Correl. coeff. (x_1, \dots, x_n) of observables

but determined experimentally!

Let's consider 2-d case

$$\text{SVD } \vec{x}_1 = (a_1, b_1)$$

$$\vec{S}_1 = \begin{pmatrix} \sigma_{a_1}^2 & \rho_{ab,1} \sigma_{a_1} \sigma_{b_1} \\ \rho_{ab,1} \sigma_{a_1} \sigma_{b_1} & \sigma_{b_1}^2 \end{pmatrix}$$

where ρ_{ab}

$$\vec{x}_2 = (a_2, b_2)$$

$$\vec{S}_2 = \begin{pmatrix} \sigma_{a_2}^2 & \rho_{ab,2} \sigma_{a_2} \sigma_{b_2} \\ \rho_{ab,2} \sigma_{a_2} \sigma_{b_2} & \sigma_{b_2}^2 \end{pmatrix}$$

where ρ_{ab} is $-1 \leq \rho_{ab} \leq 1$ is the

correlation coefficient, \vec{S} is the error matrix.

Define a weight matrix

$$\vec{W} = (\vec{S})^{-1}$$

that is just the inverse of the error matrix.

why? Let's consider error propagation

1-d case

$$\bar{x} = \frac{\frac{x_1}{\sigma_1^2} + \frac{x_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} = \frac{\frac{x_1}{s_1} + \frac{x_2}{s_2}}{\frac{1}{s_1} + \frac{1}{s_2}} = \frac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$$

where, recall, $w = \frac{1}{s}$ is "weight", i.e., weighted average!

n-d case

$\langle \vec{x} \rangle$ (\vec{x})

$$\langle \vec{x} \rangle = \frac{\vec{w}_1 x_1 + \vec{w}_2 x_2}{\vec{w}_1 + \vec{w}_2}$$

vector

with $\vec{w}_{tot} = \vec{w}_1 + \vec{w}_2$, i.e.,

$(\vec{w}_1 + \vec{w}_2)^{-1} \cdot (\vec{w}_1 x_1 + \vec{w}_2 x_2)$
Matrix \cdot vector

$$S_{\langle \vec{x} \rangle} = (\vec{w}_{tot})^{-1} = (\vec{w}_1 + \vec{w}_2)^{-1}$$

↳ error on mean

⊗ **Computer project** !!

Give \vec{x}_1, \vec{s}_1 and $\langle \vec{x} \rangle, S_{\langle \vec{x} \rangle}$ and ask to find \vec{x}_2, \vec{s}_2 and from S_2 each $\sigma_{a,2}, \sigma_{b,2}, \rho_{ab,2}$

Generalized propagation of errors.

Problem

$$\vec{x}_1 = (4.646, 0.439) \quad \sigma_1^a = 0.047 \quad \sigma_1^b = 0.042 \quad \rho_1^{ab} = -0.402$$

$$\langle \vec{x} \rangle = (4.591, 0.454) \quad \sigma_{\langle x \rangle}^a = 0.031 \quad \sigma_{\langle x \rangle}^b = 0.038 \quad \rho_{\langle x \rangle}^{ab} = -0.405$$

where $\langle \vec{x} \rangle$ is statistical average of \vec{x}_1 and \vec{x}_2

find \vec{x}_2 and $\sigma_2^a, \sigma_2^b, \rho_2^{ab}$

↙	↓	↓	↓	↓
4.547	0.481	.044	.094	-0.514

Cerenkov Radiation

What happens when a particle goes faster than the speed of light in a medium.

$$c_m = \frac{c_0}{n}$$

$n = 1.33$ for H_2O

$$\left[\begin{aligned} v > \frac{c_0}{n} &\Rightarrow \beta c_0 > \frac{c_0}{n} \\ &\Rightarrow \beta > \frac{1}{n} \end{aligned} \right]$$

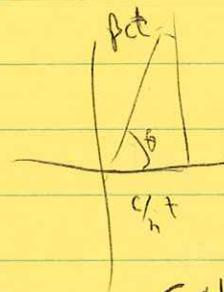
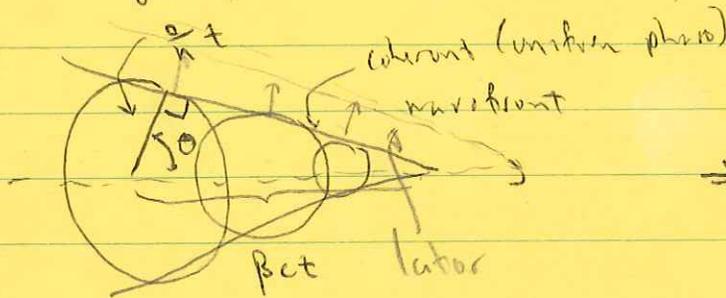
For electrons, for example $\beta > \frac{c_m}{c_0} = \frac{1}{n}$ for $E_e = 1 \text{ MeV}$ or so

As for ships going faster than water waves generate a "bow wave", and plane supersonic planes generate a shock sonic boom, particles going faster than c_m will generate a coherent wavefront at a special angle, known as θ_c radiation.

$$\beta_{\text{min}} = \frac{1}{n}$$

minimum ~~value~~ β for θ_c radiation.

What is the angle at which the maximum is emitted?



(Threshold crossing)

$$\cos \theta_c = \frac{c/n t}{\beta c t} = \frac{1}{\beta n}$$

particle ID from angular dependence! $\left\{ \begin{array}{l} \text{above } n \text{ threshold} \\ \text{angular dependence (rig in rig)} \end{array} \right.$

momentum range w/ n
 $n=1 \Rightarrow \begin{cases} \sim 5 \text{ liquids/glasses } \theta_c \text{ large} \\ 10^{-3} \text{ gasses } \theta_c \sim 50 \text{ mrad} \end{cases}$



A phototube has a QE of 10% for $200\text{nm} < \lambda < 550\text{nm}$. Assume all \hat{e} light from a $\beta \approx 1$ particle is collected onto the phototube, has long wires & hydrogens \hat{e} counts to measure 99% efficiency for counting that particle? What are the $\beta \approx 1$ models for this counter (assume STP)

Calculation of the intensity & spectrum of \hat{e} radiation is a classical problem (cf Jackson, Ch 14), yielding

$$\frac{dE}{d\omega dx} = z^2 \frac{4\pi}{c} \omega \sin^2 \theta_c \left[1 - \frac{1}{\beta^2 n^2(\omega)} \right] = z^2 \frac{4\pi}{c} \omega \sin^2 \theta_c$$

no need

When we must keep in mind that θ_c is a function of ω since is general so is n . More relevant for HEP applications is the # of photons emitted, gotten by dividing through by $\hbar\omega$:

$$\frac{dN}{d\omega dx} = \frac{z^2 q}{\beta} \frac{4\pi}{c} \sin^2 \theta_c \rightarrow \text{Plot is frequency (energy) space}$$

or in terms of wavelength

$$\lambda = \frac{2\pi c}{\omega}$$

quanta number

$$\frac{dN}{d\lambda dx} = \frac{2\pi z^2 q}{\lambda^2} \sin^2 \theta_c$$

For range $\sim 190\text{nm} < \lambda < 900\text{nm}$
 $\frac{dN}{d\lambda} \sim 5$ photons/cm is gas
 $(n \sim 10^{-3})$

NOTE:
 $\sin \theta_c$
 slowly
 varies
 w/ β
 for $\beta \gg 1$

When we have used the Jacobian $d\omega = 2\pi c d\left(\frac{1}{\lambda}\right) = -\frac{2\pi c}{\lambda^2} d\lambda$

When is typically presented at this point is the integral from $350\text{nm} < \lambda < 550\text{nm}$, etc

Note that the peaks for short wavelength, and in fact is divergent until cut off by the sensitivity of the detecting device. In the case of the eye, this cutoff is blue/purple, and hence, we conclude that \hat{e} light is the source of the "orange" "Boris blue glow" associated w/ intense radioactive sources

Increased in H_2O .

What is typically done is to ~~probe~~ present the intensity integrated over the range $350\text{nm} < \lambda < 550\text{nm}$, giving the yield

$$\frac{dN}{dx} = 475 Z^2 \sin^2 \theta_e \text{ photons/cm.} \quad (\lambda \text{ is incident charge)}$$

For a typical gas radiator, $n \approx 1 + 10^{-3}$, yielding for $\beta \rightarrow 1$

$$\theta_e = \arccos\left(\frac{1}{n}\right) \approx 45 \text{ mrad} \sim 50 \text{ mrad}$$

$$\text{so so } \frac{dN}{dx} \approx 1/\text{cm}$$

In fact, fused silica or quartz window phototubes go down below 200nm, and you are ~~usually~~ ^{often} limited by the gas transparency, which ~~is associated with the~~ ~~transparency~~ in the range 200-250nm - but note that $n(\lambda)$ changes greatly in the region of an absorption peak! Thus, 58/cm ~~is~~ a more typical yield - but then there's the QE price (~20% is good).

Neutron Moderation

For slow neutrons ($KF < \sim 100\text{meV}$), inelastic scattering is not kinematically allowed, and so energy loss is via elastic scattering. However, $n \rightarrow \gamma$ scattering is very limited in energy transfer to Z unless $Z \rightarrow 1$. Thus, neutron moderators are low Z - typically, hydrocarbons w/ a lot of H nuclei \Rightarrow paraffin is popular!

Transition Radiation

Occurs at transition between different media, related to plasma frequency ω of material (related to n)...

Total energy:

$$E \approx \left(\frac{2}{3}\right) \alpha \delta \text{ [thru plasma]}$$

← typically ~ 70 eV

} from $n \approx 1$
to $n > 1$
transition I'd guess.

Spectrum:

Peaks about ~ 20 keV

$\Rightarrow \delta > 1,000$ for appreciable effect

Good way to discriminate electrons from hadrons in the 1 GeV - 100 GeV range (ideal for LHC)