PHYSICS 221A FALL 2023 – HOMEWORK 2

Due in class Tuesday, November 7, 2023

Complementary reading: Burcham and Jobes chapters 9 and 10; and if interested, glance at Halzen and Martin,sections 2.8-2.14.
Also, it may prove illuminating to fill in all the algebraic steps in the sketch we gave in class of partial wave analysis, up to the derivation of the Breit-Wigner resonance formula.

Problem 1
The following is a concrete example of the use of the differential cross section to determine partial wave amplitudes. This problem is Burcham and Jobes, Problem 9.6.
Consider the scattering of a spinless particle when no inelastic reactions are possible. Assuming that only S ($l = 0$) and P ($l = 1$) waves contribute to the scattering, write down the explicit expression for the scattering amplitude $f(\theta)$ in terms of the partial wave amplitudes T_0 and T_1. Derive the corresponding expression for the differential cross section $d\sigma/d\Omega$, and express this in terms of the phase shifts δ_0 and δ_1. How would you go about determining these phase shifts from the measured differential cross section?

Problem 2
Consider a process with a final state containing three particles. The three-body density of final states which appears as a factor in Fermi's Golden Rule is given by

$$\rho_3(E) = (\frac{V}{\hbar^3})^3 \frac{d}{dE} \int d^3p_1 d^3p_2 d^3p_3.$$

Show that for a resonance of mass M decaying into three daughter particles,

$$\rho_3(E) = (\frac{V}{\hbar^3})^3 \int d^3p_1 d^3p_2 d^3p_3 \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)\delta(E_1 + E_2 + E_3 - M).$$

Problem 3
Show that the Lorentz-invariant two-body phase space factor for massless final state particles is a constant independent of the cms energy E

$$R_2(E) = \frac{\pi}{2}.$$

In the following five problems, you will construct the explicit wavefunction of the Λ^0, and then calculate its magnetic moment. In doing so, you will confirm the quark-exchange symmetry of the decomposition of the product of three fundamental triplets

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10_S} \oplus \mathbf{8_{M_S}} \oplus \mathbf{8_{M_A}} \oplus \mathbf{1_A},$$

as well as the explicit quark wavefunction of the proton introduced in class. These problems are roughly Burcham and Jobes, Problems 10.4-6 and 10.15.

Problem 4
The fundamental quark triplet may be regarded as a V-spin doublet (u, s) and a V-spin singlet d, or a U-spin doublet (d, s) and a U-spin singlet u. Show that $V_-|u\rangle = |s\rangle$ and $U_+|s\rangle = |d\rangle$.

Problem 5
The direct product of two quark triplets gives rise to a sextet and an anti-triplet

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \overline{\mathbf{3}}.$$

By making use of the appropriate raising and lowering operators, and the fact that states with identical quark content must be orthogonal, determine the quark wavefunctions of these nine states. (Hint: begin with the sextet, where three of the states can be written down by inspection).

Problem 6
Recall our claim in class that the product of three quark triplets decomposes according to

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = (\mathbf{6} \oplus \overline{\mathbf{3}}) \otimes \mathbf{3} = \mathbf{10_S} \oplus \mathbf{8_{M_S}} \oplus \mathbf{8_{M_A}} \oplus \mathbf{1_A},$$

where the subscripts S, M_S, M_A, and A indicate a quark wavefunction that is totally symmetric, symmetric under the exchange of the first two quarks, antisymmetric under the exchange of the first two quarks, and totally antisymmetric, respectively. Determine the quark wavefunctions of the uud states of the decouplet, $\mathbf{6} \otimes \mathbf{3}$ octet, and $\overline{\mathbf{3}} \otimes \overline{\mathbf{3}}$ octet. Verify the symmetry assignments in the decomposition of the product representation, and the form of the proton quark wavefunctions used in class. (Hint: You'll need to consult the weight diagrams, for example on p 331 of Burcham and Jobes, to figure out how the various uud states arise. Also, you know ahead of time that the decouplet must be totally symmetric - why?)

Problem 7

Determine the quark wavefunctions of the uds states of the decouplet, $\mathbf{6} \otimes \mathbf{3}$ octet, and $\mathbf{3} \otimes \mathbf{3}$ octet. (Hint: The I=1 mixed-symmetry octet states Σ^0 can be obtained from the proton wavefunction with the appropriate raising and lowering operators.) In particular, in the end you should find that

$$\Lambda^0_{M_S} = \frac{1}{2}[(sd + ds)u - (su + us)d]$$

$$\Lambda^0_{M_A} = \frac{1}{\sqrt{12}}[(sd - ds)u + (us - su)d - 2(du - ud)s].$$

Problem 8

Obtain the explicit wavefunction for the Λ^0 with spin component $S_z = 1/2$. Use this to show that the quark model predicts that the magnetic moment of the Λ^0 is μ_s, the intrinsic magnetic moment of the s quark.