$3 \otimes \bar{3}$ multiplets, nonetrics & mixing
u, d, s & charges - cultural note about non-observation
excited state mesons

Pseudoscalar mesons & the eightfold way

The decays $\pi^0 \to \gamma\gamma$ $\quad K \to \mu\nu$ etc
$\pi^+ \to \mu\nu$

suggest that the SU mesons contain no net "hadron-ness", which seems to be a globally conserved quantity (p v stable to all forces). Yet its interactions are hadron-like (strong), so we conjecture that the mesons are stable compositions of equal amounts of hadron matter & hadronic anti-matter. Thus, the simplest way to form this is to combine the two associated (3-d) fundtl representations, which we denote

$$3 \otimes \bar{3} \to \text{mesons?}$$

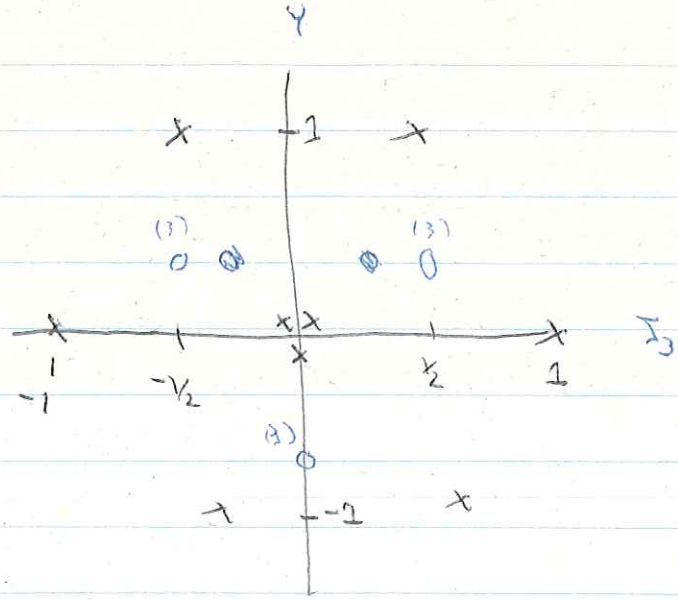
Let's try it!

By analogy w/ SU(2), to get a list of states in product representation (we won't worry about associated Clebsch-Gordon coefficients), we superimpose the $\bar{3}$ representation at each point of the 3 representation

(Do something clever w/ a xerox of graph paper here)

$3 \otimes \bar{3}$

weight diagram



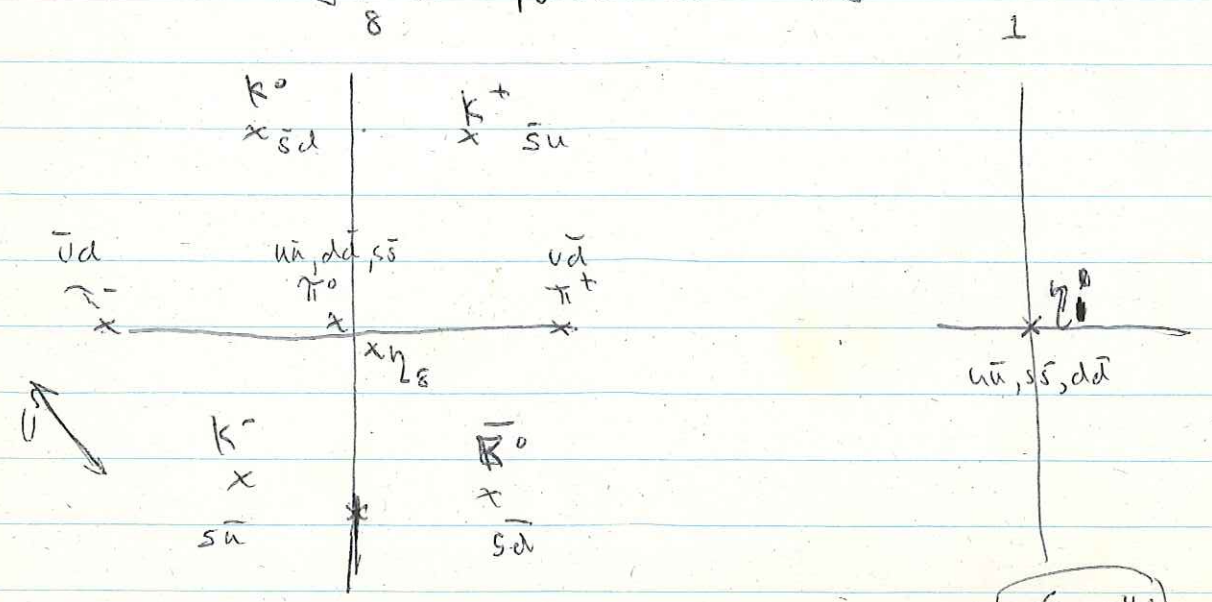
Note that there are 3 states at the origin, but only 2 can belong to the irreducible rep. containing the proplery (~~[illegible]~~ $|I, I_3\rangle = |1,0\rangle$ and $|0,0\rangle$)

Thus, this must be a sum of two irred. reps, of dimension 8 and 1

"eightfold way"

$$3 \otimes \bar{3} = 8 \oplus 1$$

and we assign our pseudoscalar mesons



now that $K^+ = \bar{s}u$ has $s=+1$?

from which we see that for mesons $(3 \otimes \bar{3})$

$$\boxed{\begin{aligned} Y &= s \\ Q &= I_3 + Y/2 \end{aligned}}$$

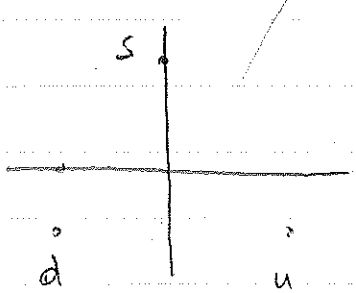
(strangeness will be modified shortly) (but $s=-1$, $\bar{s} \pm +1$ (K^+ convention))

charge operator – "Gell-Mann–Nishijima relation"

NOTE – at this point, what you'd like to do is say that the mesons have substructure, & are composed of constituents, called "quarks" with the following property

	I	I_3	s	
u (up)	$\frac{1}{2}$	$+\frac{1}{2}$	0	"isospin up"
d (down)	$\frac{1}{2}$	$-\frac{1}{2}$	0	"isospin down"
s (strange)	0	0	-1	if net strangeness is $\neq 0$, we get a strange particle

But in 1964, no one had ever observed fractionally charged particles, so are those really particles? Hmm – wait until SLAC scattering expts @ end of decade (next quarter).



Fundamental Rep Eigenvectors

$\begin{pmatrix}1\\0\\0\end{pmatrix}$ = up $\quad \begin{pmatrix}0\\1\\0\end{pmatrix}$ = down $\quad \begin{pmatrix}0\\0\\1\end{pmatrix}$ = strange

On our original diagram, we've written explicit quark wavefunc. for all but π^0, η_8, η_1. Those were easily worked out

$$\eta_1 = \frac{1}{\sqrt{3}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right)$$

can't show preference for u, d, s if symmetry is exact (which it is by assumption)

funny sign for antisymmetry in SU(2)

$$\pi^0 = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \qquad \text{Since } I=1 \quad I_3=0$$

$$\eta_8 = \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\right) \qquad \text{by orthonormality}$$

In real life, η_1, η_8 mix through an undetermined parameter Θ_{PS} (PS for "pseudoscalar) to form physical η, η' (more on this later). [SU(3) is broken when s quarks are included]

<u>High Mass Resonances</u>

If we posit that quarks are fundamental spin ½ fermions, then the "spin" of a composite state should be determined by the quark spin alignment and relative angular momentum

$$\vec{J}_{meson} = \vec{L}_{quark} + \vec{S}_{quarks}$$

and we should have quark multiplets corresponding to all L, S combos

$^{2s+1}L_J$

NOTE: '*' seems to indicate s=1 for K's

	L	S	J^P		
$^1S_0$	0	0 (anti-aligned)	0^-	pseudoscalar	K's, π's, η's
$^3S_1$	0	1 (aligned)	1^-	vector	K*'s, ρ's, ω, φ
$^3P_2$	1	1 (aligned)	2^+	tensor	K_2^*'s, a_2's, f_2's
switch these!					
$^3P_0$	1	1 (aligned)	0^+	scalar	K_0^*'s, a_0's, f_0's
$^1P_1$	1	0	1^+	axial vector	K's, a_1's, f_1's

So, this is 9 × 5 = 45 meson states. Some higher L (2,3) have also been discovered... But they get pretty heavy

m_{π^0} = 135 MeV $\qquad$ m_{a_1} = 1260

m_{ρ^0} = 770 MeV $\qquad$ m_{a_2} = 1320

$m_{a_0^0}$ = 984 MeV

do wavefunctions p 48

OCTET / SINGLET MESON MIXING IN SU(3)

We have only got time for a rough outline of this topic; for a complete description, see e.g. Burcham + Jobes, 337-340. On the other hand, it's important to go through this in order to understand the origin + behavior of the physical $I = Y = 0$ mesons, e.g. the observed η_8, η_0 mixtures for the various J^P multiplets.

~~We observe~~ For perfect SU(3) symmetry, all masses are equal in any multiplet. We do observe (up to small E+M effects) that ~~I multiplets in~~ I submultiplets in SU(3) multiplets have equal mass $\Rightarrow$ SU(3) is broken in Y. Now, it can be easily verified that
(But we don't have a Y raising + lowering operator, however,)

$$Y = U_3 + \tfrac{1}{2} Q$$

So now breaking up in terms of U_3 and Y!

and since that Q is constant w/in any U-spin multiplet. Thus, break out the symmetry-breaking terms in hamiltonian

$$H_{TOT} = H_0 + H_V + H_S$$

"U-vector", depends on U_3

"U-scalar" depends on U

← Think H atom: 200 MeV splitting on J_z energies on $\vec{J}$.

and so

$$\langle U, U_3 | H_0 + H_v + H_s | U, U_3 \rangle = m_0^2 + m_v^2 U_3 + m_s^2 U$$

where the use of m_i^2 rather than m_i is a subtlety having to do w/ the fact that mesons (bosonic) satisfy the Klein-Gordon eqn, in which mass appears quadratically.

Now, apply this to the $U=1$ triplet in the meson octet. By applying U_- to $K^0 = |\bar{s} d\rangle$ it is easily shown that

back of previous page.

$$|U=1, U_3=0\rangle = \frac{1}{2} |\sqrt{3}\, \eta_8^0 - \pi^0\rangle$$

allowing us to write down the relations based on U_3, Q assigned

$$m_{K^0}^2 = \langle K^0 | H | K^0 \rangle = m_0^2 + m_s^2 + m_v^2$$

$|U, U_3\rangle$

$|K^0\rangle = |1, 1\rangle$

$\Rightarrow m_v^2 = 0$ and so

$m_K^2 = m_0^2 + m_s^2 = \langle 1,0|H|1,0\rangle$

$$m_{\bar{K}^0}^2 = \langle \bar{K}^0 | H | \bar{K}^0 \rangle = m_0^2 + m_s^2 - m_v^2$$

$|\bar{K}^0\rangle = |1, -1\rangle$

$|U, U_3\rangle$

$m_{K^0}^2$

"$m_0^2 + m_s^2 = \langle 1,0|H|1,0\rangle =$

this are U, U_3!, i.e. $|1,0\rangle = \frac{1}{2}|\sqrt{3}\,\eta^0 - \pi^0\rangle$

$$\frac{1}{4}\langle \pi^0 | H | \pi^0 \rangle + \frac{3}{4}\langle \eta_8^0 | H | \eta_8^0 \rangle = \frac{1}{4} m_{\pi^0}^2 + \frac{3}{4} m_{\eta_8^0}^2$$

(ept) $m_v^2 = 0$, and $m_K^2 = m_0^2 + m_s^2 = \frac{1}{4} m_{\pi^0}^2 + \frac{3}{4} m_{\eta_8^0}^2$

[OK - H does not mix π_0, η_8^0 since isospin is good symmetry]

$\hookrightarrow \pi^0, \eta$'s have different Quantum Nos - don't mix!]

Since $m_{K^0} = m_{\bar{K}^0}$, we can combine this to get

$$\boxed{m_{\eta_8^0}^2 = \frac{1}{3}\left(4 m_{K^0}^2 - m_{\pi^0}^2\right)} = (570\,\text{MeV})^2$$

498 MeV 135 MeV

$m_\eta = 550$: close but not quite...

Meson octet "Gell-Mann - Okubo" mass formula

$\boxed{\text{QM 55}}$

548 MeV 958 MeV

→ neither η nor η' has this mass! What's going on? η_1, η_8 can mix!!

Now, this formula is not realized since the η_8^0 mixes w/ the η_1^0 (at different mass even for good SU(3)) to get the physical η, η' (here no ~~refer to generic~~ ~~representations~~). do pseudoscalar η's, but could do vector ω, ϕ, etc. can mix...

→ all QNs except SU(3) same, but SU(3) is broken, so

$$\eta' = \eta_1^0 \cos\theta + \eta_8^0 \sin\theta$$

(A)

$$\eta = -\eta_1^0 \sin\theta + \eta_8^0 \cos\theta$$

Since the mass eigenstates are the η, η', then

$$H\begin{pmatrix}\eta' \\ \eta\end{pmatrix} = \begin{pmatrix} m_{\eta'}^2 & 0 \\ 0 & m_\eta^2 \end{pmatrix}\begin{pmatrix}\eta' \\ \eta\end{pmatrix}$$

and so the SU(3) eigenstate mass matrix is not diagonal

$$H\begin{pmatrix}\eta_1^0 \\ \eta_8^0\end{pmatrix} = \begin{pmatrix} m_{11}^2 & m_{18}^2 \\ m_{18}^2 & m_{88}^2 \end{pmatrix}\begin{pmatrix}\eta_1^0 \\ \eta_8^0\end{pmatrix} = \begin{pmatrix} m_{\eta_1^0}^2 & m_{18}^2 \\ m_{18}^2 & m_{\eta_8^0}^2 \end{pmatrix}\begin{pmatrix}\eta_1^0 \\ \eta_8^0\end{pmatrix}$$

with the Gell-Mann Okubo constraint $m_{88}^2 = m_{\eta_8^0}^2 = \frac{1}{3}\left(4m_{K^0}^2 - m_{\pi^0}^2\right)$

We can diagonalize w/ matrix $R = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$ s.t. $\begin{pmatrix} m_{\eta'}^2 & 0 \\ 0 & m_\eta^2 \end{pmatrix} = R^\dagger \begin{pmatrix} m_{11}^2 & m_{18}^2 \\ m_{18}^2 & m_{88}^2 \end{pmatrix} R$

Upon diagonalization we find an expression for the mixing angle in terms of observables → and eliminating m_{11}, m_{18}

$$\boxed{\tan^2\theta = \frac{m_{\eta_8}^2 - m_\eta^2}{m_{\eta'}^2 - m_{\eta_8}^2}}$$ → w/ $m_{\eta_8}^2 = \frac{1}{3}\left(4m_{K^0}^2 - m_{\pi^0}^2\right)$

Plugging in masses,

Input masses	J^P multiplet	θ	Physical mesons
$\begin{pmatrix} K^0 & 494 \\ \pi^0 & 135 \end{pmatrix}$	0^- (pseudoscalar)	-11°	η, η'
$\begin{pmatrix} (892) \\ K^* \\ \rho \\ (770) \end{pmatrix}$ K^{*0}, ρ^0 → used in mass constraint	1^- (vector)	40°	ω, ϕ
$\begin{pmatrix} K_2^*(1430) \\ a_2(1320) \end{pmatrix}$ K_2^{*0}, a_2^0 → used in mass constraint	2^+ (tensor)	32°	f_2, f_2'

So what? Perhaps θ is just a non-physical parameter, and this is all just a parametrization of ignorance. But no - recall that

$$\eta_1^0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$\eta_8^0 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

Plugging in the mixing relation θ, it turns out that for $\sin\theta = 1/\sqrt{3}$, or $\theta = 35^\circ$

ω ~~ϕ~~ $q' = u\bar{u} + d\bar{d}$

ϕ ~~ω~~ $q = s\bar{s}$

$\cancel{\omega = \frac{1}{\sqrt{2}}(u\bar{u}}$

$$\omega \doteq \sqrt{\tfrac{2}{3}}\,\eta_1 + \sqrt{\tfrac{1}{3}}\,\eta_8 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$\phi \doteq \sqrt{\tfrac{1}{3}}\,\eta_1 - \sqrt{\tfrac{2}{3}}\,\eta_8 = s\bar{s}$$

Thus, if the quark model is correct, ~~then~~

$\phi_{1020} \rightarrow$ mostly $s\bar{s}$

$\omega \rightarrow$ mostly $u\bar{u}$ $d\bar{d}$

[Also, $f_2' \rightarrow KK$ $(88.8 \pm 3.1)\%$]

$B(\phi \rightarrow KK) = (83.2 \pm 0.7)\%$

[$B(\omega \rightarrow \pi^+\pi^-\pi^0, \pi^0\gamma, \pi^+\pi^-) = 99.7\%$ but shows nothing since $m_\omega < 2m_K$]

~~B(ω~~ Also, the $\omega(782)$ does not have enough mass to go to KK, but its relatively short lifetime (width = 8 MeV) indicates that it's not being suppressed by a desire to!

[The ϕ is a relatively narrow resonance decay predominantly to K^+K^- - which is a nice cross-check on particle ID systems!]

$$\Gamma_\phi = 4.3 \text{ MeV}$$

Show $Y = B + S$ $\quad B(|u,d,s\rangle) = \frac{1}{3}(|u,d,s\rangle)$

BARYONS IN THE QUARK MODEL

Given the ~~3×3~~ charge $\frac{1}{3}$ / $\frac{2}{3}$ charge of the eigenvectors of the fundtl representation (quarks), and the fact that baryons have "nut hadron-ness" (all baryon decays have at least 1 baryon decay product), we conjecture that baryons will be represented by a product of 3 fundamental representations...

$3 \otimes 3 \otimes 3 \quad = \quad$ baryons?

$Q = I_3 + Y/2$

	I_3	Y	Q
u	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	$-\frac{2}{3}$	$-\frac{1}{3}$

write

$$3 \otimes 3 \otimes 3 = 3 \otimes (3 \otimes 3) = 3 \otimes (6 \oplus \bar{3}) = 3 \otimes 6 \oplus 3 \times \bar{3}$$

~~We~~ $\boxed{\text{SHOW B\&J, p 331}}$

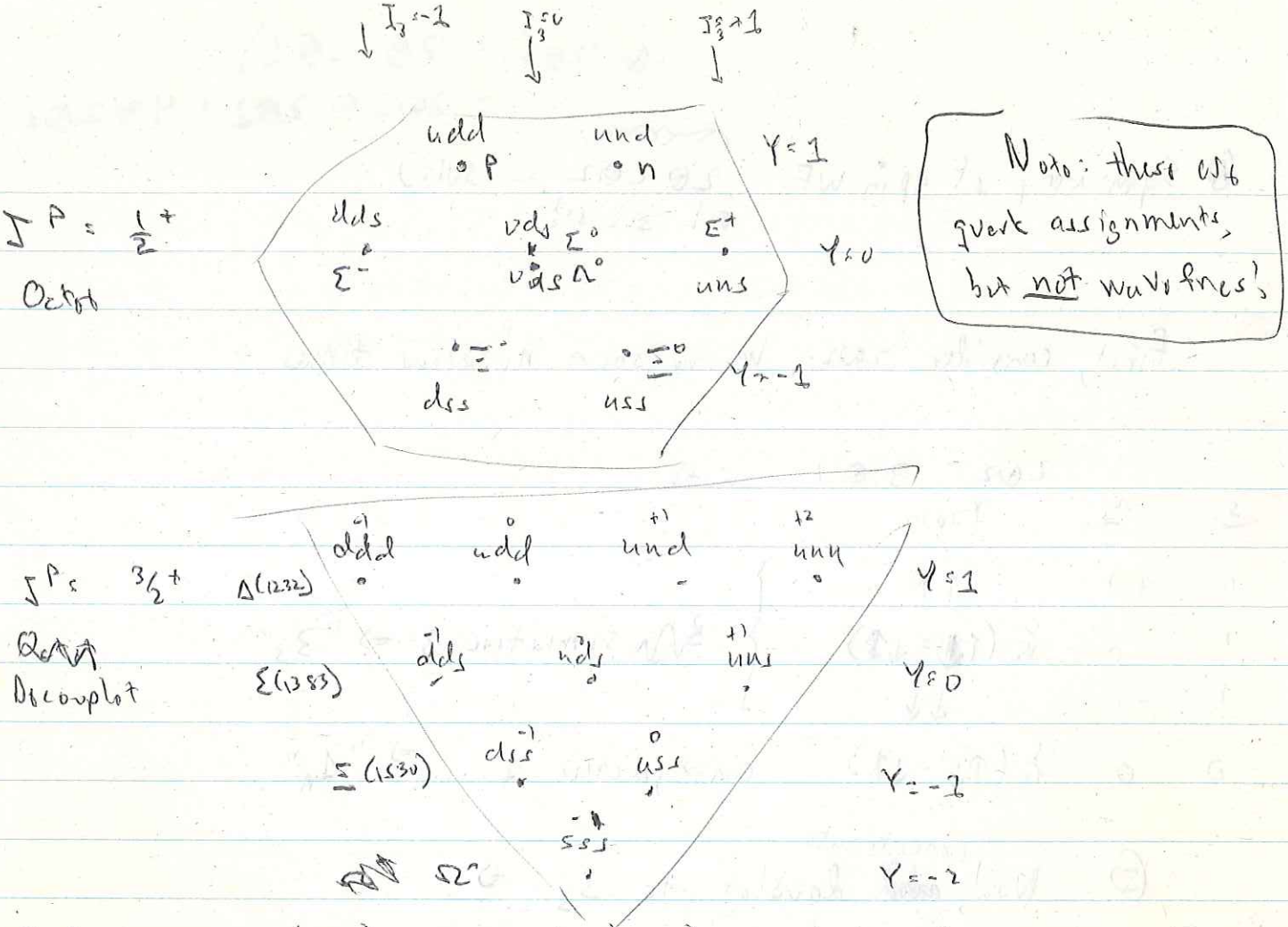
We know $3 \otimes \bar{3} = 8 \oplus 1$

he see $3 \otimes 6 = 8 \oplus 10$

so

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

At first blush - great!: These irreducible rep's incorporate our lowest lying baryons & baryonic resonances



In fact, in 1964 when this was put together, the Ω^- had not been seen yet. Its discovery within a year was perhaps the greatest success of the quark model.

But, on closer examination, we see some problems.

① $Y \neq S$

This is a small problem & easily fixed. Assign quarks a "hadronness" (baryon no.) of $\frac{1}{3}$, and set

$$\boxed{Y = S + B}$$ ← this holds true if quark level also. $B|q\rangle = \frac{1}{3}$ $B|\bar{q}\rangle = -\frac{1}{3}$

Then, these irreducible reps describe the observed spectrum, and $\boxed{\text{still } Q = I_3 + \frac{1}{2}Y}$ Also, $B = 0$ for mesons & 1 for baryons.

↑ Gell-Mann Nishijima relation

⊕ B + J ~~example~~ problem 10.4-10.7

(2) Where are the other 8, 1, 10 for $J_p = \frac{1}{2}^+$
and the other 1, 8, 8 for $J_p = \frac{3}{2}^+$?

To answer this, we must consider the symmetry of the total wave fnc. under quark exchange, which must be <u>odd</u> since baryons are fermions.

$$\overset{A}{\psi_{TOT}} = \overset{S}{\psi(\text{space})}\, \psi(\text{flavor})\, \psi(\text{spin})$$

~~By hypothesis~~ In all expectation, the lowest lying states are in a relative L=0 state, so $\psi(\text{space})$ must by symmetric. So, $\psi_f \psi_s$ must be antisymmetric (A). The symmetry of the w.f. is another good parameter of the representation, although not all rep's have consistent 3-particle symmetry

---

(B) ~~Flavor~~ Symmetry of Flavor WF - $3 \otimes 3 \otimes 3$ in SU(3)

$$3 \otimes 3 \otimes 3 = 10_S + 8_{M_S} + 8_{M_A} + 1_A \quad \left[\text{You will show!}\right]$$

where $M_S \Rightarrow$ symmetric only under exchange of first two quarks
$M_A \Rightarrow$ antisym " " " " " "

(C) SHOW THIS

(D)

~~Interesting problem:~~

~~(1) Symmetry of Spin WF $2 \otimes 2 \otimes 2$ in SU(2)~~

$$2 \otimes 2 \otimes 2 = \overset{J=3/2}{4_S} \oplus \overset{J=1/2}{2_{M_S}} \oplus \overset{J=1/2}{2_{M_A}}$$

Now, try to form $J=3/2$ multiplet...
We note that we can combine 10_S and 4_S to get a ~~symmetric flavor deco~~ $J=3/2$ flavor decouplet but not an antisymmetric one. So - things are even worse!

DECOUPLET: $(SU(3), SU(2)) = (10, 4)$ (actually, on SU(6) rep!)

~~Also, we can combine~~

Solution: add a fourth degree of freedom, $\boxed{\text{color}}$, and say all observed hadrons are color singlets. If we say there are 3 colors, then again the singlet color representation of $3 \otimes 3 \otimes 3$ is antisymmetric. So, then

$$\psi_{TOT} = \overset{S}{\psi}_{SPACE} \underbrace{\psi_{FLAV}\, \psi_{SPIN}}_{\text{AN! must be S!}} \overset{A}{\psi}_{COLOR}$$

must just state that this is totally sym

$\boxed{\text{see pg. QM66}}$

$J=3/2$ → $(10, 4)$

meaning $J=3/2$ decouplet
$J=1/2$ octet $(8, 2)$ ← $J=1/2$
$= \frac{1}{\sqrt{2}}\left[(8_{M_S}, 2_{M_S}) + (8_{M_A}, 2_{M_A})\right]$

And then everything falls into place.

WHAT IS THIS COLOR?!! This is the first hint of a new quantum no, one associated w/ the DYNAMICAL SU(3) gauge symmetry which forms the basis of

Q outer

C nrone

D ynamics

Our dynamical theory of the S.I. But this will have to wait...
In fact, it turns out to be strong charge.

$\boxed{\text{QM53}}$

No 1_A! Since no totally A spin WF!

Constituent vs Current Quark Mass

In ~~QCD~~ The mass that appears in, we ~~will write down~~ the Lagrangian, e.g. for free quarks

$$\mathcal{L} = \bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

is known as the current mass, and for u, d and our s, is light

$m_u^c \cong 4$ MeV

$m_d^c \cong 7.5$ MeV

$m_s^c \cong 150$ MeV

Clearly, though, $m_p \neq 2m_u^c + m_d^c$, etc. which also includes the energy of the inseparable gluon field associated w/ the quark, as well as binding effects. We thus define a phenomenological constituent mass as the mass associated w/ the quark when bound in a ~~hadron~~ baryon or meson (different for the two cases!)

Thus, for a baryon, you might naively say $m_u^{con} \cong m_d^{con} \cong m_p/3$. However, as we'll see, we can understand baryon masses fairly well by including a spin-spin interaction between the quarks, which we'll want to take into account when defining the constituent mass. Taking B.E. into account (see B+J discussion of baryon masses)

$M_u = 0.336$ GeV/c^2

$M_s = 0.538$ GeV/c^2

} low-energy (0-momentum transfer) limit of quark masses ⇒ appropriate for magnetic moments. From B+J, or baryon mass development just preceding...

$\boxed{\text{Qm59}}$

Baryon Magnetic Moments

For a point-like ~~spin~~ ~~the~~ spin-½ particle of charge e, relativistic QM (Dirac eqn) tells us that the magnetic moment is

$$\mu = \frac{e}{2m}$$

We thus define the nuclear magneton as

$$\mu_N = \frac{e}{2m_N}$$

where $m_N \sim m_p \sim m_n$ is the nucleon mass. NOTE that immediately, the fact that ~~μ_p~~ $\mu_p = 2.793\mu_N \neq \mu_N$ and $\mu_n = -1.913\mu_N \neq 0$ is strong evidence (in the 30's) that neutrons & protons are not pointlike. Later confirmed by Stanford / SLAC in ~~60's~~ late 50's directly.

In any regard, if we now hypothesize ~~on~~ that our quarks are point-like, having the constituent mass from before,

$$\mu_u = \frac{2}{3}\frac{e}{2m_u} = 1.863\mu_N \qquad \mu_d = -\frac{1}{3}\frac{e}{2m_d} = -0.931\,\mu_N$$

$$\mu_s = -\frac{1}{3}\frac{e}{2m_s} = -0.582\,\mu_N$$

Explicit Baryon Wavefunctions

skip?

As mentioned when deriving the baryon octet & decuplet,

$$3 \otimes 3 \otimes 3 = (\cancel{3} \oplus \cancel{6}) \; (6_S \oplus \bar{3}_A) \otimes 3 \qquad \text{(flavor)}$$

$$= 10_S \oplus 8_{M_S} \oplus 8_{M_A} \oplus 1_A$$

and

$$\cancel{2 \otimes 2 \otimes 2 = 4_S \oplus 8_M}$$

$$2 \otimes 2 \otimes 2 = (3_S \oplus 1_A) \otimes 2 = 4_S \oplus 2_{M_S} \oplus 2_{M_A}$$

where $M_{S,A}$ is symmetry of first two quarks.

Thus, to form the baryon octet, say with a totally antisymmetric wf

$$\psi_{tot} = \psi_{space} \, \psi_{spin} \, \psi_{flavor} \, \psi_{color}$$

$$A \quad = \quad S \qquad\qquad A$$

we saw that

$$\text{Octet} = \frac{1}{\sqrt{2}} \left[(8_{M_S}, 2_{M_S}) + (8_{M_A}, 2_{M_A}) \right]$$

You will show that, for example for the baryon octet uud state (proton) (there will at least make sense by inspection for now)

$$\left.\begin{aligned} |p_{m_s}\rangle &= \frac{1}{\sqrt{6}}\,|(ud+du)u - 2uud\rangle \\ |2_{m_s}\rangle &= \frac{1}{\sqrt{6}}\,|(\uparrow\downarrow+\downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow\rangle \end{aligned}\right\} (8_{m_s}, 2m_s)$$

$$\left.\begin{aligned} |p_{m_A}\rangle &= \frac{1}{\sqrt{2}}\,|(ud-du)\,u\rangle \\ |2_{m_A}\rangle &= \frac{1}{\sqrt{2}}\,|(\uparrow\downarrow-\downarrow\uparrow)\uparrow\rangle \end{aligned}\right\} (8_{m_A}, 2_{m_A})$$

Combining these, we get (you can check explicitly just by multiplying out...) [[Yes - really have to follow through carefully; dont forget $\frac{1}{\sqrt{6}}$ and $\frac{1}{\sqrt{2}}$ factor!]]

LEAVE ON BOARD

$$|p\rangle = \text{octet } uud = \frac{1}{\sqrt{2}}\left\{(8_{m_s}, 2m_s) + (8_{m_A}, 2m_A)\right\}$$

$$= \frac{1}{\sqrt{18}}\Big|\overset{(1)}{2u\uparrow u\uparrow d\downarrow} + \overset{(2)}{2d\downarrow u\uparrow u\uparrow} + 2u\uparrow d\downarrow u\uparrow$$

$$- u\downarrow u\uparrow d\uparrow - \overset{(5)}{u\uparrow u\downarrow d\uparrow} - d\uparrow u\uparrow u\downarrow$$

$$- d\uparrow u\downarrow u\uparrow - u\downarrow d\uparrow u\uparrow - u\uparrow d\uparrow u\downarrow\Big\rangle$$

A

✓ ok - not to hard. Use notation $u\uparrow u\uparrow d\downarrow \to uud$ to save time!

which, in the end, is totally symmetric, as required. Notice that the $u\uparrow u\uparrow d\downarrow$ states + permutations get a weight of 2. This will be a non-trivial statement about baryon magnetic moments.

If the quark model holds, then what does Ⓐ predict for μ_p?

$$\mu_p = \langle p \uparrow | \mu_1 + \mu_2 + \mu_3 | p \rangle$$

where μ_i is just the magnetic moment op. for the ith quark. Because of the complete symmetry of $\chi_{spin}\chi_{charge}$, we need merely consider the first quark, and then mult by 3...

just first quark!

Thus for example, in Ⓐ term j gives for μ_1

Term	Coeff	Note
1	$\frac{4}{18}\mu_u$	
2	$-\frac{4}{18}\mu_d$	negative μ_d since d is pointing down
⋮		
5	$\frac{1}{18}\mu_u$	positive μ_n since squaring
etc.		

Putting all terms together... & mult by 3

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d) = 2.79\mu_N \quad \text{Measured} = 2.793\,\mu_N$$

REMARKABLE!!