Nuclear and Particle Physics

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7 *Sub-nuclear physics: an overview*

Particle classification · The particle directory · Leptons and quarks: the fundamental particles · The fundamental interactions · Vacuum polarization · Towards a unification of the fundamental interactions

7.1 Particle classification

For the purposes of Part I of this book only a general classification of elementary particles such as the proton, electron, neutrino and photon into *fermions* and *bosons* was necessary (section 3.9). In sub-nuclear physics, however, a very large number of particles are encountered and it is useful further to subdivide these main groups according to the types of interaction in which they participate. All electrically charged particles, by virtue of their charge, can interact electromagnetically. With this caveat, some particles respond only to the weak force and such particles are collectively known as *leptons*. Among these are the familiar electron e^- and the neutrino ν_e*, and muon μ^- and its neutrino ν_μ and the more recently discovered τ lepton.[1] All leptons have intrinsic spin $\frac{1}{2}$ and are therefore fermions. Those particles which can participate in the strong interactions are known as *hadrons*. Unlike the leptons, which are all fermions, the hadron family contains both fermions and bosons. The hadrons with half-integer spin are also known as *baryons* amongst which the neutron and proton are the most familiar. The *mesons*, originally named because they had masses intermediate between the light or zero-mass leptons and the heavier baryons, are bosons.

* We shall see later that each charged lepton has a distinct neutrino associated with it which is therefore designated with the appropriate subscript ν_e or ν_μ. The τ neutrino ν_τ has not yet been observed directly.

These broad categories of particle are summarized in table 7.1.

Table 7.1
Broad classificaion of sub-nuclear particles

Fermions (half-integer spin)	Leptons (weak and electromagnetic)	
	Baryons	Hadrons (strong, weak and electromagnetic)
Bosons (integer spin)	Mesons	
	Photon (electromagnetic)	
	Z^0, $W^{\pm}$ (weak)	

All electrically charged particles can interact electromagnetically. Some hadrons decay via the weak interaction.

7.2 *The particle directory*

A list of the so-called stable particles, together with some of their properties, is given in table 7.2. By stable in this context is meant particles which are truly stable, such as the electron and its neutrino, or those which are stable against decay via the strong interactions. Those which do decay do so via the weak interaction with relatively long lifetimes, about 10^{-10} s, or via the electromagnetic interaction with much shorter lifetimes, about 10^{-16} s, such as the electromagnetic decay of the neutral pion, $\pi^0 \rightarrow \gamma\gamma$. Within the broad categories of leptons, mesons and baryons, defined in the previous section, the particles are arranged in order of increasing mass. Within each category one sees a variety of particle multiplets such as the triplet of π mesons, π^+, π^0 and π^-, with electric charge $+1$, 0 and -1 times the electron charge and roughly the same mass. There are particle doublets such as the xi or cascade particles Ξ^0 and Ξ^- and various singlets such as the neutral lambda (Λ) and the omega-minus (Ω^-) particles for which only one charge state exists. Why this is so will become apparent in due course.

One also notices in table 7.2 various adjectives like strange, charmed and bottom used in describing the hadrons. In addition to the more obvious properties such as charge, rest mass, intrinsic spin and parity, the hadrons possess these other attributes or quantum numbers, strangeness, charm and 'bottomness', in varying degrees. For the time being, let these quantum numbers merely serve as an indication of the richness of particle types in the sub-nuclear 'zoo'.

Already, on rather general grounds, this would appear to be a relatively large number of 'elementary' particles if indeed they really are elementary. In addition, to every particle there corresponds an antiparticle, although

Table 7.2
Stable particle table

Particle	Spin–parity J^P	Mass/MeV	Principal decay modes	Mean lifetime/s
Leptons				
ν_e	$J = \frac{1}{2}$	$<7.3 \times 10^{-3}$	–	Stable
e	$J = \frac{1}{2}$	0.511	–	Stable
ν_μ	$J = \frac{1}{2}$	<0.27	–	Stable
μ	$J = \frac{1}{2}$	105.66	$e\nu\bar{\nu}$	2.20×10^{-6}
ν_τ	$J = \frac{1}{2}$	<35	–	Stable
τ	$J = \frac{1}{2}$	1784.1	$\mu\nu\bar{\nu}$, $e\nu\bar{\nu}$, hadrons	3.1×10^{-13}
Non-strange mesons				
$\pi^\pm$	0^-	139.57	$\mu\nu$	2.6×10^{-8}
π^0	0^-	134.97	$\gamma\gamma$	0.83×10^{-16}
η	0^-	547.5	$\gamma\gamma$, $3\pi^0$, $\pi^+\pi^-\pi^0$	$\Gamma = 1.19 \pm 0.11$ keV
Strange mesons				
$K^\pm$	0^-	493.65	$\mu\nu$, $\pi^\pm\pi^0$, 3π	1.24×10^{-8}
$K^0\bar{K}^0$	0^-	497.67	50% K^0_S, 50% K^0_L	
K^0_S	0^-		$\pi^+\pi^-$, $\pi^0\pi^0$	0.89×10^{-10}
K^0_L	0^-		$3\pi^0$, $\pi^+\pi^-\pi^0$, $\pi^\pm\mu^\mp\nu$, $\pi^\pm e^\mp\nu$	5.17×10^{-8}
Charmed non-strange mesons				
$D^\pm$	0^-	1869.3	eX, KX, K^0X, $\bar{K}^0X$	10.7×10^{-13}
$D^0\bar{D}^0$	0^-	1864.5	eX, μX, KX, K^0X, $\bar{K}^0X$	4.2×10^{-13}
Charmed strange meson				
$F^\pm$ (now $D_s^\pm$)	0^-	1971	KX, K^0X, $\bar{K}^0X$, non-$K\bar{K}X$, eX	4.5×10^{-13}
Bottom mesons				
$B^\pm$	0^-	5279		
			DX, $D^0/\bar{D}^0X$, D^*X, FX FD, F*D, FD*, F*D*	$(12.9 \pm 0.5) \times 10^{-13}$
$B^0\bar{B}^0$	0^-	5279		
Non-strange baryons				
p	$\frac{1}{2}^+$	938.3		Stable ($>10^{32}$ a)
n	$\frac{1}{2}^+$	939.6	$pe^-\bar{\nu}$	889.1 ± 2.1
Strangeness − 1 baryons				
Λ	$\frac{1}{2}^+$	1115.6	$p\pi^-$, $n\pi^0$	2.6×10^{-10}
Σ^+	$\frac{1}{2}^+$	1189.4	$p\pi^0$, $n\pi^+$	0.8×10^{-10}
Σ^0	$\frac{1}{2}^+$	1192.6	$\Lambda\gamma$	7.4×10^{-20}
Σ^-	$\frac{1}{2}^+$	1197.4	$n\pi^-$	1.5×10^{-10}
Strangeness − 2 baryons				
Ξ^0	$\frac{1}{2}^+$	1314.9	$\Lambda\pi^0$	2.9×10^{-10}
Ξ^-	$\frac{1}{2}^+$	1321.3	$\Lambda\pi^-$	1.6×10^{-10}

(continued)

Table 7.2 (Continued)

Particle	Spin–parity J^P	Mass/MeV	Principal decay modes	Mean lifetime/s
Strangeness − 3 baryon				
Ω^-	$\frac{3}{2}^+$	1672.4	ΛK^-, $\Xi^0\pi^-$, $\Xi^-\pi^0$	0.82×10^{-10}
Charmed baryons				
Λ_c^+	$\frac{1}{2}^+$	2282.0	ΛX, $pK^-\pi^+$, pK^0	1.9×10^{-13}
$\Sigma_c(2455)$	$\frac{1}{2}^+$	2453	$\Lambda_c^+\pi$	–
Ξ_c^+	$\frac{1}{2}^+$	2466	$\Lambda K^-\pi^+\pi^+$, $\Sigma^+K^-\pi^+$ $\Sigma^0K^-\pi^+\pi^+$, $\Xi^-\pi^+\pi^+$	$\approx 3 \times 10^{-13}$
Ξ_c^0	$\frac{1}{2}^+$	2473	$\Xi^-\pi^+$, $\Xi^-\pi^+\pi^+\pi^-$, $pK^-K^*(892)^0$	$\approx 0.8 \times 10^{-13}$
Bottom baryon				
Λ_b^0	$\frac{1}{2}^+$	≈ 5641	$J/\psi(1S)\Lambda$, $pD^0\pi^-$, $\Lambda_c^+\pi^+\pi^-\pi^-$	–
Gauge bosons				
γ	1^-	0	–	Stable
$W^\pm$	1	80.22 ± 0.26 GeV	$e\nu$, $\mu\nu$, $\tau\nu$	$\Gamma = 2.12 \pm 0.11$ GeV
Z^0	1	91.173 ± 0.020 GeV	e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$ $\nu\bar{\nu}$, hadrons	$\Gamma = 2.487 \pm 0.01$ GeV
g (gluon)	1^-	0	–	Stable

Adapted from Particle Data Group 1992 'Review of particle properties', *Phys Rev* **D45** June. Sometimes the width Γ of a state is quoted instead of the lifetime: the width and lifetime are related through the uncertainty principle, $\Gamma \approx \hbar/\tau$ (see section 3.10). In the entries for the charmed and bottom mesons and the charmed baryons X stands for any particles consistent with the appropriate conservation laws. The spin-parity assignments for the charmed and bottom baryons are quark model predictions.

in some cases the particle and its antiparticle are indistinguishable. One can also state with confidence that new stable particles have yet to be discovered, so the list is not complete. One would expect that a truly elementary particle should have no internal structure, for if it had, the constituent parts would necessarily be 'more elementary' than the whole.

In addition to the list of stable particles, many other hadrons have been discovered in experiments using the high energy accelerators described in chapter 2. These hadrons have lifetimes of approximately 10^{-23} s, i.e. many orders of magnitude smaller than the stable particles, but in other respects they are similar to the stable particles. The extremely short lifetime is indicative of the fact that these states decay via the strong interaction. If the stable particles are considered as elementary then these new states, sometimes called resonances, could be placed on the same footing. One such state is the N(1520) where the number in brackets is the mass of the particle in mega-electronvolts and the particle symbol indicates that its basic properties are similar to the nucleon. The principal decay mode of this state is single pion emission, $N(1520) \rightarrow N\pi$, as shown

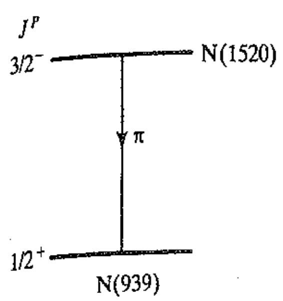


Figure 7.1
The nucleon N(939) and its first excited state N(1520).

in figure 7.1, and thus it seems more appropriate to regard the N(1520) as an excited state of the ground-state nucleon. That such an excited state exists suggests that the nucleon has some internal structure and should not be considered elementary. This viewpoint is strengthened by the parallel situation concerning the other stable baryons, the excited states of which, along with those of the nucleon, are shown in figure 7.2. Historically, the $\Delta(1232)$ was the first resonance to be discovered. It differs from the other ground states in figure 7.2 in that it decays strongly to a nucleon and a pion, $\Delta(1232) \rightarrow N\pi$, with a width of 115 MeV.* Strong decays of the other ground states are forbidden by various conservation laws. For instance, if the mass of the Σ were about 60 MeV greater than it is the Σ would be a resonant state decaying via the strong interaction to $\Lambda\pi$ with a width comparable with that of the $\Delta(1232)$. A similar situation exists in the case of the mesons. In conclusion, if we take as our definition of a fundamental or elementary particle one which is structureless and which therefore cannot exist in an excited state, then none of the hadrons is elementary.

7.3 *Leptons and quarks: the fundamental particles*

Up to the present limits of resolution, approximately 10^{-18} m, the leptons are structureless and, although specific investigations have been undertaken, none of them has been observed in an internally excited state. They are therefore regarded as fundamental particles.

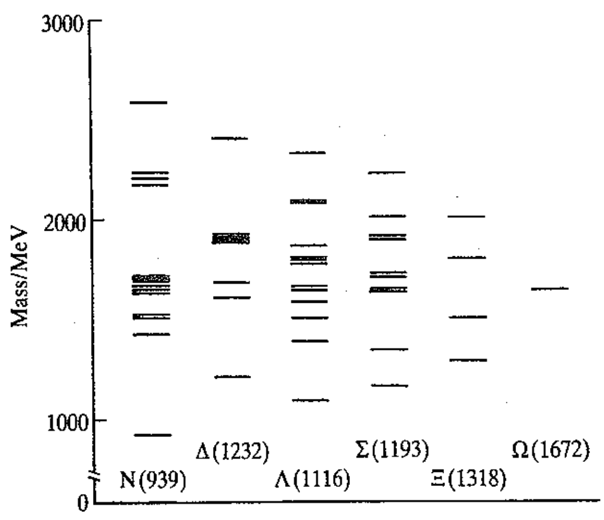


Figure 7.2
The spectrum of the strange and non-strange baryons. Only those states which are well established experimentally are shown.

* In the case of strongly decaying particles the width Γ of the state is measured directly and it is more usual to quote the width rather than the lifetime τ to which it is related by the uncertainty principle $\Gamma\tau \approx \hbar$ (see section 3.10).

In 1964 Gell-Mann[2] and Zweig, independently, suggested that the static properties of all the hadrons known at that time could be accounted for by the existence of particles known as quarks. By 1964 the strange mesons and baryons had been discovered but nothing was known about the particles labelled charmed and bottom in table 7.2. It is not necessary to consider the latter types of particle to understand the basis of the Gell-Mann–Zweig proposal but later in the book we will see how they can be incorporated into the scheme. In what follows then, we consider only the non-strange and strange mesons and baryons and, specifically, the set of $J^P = 0^-$ mesons

$$\pi^+ \; \pi^- \; \pi^0 \; \eta^0 \; \mathrm{K}^+ \; \mathrm{K}^- \; \mathrm{K}^0 \; \bar{\mathrm{K}}^0$$

and the set of $J^P = \frac{1}{2}^+$ baryons

$$\mathrm{p} \; \mathrm{n} \; \Lambda \; \Sigma^+ \; \Sigma^- \; \Sigma^0 \; \Xi^0 \; \Xi^-.$$

Before proceeding further it is necessary to discuss the concepts of baryon number, isobaric spin or isospin and strangeness.

Quite simply, each baryon is assigned a baryon number B of $+1$ and each antibaryon has $B = -1$. In all types of interaction, the baryon number is conserved so that baryons can only be created in pairs, i.e. baryon plus antibaryon. For instance, provided there is sufficient energy, the process

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{p} + \bar{\mathrm{p}}$$
$$B\colon 1 + 1 \rightarrow 1 + 1 + 1 + (-1)$$

is allowed since in both the initial and final states the total baryon number, which is an additive quantum number, is 2. On the other hand a process such as

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \bar{\mathrm{p}} + \pi^+ + \pi^+$$
$$B\colon 1 + 1 \rightarrow 1 + (-1) + 0 + 0$$

even if energetically possible, is forbidden by the conservation of baryon number. Any particle which is not a baryon has baryon number zero, so all the leptons and bosons have $B = 0$.

The concept of *isospin* was introduced in section 3.8 in connection with neutrons and protons. It was pointed out that because the strong interaction does not distinguish between the behaviour of these two particles it is useful to regard the nucleon as having an internal degree of freedom described by an isospin vector $\boldsymbol{I}$ in a fictitious space. Taking $I = \frac{1}{2}$ there would be two possible orientations of the vector with $I_3 = \pm\frac{1}{2}$ along the quantization axis corresponding to the proton and neutron respectively. The strong interaction then does not distinguish between these two orientations of the isospin vector. Since this is a general property of the strong interactions the concept of a conserved isospin $\boldsymbol{I}$ extends to all particles which can interact strongly, i.e. the hadrons. All hadrons belong to isospin multiplets. As in the case of an ordinary spin s for which there are $2s + 1$ different orientations of the spin vector, so for an isospin I there are $2I + 1$ different orientations of the isospin vector corresponding to the different particles in the isospin multiplet. The three charge states of the pion imply that $I_\pi = 1$ and $I_3 = +1, 0, -1$ for the π^+, π^0 and π^- respectively. The η^0 has no charged partners and is thus an isospin singlet with $I = 0$ and $I_3 = 0$. There is evidently a relation between the charge Q, expressed in units of $|e|$, where e is the charge of the electron, and the third component of isospin. For the π and η mesons this is simply

$$Q = I_3 \tag{7.1}$$

but this relation does not hold for the nucleon. In this case (cf. equation (3.25))

$$Q = I_3 + \frac{B}{2} \tag{7.2}$$

where B is the baryon number, and since for the mesons $B = 0$ this more general relation remains valid.

Lack of space prevents us from describing in detail the fascinating story surrounding the discovery and elucidation of the role of the so-called strange particles. Their existence was totally unexpected and originally their properties strange. The Λ^0 is one such particle which can be copiously produced for instance in collisions between negative pions and protons:

$$\pi^- + \mathrm{p} \to \Lambda^0 + \cdots. \tag{7.3}$$

The cross-section for this process is typical of that for a strong interaction. On the other hand, the 'reverse' process

$$\Lambda^0 \to \mathrm{p} + \pi^- \tag{7.4}$$

has a rate which is characteristic of a weak interaction. This apparently strange behaviour, production via the strong interaction and decay via the weak, is neatly accommodated in the hypothesis of 'associated production' put forward by Pais.[3] Gell-Mann[4] and Nishijima[5] introduced a 'strangeness' quantum number S which was assumed to be conserved in the strong interaction but not in the weak. The Gell-Mann–Nishijima scheme modified equation (7.2) to incorporate the strange particles. Accordingly

$$Q = I_3 + \frac{B + S}{2}. \tag{7.5}$$

Equation (7.5) is satisfied for the proton and pion provided both have strangeness $S = 0$. The Λ^0 with $I = I_3 = 0$ and $B = 1$ must have strangeness $S = -1$. Conservation of strangeness then prohibits the decay (7.4) via the strong interaction but allows the decay to proceed weakly. In the process (7.3) the Λ^0 must be produced in association with a particle with strangeness $S = +1$. In order to satisfy the conservation of charge and baryon number this particle must be a meson; the K^0 is a suitable candidate. According to (7.5) I_3 for the K^0 is $-\frac{1}{2}$ and in fact the K^0 and K^+ mesons form an isospin doublet: the antiparticles $\bar{K}^0$ and K^- also form an isospin doublet. We leave it as an exercise for the reader to show that the sigma baryons Σ have $S = -1$ and the cascade baryons Ξ have $S = -2$. The combination $B + S$ is known as the hypercharge, Y, and strange baryons are sometimes called hyperons.

The proposal of Gell-Mann and Zweig was that the hadrons are composite objects being bound states of spin $\frac{1}{2}$ fermions called quarks. To accommodate the rich spectrum of hadrons they introduced three types or flavour of quark, 'up', 'down' and 'strange'. The up and down quarks, u and d, form an isospin doublet with the u quark having $I_3 = +\frac{1}{2}$ and the d quark $I_3 = -\frac{1}{2}$. The strange or s quark is an isospin singlet with strangeness -1. The u and d quarks have strangeness 0. In their scheme, the baryons are bound states of three quarks and the most democratic way to produce a baryon number $B = 1$ is to assign each quark flavour a baryon number $B = \frac{1}{3}$. The properties of the quarks are summarized in table 7.3.

An important consequence of the choice of $B = \frac{1}{3}$ for the quarks is that they have fractional electric charge. Application of the Gell-Mann–Nishijima formula (7.5) gives the values of Q shown in table 7.3. For each quark there exists a corresponding antiquark for which the conserved quantities, charge, baryon number and strangeness, have the opposite sign. Consequently the values of I_3 also have the opposite sign. The properties of the antiquarks are summarized in table 7.4.

Within this framework a quark–antiquark bound state most economically produces the required $B = 0$ for mesons.

Table 7.3
Properties of the quarks proposed by Gell-Mann and Zweig

Flavour	I	I_3	S	B	Q
u	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\frac{1}{3}$	$+\frac{2}{3}$
d	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	0	-1	$\frac{1}{3}$	$-\frac{1}{3}$

Table 7.4
Properties of the antiquarks

Flavour	I	I_3	S	B	Q
$\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$
$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$
$\bar{s}$	0	0	$+1$	$-\frac{1}{3}$	$+\frac{1}{3}$

Table 7.5
Summary of the quantum numbers of the $J^P = 0^-$ mesons and the $J^P = \frac{1}{2}^+$ baryons

Meson	I	I_3	S	B	Y	Baryon	I	I_3	S	B	Y
K^+	$\frac{1}{2}$	$+\frac{1}{2}$	$+1$	0	$+1$	p	$\frac{1}{2}$	$+\frac{1}{2}$	0	1	$+1$
K^0	$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	0	$+1$	n	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	$+1$
π^+	1	$+1$	0	0	0	Σ^+	1	$+1$	-1	1	0
π^0	1	0	0	0	0	Σ^0	1	0	-1	1	0
π^-	1	-1	0	0	0	Σ^-	1	-1	-1	1	0
η^0	0	0	0	0	0	Λ^0	0	0	-1	1	0
$\bar{K}^0$	$\frac{1}{2}$	$+\frac{1}{2}$	-1	0	-1	Ξ^0	$\frac{1}{2}$	$+\frac{1}{2}$	-2	1	-1
K^-	$\frac{1}{2}$	$-\frac{1}{2}$	-1	0	-1	Ξ^-	$\frac{1}{2}$	$-\frac{1}{2}$	-2	1	-1

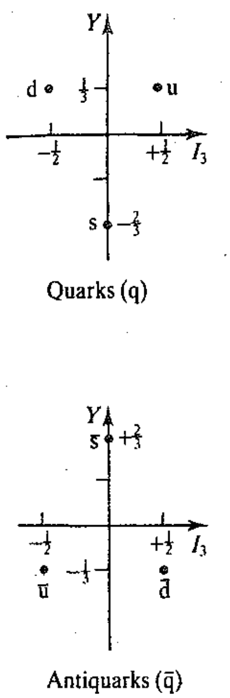


Figure 7.3
The basic quark and antiquark triplets.

It is sometimes convenient to visualize the properties of the quarks and antiquarks by plotting the hypercharge $Y = B + S$ against the third component of isospin I_3 as shown in figure 7.3.

In order to see how successfully the quark model reproduces the properties of the hadrons we summarize in table 7.5 the quantum numbers of the octet of $J^P = 0^-$ mesons and the octet of $J^P = \frac{1}{2}^+$ baryons.

The plots of hypercharge versus I_3 for these mesons and baryons are shown on the left-hand side of figure 7.4 while on the right-hand side are the quark flavour contents of the particles. The reader may readily verify the agreement between the properties of the physical particles and those predicted from the quark flavour assignment. Additionally, the quark spins may be coupled appropriately to give the correct spin-parity for the meson and baryon states. In the case of the mesons the quark–antiquark pair are bound with zero relative orbital angular momentum and spins antiparallel to give a total spin of zero. The negative parity is guaranteed by the opposite intrinsic parity of fermion and antifermion. (In the case of integer spin bosons the intrinsic parity of particle and antiparticle are the same.) The situation regarding the baryons is only slightly more complicated. The $\frac{1}{2}^+$ baryons are the lowest-lying, i.e. lightest, baryons and it is reasonable to assume that the relative orbital angular momenta l and l' in the three-quark system depicted in figure 7.5 are both equal to zero. Two of the individual quark spins can be coupled to give a resultant of zero so that the net spin-parity of the states is simply that of the unpaired quark, namely $\frac{1}{2}^+$. It is clear that by introducing orbital angular momentum into the quark systems a whole series of hadrons with different J^P values can be constructed from only three quark flavours.

An interesting and very important situation arises in the baryon spectrum when the quark spins are aligned, again with $l = l' = 0$, to give

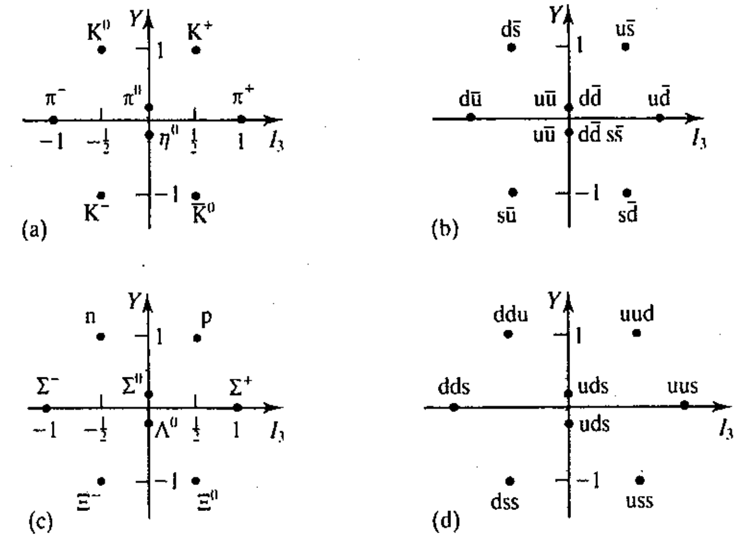


Figure 7.4
(a) The octet of 0^- mesons; (b) quark flavour assignments for the 0^- mesons; (c) the octet of $\frac{1}{2}^+$ baryons; (d) quark flavour assignments for the $\frac{1}{2}^+$ baryons.

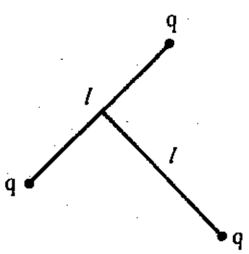


Figure 7.5
Orbital angular momentum in a three-quark system. l is the relative orbital angular momentum between two of the quarks and l' that between the di-quark system and the remaining quark.

$J^P = \frac{3}{2}^+$. There are ten $J^P = \frac{3}{2}^+$ baryons known in nature and these states are shown in figure 7.6 together with the flavour content of each state. When one focuses attention on either the $\Delta^{++}(1232)$ state with quark content uuu or the Ω^- state with three s quarks an apparent problem arises. The problem stems from the principle of indistinguishability of identical particles. Given a system of identical particles, either bosons or fermions, interchange of any pair of particles in the system can only multiply the wavefunction of the system by $+1$ or -1. For a second interchange of the two particles must result in a reversion to the original system. For bosons the factor is $+1$ and the wavefunction is symmetric 7·4
with respect to particle interchange, while for fermions the factor is -1 and the wavefunction is antisymmetric, or, in other words, identical fermions must obey the Pauli exclusion principle. This appears not to be so in the case of the $\Delta^{++}(1232)$ and Ω^-, in which three identical spin $\frac{1}{2}$ quarks, whose spatial positions are on average the same, occupy the same spin state. Many ideas were put forward to explain this conflict but the simplest one has stood the test of time. In addition to flavour, quarks

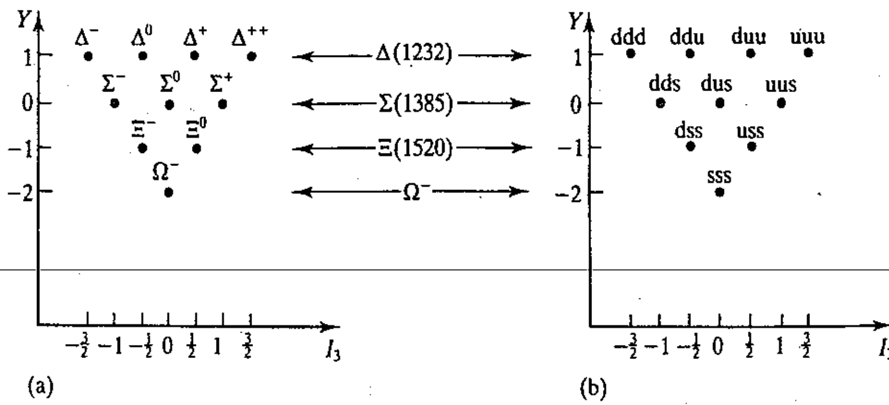


Figure 7.6
(a) The $\frac{3}{2}^+$ baryon decuplet and (b) its quark flavour content.

have another internal degree of freedom or quantum number, 'colour'. This has nothing to do with the ordinary notion of colour and may be thought of as a property somewhat akin to electric charge, except that there are three types of 'strong charge', or colour, compared with just one type of electric charge. The three colours are conventionally chosen as red, green and blue, and each quark can exist with equal probability in any of these states. The assertion is made that every hadron is a colour singlet, i.e. has no net colour or is 'white'. Then, in the $\Delta^{++}(1232)$, the three u quarks are no longer identical but instead are $u_R u_G u_B$. Since red + green + blue ≡ white, the $\Delta^{++}(1232)$ is a colour singlet as required. If this condition were not imposed there would be a proliferation of hadrons which is not observed experimentally. Just as both signs of electric charge exist so there are complementary colours, cyan, magenta and yellow or 'anti-red' $\bar{R}$, 'anti-green' $\bar{G}$ and 'anti-blue' $\bar{B}$. The antiquarks are assigned these anti-colours and colour singlets can be constructed from equal mixtures of the three colour states, as in the $\Delta^{++}(1232)$, equal mixtures of anti-colour ($\bar{R}\bar{G}\bar{B}$) or equal mixtures of colour and anti-colour, $R\bar{R}$, $G\bar{G}$ or $B\bar{B}$. These constructions produce baryons, antibaryons and mesons respectively.

According to the present available evidence, quarks are structureless particles and, along with the leptons, are considered to be elementary.

7.4 *The fundamental interactions*

The four fundamental interactions known in nature, namely the gravitational, electromagnetic, weak and strong interactions, at first sight appear to be totally unrelated, but as our subject develops we will see that not only are there striking similarities between them but that it may be possible to embrace them within a single unified theory. Although complete unification may be a long way off, important steps, which are outlined below, have already been taken towards this ultimate goal.

In 1928, Dirac[6] formulated a relativistic quantum wave equation for the electron, thus bringing together two of the essential ingredients of modern theoretical physics, special relativity and quantum mechanics, in a single theory. The complete unification of special relativity and quantum mechanics is possible only within the framework of a quantum field theory. The development of such a theory began in the late 1920s with the work of Dirac, Heisenberg and Pauli and is a general formalism which can in principle be applied to all four fundamental interactions. In the late 1940s Feynman, Schwinger, Dyson and Sin-itiro Tomonaga, working independently, developed the theory of quantum electrodynamics (QED), a field theory which permitted calculations to be made which agreed with experimental results to an unprecedented degree of accuracy. Over 50 years ago Fermi[7] proposed a theory of the weak

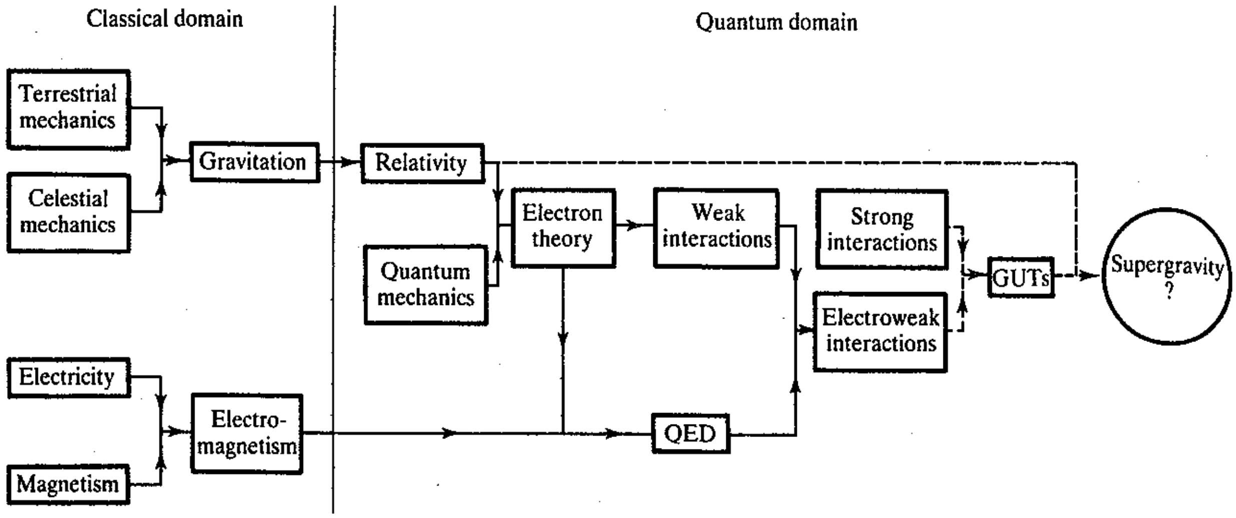


Figure 7.7 Block diagram showing the progress towards unification of the fundamental forces. The broken lines indicate steps which have yet to be completed.

interactions (section 5.2.3) which, although not quite capable in its original form of accommodating the full range of weak interaction phenomena which were later discovered, nonetheless provided a framework which is readily recognizable in the modern theory of the weak interactions. A major step along the road to unification was made by Glashow, Weinberg and Salam[8] when they unified the weak and electromagnetic interactions. Just as the photon, the quantum of the electromagnetic field, transmits the electromagnetic force between electrically charged particles, so the weak force is transmitted via the exchange of massive 'intermediate bosons', the W^+, W^- and Z^0. The 'standard theory' of the electroweak interactions, like QED, belongs to the class of theories known as gauge field theories, and the four field quanta, the γ, W^+, W^- and Z^0, are known as gauge bosons. They are all spin 1 particles and should be added to the quarks and leptons in the list of fundamental particles. It is now known that the strong interactions can also be described by a quantum gauge theory known as quantum chromodynamics (QCD) in which colour, introduced in the last section, plays a central role. Indeed, there are so-called grand unified theories (GUTs) which, although not entirely satisfactory, are a first step towards the unification of the strong, weak and electromagnetic interactions: the ultimate goal is to include gravitation. A field theoretical approach is necessary because of the need to include the possibility that particles can be created and destroyed; 'action at a distance' is achieved through the exchange of particles, the field quanta or gauge bosons referred to above. The various stages in the process of unification of the fundamental interactions are summarized in figure 7.7.

As was implicit in the previous sections in this chapter, it is a common belief that the interactions are describable by a limited number of

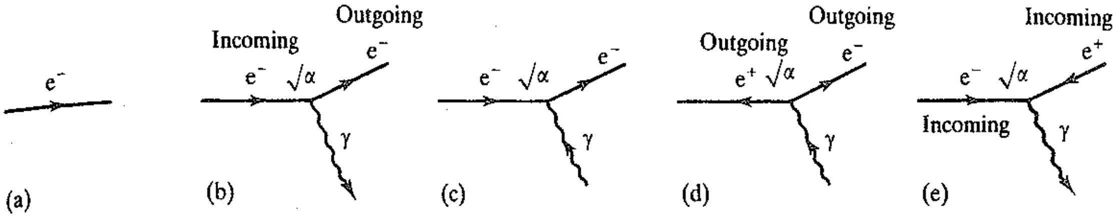


Figure 7.8 The basic Feynman diagrams of quantum electrodynamics: (a) a free electron; (b) photon emission; (c) photon absorption; (d) pair production; (e) e^+e^- annihilation.

fundamental particles interacting according to the general principles of quantum mechanics and relativity. These interactions or 'couplings' have a strength, or probability of occurrence, which is characterized by a 'coupling constant' and these span many orders of magnitude, ranging from about 10^{-39} for the gravitational interaction, which is by far the weakest, to about 1 for the strong interactions.

7.4.1 *The electromagnetic coupling*

The fundamental particles which couple to the electromagnetic field are the charged leptons and, by virtue of their electric charge, the quarks. By way of illustration let us consider the electron and the photon.

Following the approach due to Feynman a free non-interacting electron is represented as a line (figure 7.8(a)). There is some probability, given by the square of the amplitude for the process, that an electron emits a photon and this coupling of the electron to the electromagnetic field is represented by figure 7.8(b) in which the photon is shown as a wavy line. One can think of the incoming electron being absorbed at the vertex, where the outgoing electron and photon are created. This Feynman diagram, as it is called, represents the basic electromagnetic process $e^- \to e^-\gamma$ and the amplitude for this process is proportional to the charge on the electron, or more precisely to $\sqrt{\alpha}$ where α, the fine-structure constant, is given by

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}.$$

This is a small number and indicates that electromagnetic interactions are relatively weak. In Feynman diagrams of this sort it is possible to change the direction of the arrows provided that the particle is replaced by its antiparticle. Thus figure 7.8(c) represents the reverse process $e^-\gamma \to e^-$, corresponding to the absorption of a photon by the incoming electron. (The photon and its antiparticle are indistinguishable.) Similarly, by reversing the arrow on the incoming electron and changing it to the antiparticle, the positron, we arrive at figure 7.8(d), which represents electron–positron pair production. Finally, if all the arrows in figure 7.8(d) are reversed we get figure 7.8(e) which corresponds to electron–positron annihilation. The coupling is the same in each diagram. In the mathematical treatment the interaction is described by a Lagrangian L and in fact each of the diagrams in figure 7.8 represents one term in the Lagrangian. On the understanding that diagrams 7.8(c), 7.8(d) and 7.8(e) follow from 7.8(b) only the latter is included in the Lagrangian.

Of course none of the processes 7.8(b)–7.8(e), as they stand, conserves energy and momentum and they are called *virtual* processes. According to the uncertainty principle, however, energy conservation can be violated for a short time so that in the presence of a second electron for instance, the photon in figure 7.8(b) can be absorbed and energy and momentum conserved in the overall process. The corresponding Feynman diagram is shown in figure 7.9(a) which represents one contribution to the elastic scattering of one electron by another via virtual photon exchange. The coupling at the second vertex is again $\sqrt{\alpha}$. The amplitude for this process is essentially the product of three factors: (i) the amplitude that the incoming electron emits a photon at vertex 1, (ii) an amplitude that the photon propagates from the space–time point 1 to the space–time point vertex 2, the photon 'propagator', and (iii) the amplitude that the photon is absorbed by the second incoming electron at vertex 2. The amplitude for the overall process is thus proportional to α and is described as a first-order process. Other more complicated diagrams can contribute to the process $e^-e^- \to e^-e^-$; for instance by combining figure 7.8(b) with figure 7.8(d) one gets the second-order diagram, with an amplitude proportional to α^2, shown in figure 7.9(b). Each vertex contributes a factor $\sqrt{\alpha}$ to the amplitude. Proceeding in this fashion one can construct diagrams proportional to α^3, α^4 and so on, and indeed the total amplitude for the process $e^-e^- \to e^-e^-$ is an infinite sum of diagrams of increasing order, and hence decreasing amplitude, because $\alpha \ll 1$. Perturbation theory can be used to calculate the total amplitude, and hence the cross-section for the process $e^-e^- \to e^-e^-$, to any degree of accuracy required. All of this is known as quantum electrodynamics. There are precise rules, the so-called Feynman rules, for calculating the contribution to the amplitude for each diagram. The enumeration of the Feynman rules is beyond the scope of this book but we will nevertheless make frequent use of Feynman diagrams in our description of the interactions involving sub-nuclear particles.

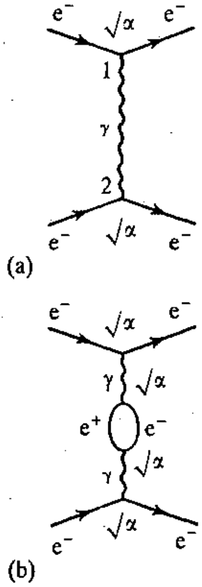


Figure 7.9
Feynman diagrams for electron–electron scattering: (a) a first-order process; (b) a second-order process.

7.4.2 *The strong coupling*

In section 1.4 it was pointed out that a short-range attractive force is required to bind the nucleons inside a nucleus. In 1935 Yukawa proposed, in analogy with QED, that the nuclear force was due to the exchange of a particle with non-zero mass: a meson. He envisaged the fundamental coupling shown in figure 7.10(a) and, following the argument used in the

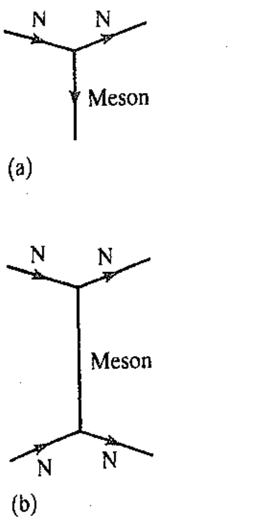


Figure 7.10
Yukawa's scheme for the strong interaction. He proposed (a) that the fundamental coupling was between the nucleon and a meson and (b) that the nucleon–nucleon force was produced by virtual meson exchange.

previous section, that the interaction between two nucleons would proceed via the exchange of a virtual intermediate meson (figure 7.10(b)).* A simple calculation based on the uncertainty principle shows that for a range of 1.4×10^{-15} m for the strong force the exchanged meson must have a mass of about 140 MeV/c^2. (For an explanation of the system of units commonly employed in elementary particle physics see section 1.5 and appendix F.) This is in contrast to the infinite range of the electromagnetic force which is due to the fact that the mass of the photon is zero.

As pointed out in section 7.3 neither the nucleon nor the mesons are in fact fundamental particles so that figure 7.10 does not represent the basic strong coupling. We recall that the hadrons are composite objects consisting of quarks which carry, in addition to electric charge, a 'strong charge', or colour, which plays a role in the strong interactions analogous to the role of electric charge in the electromagnetic interactions. We neglect, for the time being, the fact that there are three colours. The field quanta, or gauge bosons, of the strong interactions are called gluons which, like the quarks, carry colour. The fundamental coupling for the strong interactions is then between quarks and gluons (figure 7.11(a)) with a strength proportional to $\sqrt{\alpha_s}$, where α_s is the strong coupling constant, and a quark–quark interaction proceeds via the exchange of a virtual intermediate gluon (figure 7.11(b)). The gluons have zero mass and this would appear to contradict the fact that the strong or colour force is a short-range force. We return to this point in section 7.5.

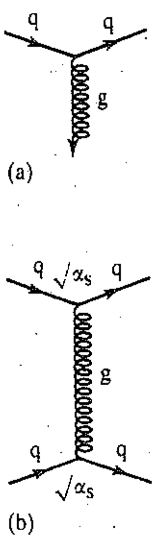


Figure 7.11
(a) The basic strong coupling between coloured quarks and gluons and (b) a Feynman diagram for quark–quark scattering via the exchange of a virtual intermediate gluon.

The analogy with the electromagnetic interactions is not as close as would appear from the discussion so far. The photon does not carry electric charge whereas the gluons are coloured. This implies that when two quarks interact, there is a flow of colour between them. In the electromagnetic interactions electric charge is conserved not only in the overall process but at each vertex in a Feynman diagram. Similarly, the colour quantum number must be conserved at each vertex in the strong interaction diagrams. This implies that the gluons are in fact bi-coloured. To be specific let us assume that the incoming quark in figure 7.11(a) is red. It cannot get rid of its colour simply by emitting a red gluon since conservation of colour would then imply that the outgoing quark is colourless and colourless quarks do not exist. If on the other hand it gets rid of its redness by emitting a red–anti-blue gluon, colour can be conserved if the outgoing quark is blue, as shown in figure 7.12(a). We recall that in a Feynman diagram if the direction of a line is reversed the particle must be replaced by its antiparticle. The same is true for colour:

* It is perhaps more readily acceptable that a *repulsive* force can arise from the exchange of a particle. In quantum field theory an *attractive* force can also arise from the same mechanism because the impulse of the exchanged particle is not necessarily in its direction of motion.

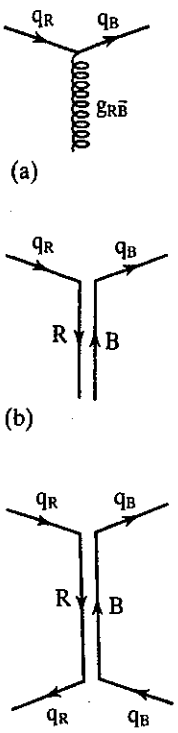


Figure 7.12
(a) A basic colour interaction. The incoming red quark q_R is absorbed at the vertex and a bi-coloured gluon $g_{R\bar{B}}$ and a blue quark q_B are created. Colour is conserved at the vertex.
(b) The flow of colour in (a).
(c) A colour flow diagram for a quark–quark interaction.

on reversing a colour line we replace it by its anti-colour and arrive at the colour flow diagram (figure 7.12(b)). The colour flow diagram corresponding to the quark–quark interaction of figure 7.11(b) is shown in figure 7.12(c), where it can be seen that the net result is that the quarks have exchanged colour.

Our specific choice of colour in the above discussion was quite arbitrary: with three colours and three anti-colours there are nine different bi-coloured combinations consisting of a colour and an anti-colour, namely

$$R\bar{R},\ R\bar{G},\ R\bar{B},\ G\bar{R},\ G\bar{G},\ G\bar{B},\ B\bar{R},\ B\bar{G},\ B\bar{B}.$$

The colour wavefunctions of the gluons can in general be linear combinations of these states and the totally symmetric combination $R\bar{R} + G\bar{G} + B\bar{B}$ is a colour singlet, i.e. it is colourless, and therefore plays no part in the strong interactions. Thus, in contrast to the electromagnetic case in which there is only one type of field quantum, the photon, there are eight coloured gluons. Furthermore, since the gluons themselves are coloured they can couple to each other and the Feynman diagrams for colour interactions may contain triple gluon vertices as shown in figure 7.13.

7.4.3 *The weak coupling*

The nuclear β decay discussed in section 5.2 is the best known example of a weak interaction. At a fundamental level neutron decay (equation (5.13b)) must require the quark change $d \to u + e^- + \bar{\nu}_e$ in which a down quark in the neutron changes flavour to become an up quark in the proton. The reader may be puzzled to see leptons apparently coupling to quarks, the fundamental particles involved in the strong interactions, in view of the statement in section 7.1 that the leptons can interact only via the weak interactions or, in the case of the charged leptons, via the electromagnetic interactions. The solution to the apparent paradox may be understood with the aid of figure 7.14. At the top vertex the incoming d quark in the neutron is absorbed and a u quark is emitted along with a weak gauge boson, in this case the W^-. The amplitude for this process is proportional to $\sqrt{\alpha_W}$; in analogy with the strong and electromagnetic processes $\sqrt{\alpha_W}$ may be thought of as a 'weak charge'. The W^- then propagates to the bottom vertex where it is absorbed and an electron and antineutrino are emitted. In this two-stage process, the leptons couple to the weak gauge boson and not directly to the quarks. Implicit in this viewpoint is the assumption that the quarks carry a weak charge in addition to the electric charge and colour or strong charge. The amplitude for the overall process is proportional to α_W. The mass of the W^- is roughly 100 times the mass of the proton, implying that the range of the weak interaction is extremely short.

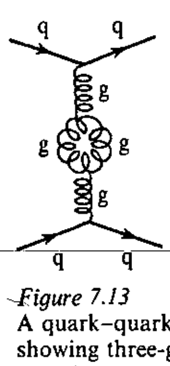


Figure 7.13
A quark–quark interaction showing three-gluon couplings.

7.5 *Vacuum polarization**

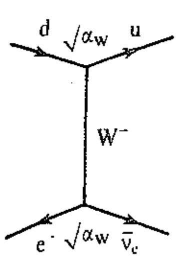


Figure 7.14
Feynman diagram for the β decay of a d quark.

It has tacitly been assumed above that the charge on the electron, the colour charge and the weak charge, or equivalently the 'fine-structure constants' α, α_s and α_W, are in fact constants. More properly, however, they should be regarded as functions of distance because of an effect which, because of analogy with the electrostatics of dielectrics, has become known as vacuum polarization.

To see this we recall that if a medium containing molecules with a permanent electric dipole moment is introduced into an electric field, that field is reduced within the dielectric by a factor ϵ, where ϵ is the dielectric constant. If the field is due to a pair of charges $\pm q$ placed within the medium (figure 7.15), the molecules will align along the field direction and produce a screening or reduction in the effective value of the two charges. In the immediate vicinity of the charges the effective charge is the bare charge q reduced by the induced 'surface' charge. The electrostatic potential energy may be written

$$V = \frac{1}{4\pi\epsilon_0\epsilon(r)}\frac{q^2}{r} \tag{7.6}$$

because over short distances the dielectric constant is a function of r. For values of r much larger than the molecular diameter d, $\epsilon(r) \to \epsilon$, the dielectric 'constant', while for $r \ll d$, $\epsilon(r) \to 1$. The effective charge $q/\sqrt{[\epsilon(r)]}$ varies with distance as shown in figure 7.16. For distances small

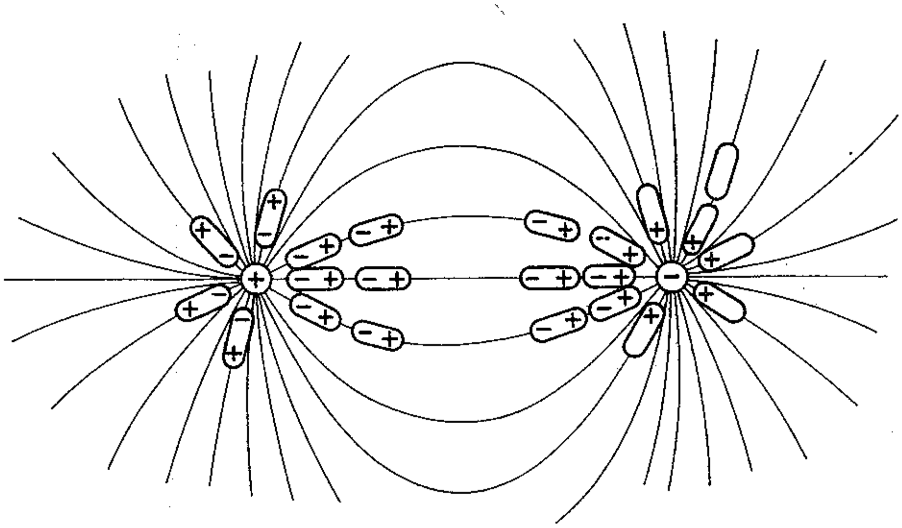


Figure 7.15
Schematic representation of macroscopic charge screening in a dielectric medium.

* A very readable and authoritative account of the vacuum in particle physics has been given by Aitchison.[9]

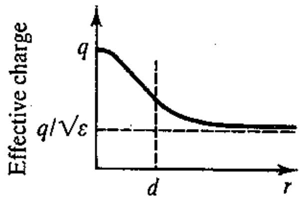


Figure 7.16
Sketch of the effective charge as a function of separation distance. d is the molecular diameter.

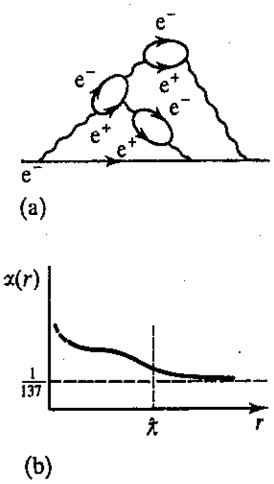


Figure 7.17
(a) Vacuum polarization in QED; (b) variation of the fine-structure constant α as a function of distance.

compared with the molecular diameter the screening effect diminishes; the effective charge increases and approaches the value of the full bare charge q. Screening effects similar to the above are evident when one considers the charge on the electron. As discussed in section 7.4.1, QED shows us that the electron can appear in many guises; it can spontaneously emit a photon which may materialize into an electron–positron pair, the electron and positron may emit further photons and so on. The electron is thus surrounded by a fluctuating 'cloud' of virtual electron–positron pairs with 'radius' of the order of the Compton wavelength, $\lambda\!\!\!^{-} = \hbar/mc$, as shown in figure 7.17(a). Because of Coulomb attraction, the positrons will on average be closer to the bare electron, and therefore the 'vacuum' will be polarized. At distances large compared with the Compton wavelength the effective charge is that which is normally quoted as the charge on the electron $e = 1.602 \times 10^{-19}$ coulomb, corresponding to a value for the fine-structure constant α of $e^2/4\pi\epsilon_0\hbar c \approx 1/137$. In analogy with the macroscopic effect discussed above, as the charge is probed at increasingly shorter distances the effective charge increases, corresponding to an increase in the fine-structure constant, as shown in figure 7.17(b). The effect is small, about 1 per cent, but nevertheless detectable. It must be noted that because the photon carries no charge there is no direct photon–photon coupling in QED. This leads to what is known as an *Abelian* field theory.

Vacuum polarization also affects the values of the strong and weak charges but in a crucially different way. The difference stems from the fact that the field quanta, the gluons and the weak bosons, themselves carry colour and weak charge respectively, unlike the photon which has no electric charge. As pointed out in section 7.4.2, the gluons can couple together and the same is true of the weak bosons. This coupling leads to what are known as *non-Abelian* field theories. The bare colour charge of a quark is modified by vacuum polarization described in QCD by diagrams like those shown in figure 7.18. If the polarization were described only by diagrams of the type shown in figure 7.18(a) the behaviour of $\alpha_s = e_s^2/4\pi\epsilon_0^s\hbar c$, as a function of distance, would be a replica of the electromagnetic case. Here, e_s is the strong charge and ϵ_0^s is the dielectric constant of the vacuum in response to the strong interactions. It was discovered independently by Gross and Wilczek[10] and by Politzer[11] that diagrams of the type shown in figure 7.18(b), which include gluon self-coupling vertices, have an 'anti-screening' effect. Roughly speaking, as we saw in section 7.4.2, the gluons tend to carry colour away from a quark or dilute the colour charge, so that in contrast to the electromagnetic case the colour charge decreases as it is probed to shorter and shorter distances. In fact, provided the total number of quark flavours u, d, s . . . is not too large, the strong coupling constant α_s approaches zero as $r \to 0$. This is known as 'asymptotic freedom' and implies that at very small distances, which would be achieved using a high energy probe, the quarks behave as free particles. The behaviour of α_s as a function of distance is shown schematically in

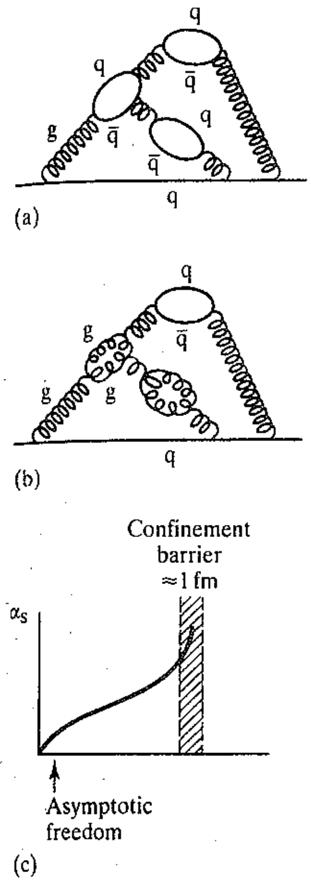


Figure 7.18
Vacuum polarization in QCD: (a) a contribution from $gq\bar{q}$ coupling; (b) a contribution including triple-gluon coupling; (c) variation of α_s with distance.

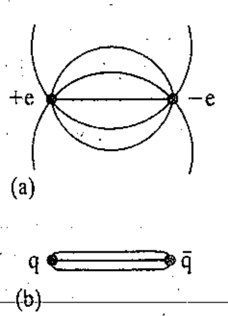
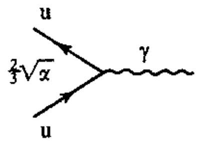
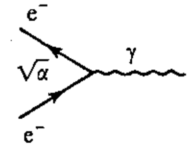
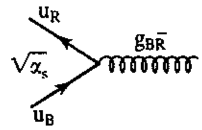
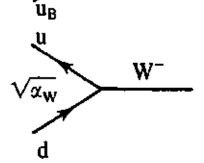
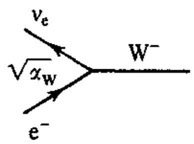


Figure 7.19
Lines of force for (a) a Coulomb field and (b) a colour field.

figure 7.18(c). One important consequence of asymptotic freedom is that, at least at high energies, perturbative methods can be used in the evaluation of QCD diagrams since $\alpha_s < 1$. The situation regarding the weak coupling constant, $\alpha_W = e_W^2/4\pi\epsilon_0^W \hbar c$, where now e_W is the weak charge and ϵ_0^W is the dielectric constant for the vacuum response to the weak force, is similar to the above with the gluons replaced by the weak gauge bosons. When defined in this way the relative values of the coupling constants, at distances which can be probed with today's high energy accelerators (see chapter 2), i.e. 10^{-15}–10^{-18} m, are $\alpha_W \approx 4\alpha$, $\alpha_s \approx 100\alpha$. The reader may be surprised to see that the weak coupling constant is larger than the electromagnetic coupling constant since, for example, the decay rate for a typical weak process $\pi^- \to \mu^-\bar{\nu}_\mu$ is much slower than that for the electromagnetic process $\pi^0 \to \gamma\gamma$ (see table 7.2). The apparent weakness of the weak interaction is in fact due to the extremely short range of the force caused by the very large mass of the weak bosons.* In section 7.4.2 it was stated that the gluons, like the photon, have zero mass and yet the range of the strong interaction is extremely short, of the order of 10^{-15} m. The explanation of this apparent contradiction is almost certainly connected with the variation of α_s with distance. Specifically, let us consider the colour force between a quark and an antiquark, bound by gluon exchange as in a meson. If, as indicated in figure 7.18(c), α_s increases with increasing distance, no finite amount of energy supplied to the system can liberate the quark or antiquark, or for that matter, the gluons; as the separation distance increases the colour force between the quark and antiquark becomes stronger. As indicated above this effect is due to the intrinsic non-linear character of QCD caused by the gluon self-coupling. Through the coupling of gluons to one another the colour field lines of force between the quark and antiquark are compelled to form a 'tube' as though there were attractive forces between the field lines, as shown in figure 7.19. This is in marked contrast to the Coulomb field; there is no self-interaction between the photons which prevents the field lines from spreading out. As the quark–antiquark distance r increases the potential energy of the system increases in proportion to r and so the quarks and gluons can never be freed. This 'infra-red slavery', as it is called, is believed to give rise to the total confinement of quarks and gluons inside hadrons, which, we recall, are 'colour neutral' or singlets. If one continues to stretch the colour flux tube there comes a point, beyond the 'confinement barrier' (see figure 7.18(c)), at which it is energetically

* In section 7.4.1 it was pointed out that the amplitude for a particular process depends not only on the coupling constants at the vertices but also on the propagator for the virtual exchanged particle. Crudely speaking this is $(p^2 - m^2)^{-1}$ where p is the four-momentum of the exchanged particle and m is its mass. For values of $p^2 \ll m^2$ this is approximately $1/p^2$ for electromagnetic interactions ($m_\gamma = 0$) and approximately $1/m_w^2$ for the weak interactions.

Table 7.6
Some basic characteristics of the electromagnetic, strong and weak interactions

Interaction	Field quanta	Basic coupling: Quarks	Basic coupling: Leptons	Coupling constants	Range/m	Typical reaction time/s	Typical cross-section/mb
Electromagnetic	γ			$\alpha \approx 1/137$	∞	10^{-16} or less	10^{-3}
Strong	Gluons			$\alpha_S \approx 100\alpha$	10^{-15}	10^{-23}	10
Weak	$W^{\pm}, Z^0$			$\alpha_W \approx 4\alpha$	10^{-18}	10^{-12} or more	10^{-11}

more favourable to rupture the tube. In doing so, another quark–antiquark pair, a meson, is created.* This gives rise to the so-called bag model of hadrons. The quarks in a hadron are confined to a 'bag' with a radius of the order of 1 fm, the confinement barrier, and the colour content of the quarks is such that the bag is colour neutral. This bag model of the hadrons then implies that if two protons, for instance, are to interact via the strong or colour force, since the bags have no net colour they must overlap before the quarks in one bag can experience the colour force due to the quarks in the other. The range of the interaction is then roughly equal to the bag radius.

Thus we see that colour, which was originally introduced to explain a statistics problem associated with the quark content of the baryons, plays a fundamental role in sub-nuclear physics. We present in table 7.6 the basic characteristics of the fundamental interactions, as modified by the existence of vacuum polarization.

7.6 Towards a unification of the fundamental interactions

In the discussion so far, the three quarks with flavours up, down and strange, together with the six leptons (e, ν_e), (μ, ν_μ) and (τ, ν_τ), have been

* This is rather like attempting to isolate the poles in a bar magnet by cutting it in half.

considered as the fundamental particles; they are all spin $\frac{1}{2}$ fermions. Their various interactions have been described in the context of quantum field theories in which the spin 1 gauge bosons γ and $W^{\pm}$, Z_0 and the eight coloured gluons are the carriers of the electromagnetic, weak and strong force respectively.

Listed among the stable particles in table 7.2 are states labelled 'charm' and 'bottom', which, like the other hadrons, are composites of quarks but with the new flavours charm (c) and bottom (b). A sixth quark flavour, top (t), is confidently expected to exist. At the present time, therefore, there appear to be 12 fundamental fermions, the six quarks and the six leptons, which can be arranged into three 'generations' of increasing mass:

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} (t) \\ b \\ \tau \\ \nu_\tau \end{pmatrix}.$$

The parentheses around the top quark indicate that as yet it has not been discovered experimentally. The parallelism between the quarks and leptons is striking; the distinct generations appear to be replicas of one another. The electromagnetic and weak interactions of the quarks and leptons are similar in many respects. For instance, the annihilation process $u\bar{u} \rightarrow e^+e^-$ can proceed by either of the mechanisms shown in figure 7.20. Does this imply some fundamental link between the quarks and leptons? The notion of a multiplet implies that transitions can take place between members of the multiplet. Does the above electroweak connection imply that the quarks and leptons legitimately belong to the same multiplet? If so this would be tantamount to a unification of the weak and electromagnetic interactions; the quarks and leptons would be sources of a single unified electroweak field. Can the strong interaction field be related to this electroweak field? In other words can the electroweak and strong fields be combined into one 'grand unified field'?

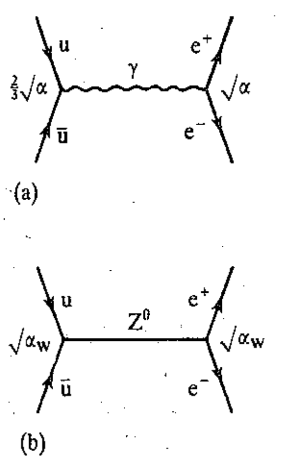


Figure 7.20
Feynman diagrams for the annihilation processes (a) $u\bar{u} \rightarrow \gamma \rightarrow e^+e^-$ and (b) $u\bar{u} \rightarrow Z^0 \rightarrow e^+e^-$.

The steps towards this grand unification began long ago in the classical domain (see figure 7.7) when Newton unified celestial and terrestrial mechanics; the same gravitational laws explained both sets of phenomena. Maxwell unified electricity and magnetism; if the electric and magnetic charges were distinct the force on a particle with electric charge e_E and magnetic charge e_M, say, would be

$$\boldsymbol{F} = e_E \boldsymbol{E} + e_M \boldsymbol{v} \wedge \boldsymbol{B}.$$

The unification is brought about through the realization that $e_E = e_M = e$, the 'electromagnetic' charge. Maxwell's theory is a relativistic theory and the unification introduces a scale, the velocity of light c, which governs the relative strength of the two interactions. At low velocities the 'electric' force dominates but at high velocities the electric and magnetic forces are comparable. In the quantum domain we have seen that the strong, weak and electromagnetic fine-structure constants have a functional dependence on distance. At short distances, or equivalently, high energies, the electromagnetic coupling increases in strength while the strong and weak coupling decreases in strength. It is conceivable that at some unifying energy scale, or interparticle separation, the three become equal as indicated in figure 7.21. Detailed calculations show that the change in α is extremely slow at short distances, in fact logarithmic, and the unification scale is $r \approx 10^{-31}$ m. At this distance the quarks and leptons would be the sources of the 'grand unified field' mentioned earlier. As with the other fields, which are distinct at the distances which can be achieved with today's accelerators, the particles will interact through the exchange of field quanta, spin 1 X bosons. The range of the interactions implies that $M_X \approx 10^{15}$ GeV, an extremely high mass. Such ultra-high energies would be commonplace in the earliest epoch, after the 'big bang', when the universe was extremely hot and dense, but are utterly unattainable in the laboratory. However, an important feature of grand unification is that the quarks and leptons would belong to one 'super-multiplet' and therefore transitions could take place between them since they would all couple to the X bosons. This has the important and dramatic implication that baryon number would not be conserved and the proton would be unstable. A possible mechanism for the decay $p \rightarrow e^+\pi^0$ is shown in figure 7.22 in which an X boson is exchanged between an up and down quark in the proton to produce a positron and a π^0. The proton of course is known to have a lifetime which is many orders of magnitude bigger than the age of the universe; detailed calculations show that the proton lifetime is greater than 10^{31} years. This seems extremely long in the context of a unified theory which puts the quarks and leptons on the same footing but it can be qualitatively understood in the same way that at low energies the weak interactions are understood to be much weaker than the electromagnetic interactions, even though $\alpha_W \approx 4\alpha$. The weak interactions are inhibited by the high mass of the W and Z bosons. Analogously, the proton decay is inhibited by the extreme mass of the X boson. It is clear that proton decay experiments are of vital significance for the ideas of grand unification.

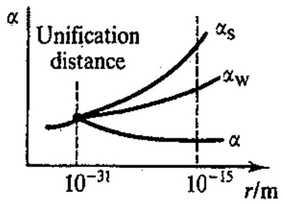
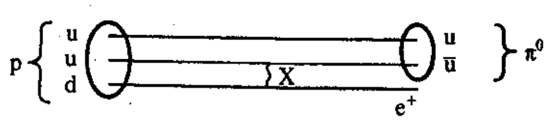


Figure 7.21
Schematic representation of the variation of the fine-structure constants showing convergence towards a single value at the 'grand unification distance' $r \approx 10^{-31}$m.

What about gravity? Because the gravitational force is so weak compared with the other three forces it can safely be neglected in most considerations concerning elementary particles. But, as we have seen, the

Figure 7.22
A possible mechanism for the decay $p \rightarrow \pi^0 e^+$.



relative strengths of these forces depend on the distance or, equivalently, the energy. This is also true of the gravitational force – except that in this case the strength increases as some power of the energy rather than logarithmically. Gravitational effects must be taken into account when the energy is of the order of the Planck mass, m_P, which is given by the expression

$$m_P = \sqrt{\left(\frac{\hbar c}{G}\right)} \approx 10^{19}\ \text{GeV}$$

where G is Newton's gravitational constant. Since this is much greater even than the grand unification mass scale, $M_X \approx 10^{15}$ GeV, gravity can normally be safely neglected. Attempts are being made, however, to incorporate gravitation into the scheme of unification. One such attempt, known as supergravity, embraces the Einstein gravitational field equations and the field equations of the strong and electroweak interactions in the same theory. The quantum of the supergravity field is the spin 2 graviton, which is placed in the same multiplet as certain fermions and bosons of lower spin. Such a theory takes us well beyond our present knowledge of the quarks and leptons, but whether or not it forms the basis of the unification of the four fundamental interactions it is the cherished belief that a single unified theory can be constructed.

REFERENCES 7

1 **Perl M L** *et al.* 1975 *Phys Rev Lett* **35** (1489)

2 **Gell-Mann M** 1964 *Phys Lett* **8** (214)

3 **Pais A** 1952 *Phys Rev* **86** (663)

4 **Gell-Mann M** 1956 *Nuovo Cimento* Suppl **4** (2848)

5 **Nishijima K** 1955 *Prog Theor Phys* (Kyoto) **13** (285)

6 **Dirac P A M** 1928 *Proc Roy Soc* **A117** (610)

7 **Fermi E** 1934 *Zeit Physik* **88** (161)

8 **Weinberg S** 1967 *Phys Rev Lett* **19** (1264); see also **Salam A** 1968 in Svartholm N (ed) *Elementary Particle Theory; Relativistic Groups and Analyticity* (8th Nobel Symposium) Almqvist and Wiksell

9 **Aitchison I J R** (1985) *Contemp Phys* **26** (333)

10 **Gross D J and Wilczek F** 1973 *Phys Rev Lett* **30** (1343)

11 **Politzer H D** 1973 *Phys Rev Lett* **30** (1346)