PHYSICS 221A FALL 2023 – HOMEWORK 1

Due in class Thursday, October 19, 2023

Complementary reading: Burcham and Jobes, chapters 7 and 8; also if interested, take a peek at Halzen and Martin, chapter 1, and sections 2.1-2.7.

Problem 1

State whether the following reactions are forbidden or allowed. If there is more than one particle in the initial state, assume that the kinetic energy is great enough to allow the transition to occur should it not be ruled out by other conservation laws. If forbidden, state why. If allowed, classify as strong, electromagnetic, or weak, and briefly state your reasoning. Draw the lowest-order diagram for each allowed process. See the PDG handbook for particle properties. (Hint: (j) is forbidden for the strong force; why? Don't worry about non-strong decays for this mode.)

(a) $K^+ + n \to \Sigma^+ + \pi^0$
(b) $\Sigma^+ \to \pi^+ + \pi^0$
(c) $e^+ + e^- \to \mu^+ + \mu^-$
(d) $D^0 \to K^- + \pi^+$
(e) $\pi^+ + p^+ \to \Sigma^+ + K^+ + \pi^0$
(f) $\Lambda^0 \to p^+ + \pi^-$
(g) $\pi^0 \to \gamma + \gamma$
(h) $\pi^+ \to \mu^+ + \gamma$
(i) $K^0 \to K^+ + e^- + \overline{\nu}_e$
(j) $\rho^0 \to \pi^0 + \pi^0$

The rotation matrix elements $d^{(j)}_{m,m'}$ are the (m',m)th matrix elements of the matrix $\exp(-i\theta J_y)$, where J_y is the operator for the y-component of the j-representation of angular momentum. In the following two problems, you'll calculate these matrix elements for the representations j = 1/2 and j = 1.

Problem 2

For j = 1/2, express J_y in terms of the appropriate Pauli spin matrix. Exponentiate this expression to show that $d^{(1/2)}_{-1/2,1/2} = -d^{(1/2)}_{1/2,-1/2} = \sin(\theta/2)$ and $d^{(1/2)}_{1/2,1/2} = d^{(1/2)}_{-1/2,-1/2} = \cos(\theta/2)$.

Problem 3
For j = 1, write down the 3x3 matrices J_+ and J_-. Combine these to find J_y, and show that $J_y^{2n+1} = J_y$; $J_y^{2n} = J_y^2$. Exponentiate to show that

$$d^{(1)}_{0,1} = -d^{(1)}_{1,0} = -d^{(1)}_{0,-1} = d^{(1)}_{-1,0} = (1/\sqrt{2})\sin(\theta)$$

$$d^{(1)}_{1,1} = d^{(1)}_{-1,-1} = 1/2(1+\cos(\theta))$$

$$d^{(1)}_{-1,1} = d^{(1)}_{1,-1} = 1/2(1-\cos(\theta))$$

$$d^{(1)}_{0,0} = \cos(\theta)$$

Problem 4
As we will see in 221B, in the decay of the spin-1 W^- boson $W^- \to l^- + \overline{\nu}_l$, the antineutrino always emerges with right-handed helicity, i.e., polarized in its direction of motion, while the electron always emerges left-handed. For a W^- polarized in the $\hat{z}$ direction, derive the polar-angle distribution in the W rest frame of the electrons from $W^- \to e^-\overline{\nu}_e$ decays. Assume that the electron and neutrino are in an $l = 0$ orbital state.

Problem 5
Consider the decay of the vector ρ meson $\rho^0 \to \pi^+ + \pi^-$, with the ρ polarized in the $\hat{z}$ direction ($J_z = +1$). Derive the polar angle distribution of the π^+ in the ρ rest frame. What about the π^-? What would these distributions be for a ρ spin projection $J_z = 0$?

Problem 6
A common two-body decay mode involves the decay of a pseudoscalar meson into a vector and a second pseudoscalar meson, with the vector meson then decaying quickly into two pseudoscalar mesons; for example, $B^- \to \rho^0 + \pi^-$, with $\rho^0 \to \pi^+ + \pi^-$, is such a decay. Find the angular distribution of the pseudoscalar decay products of the vector meson in the rest frame of the vector meson, in terms of the angle θ measured relative to the flight axis of the vector meson. This 'helicity angle' is a frequently used experimental handle in the isolation of these useful but somewhat rare decay modes. (Hint: You'll need to invoke a classical argument about the possible orientations of orbital angular momentum in one of the decays).

Problem 7
The vector $K^*(892)$ decays via the strong force essentially 100% of the time to a $K - \pi$ combination, where 'K' denotes one of the four pseudoscalar kaon states K^+, K^0, and their charge conjugates. Write down the charge-specific decay modes of the K^{*+} ($\overline{s}u$), and determine their branching fractions. Do the same for the K^{*0} ($\overline{s}d$).

Problem 8
The reactions

$$K^- + p \to \Sigma^0 + \pi^0$$

$$K^- + p \to \Sigma^+ + \pi^-$$

can occur for two different values of the total isospin vector I. What are they? (We say that the reactions proceed via these two separate 'isospin channels'.) Assuming that the reactions proceed entirely through the channel with larger I, what is the ratio of the cross sections for these two processes? What about if the reaction proceeds via the channel with smaller I?

Problem 9
Classify the following reactions at strong, electromagnetic, or weak. What quantum numbers, if any, are violated in each case? What are the possible values of orbital angular momentum in each final state?

(a) $N^+(1535) \to p + \eta^0$
(b) $\Sigma^+(1189) \to p + \pi^0$
(c) $\rho^0(770) \to \pi^0 + \gamma$

Problem 10
Which of the following decays are forbidden by C-invariance?

(a) $\omega^0 \to \pi^0 + \gamma$
(b) $\eta' \to \rho^0 + \gamma$
(c) $\pi^0 \to \gamma + \gamma + \gamma$
(d) $J/\psi \to \overline{p} + p$
(e) $\rho^0 \to \gamma + \gamma$