PHYSICS 221A FALL 2021 – HOMEWORK 3

Due in class Tuesday, November 30, 2021

In addition to thinking about radiation and detectors, this problem set will give you a little practice with a few basic concepts in probability and statistics.

Background: W. R. Leo, Chapters 2,6,10; Particle Data Group resources, sections on radiation and detectors, and probability and statistics.

Note that for Problems 1-3, I am trying to give you some practice in quickly estimating answers using information in resources such as the Particle Data Group book rather than have you derive precise answers from derived formulae.

**Problem 1** [10 pts]
For a charged pion entering a block of iron, at roughly what energy will the pion lose on average 10% of its incident energy before suffering an inelastic nuclear collision?

**Problem 2** [10 pts]
An extensive cosmic ray shower strikes the earth, which we assume to be uniform quartz ($SiO_2$) rock of density $3g/cm^3$. The shower has a core of 1000 GeV muons, and a broad distribution of electrons of energies between 10 GeV and 100 GeV.

a) Assuming that the muons ionize 'minimally' until they stop, how deeply do the muons penetrate?

b) At this depth, what fraction of its original energy remains in the electron component of the shower?

**Problem 3** [10 pts]
What thickness of aluminum attenuates a 3 MeV beam of gamma rays by 90%? You may find it easiest to search through the PDG's 'Passage of Radiation through Matter' to find a plot that provides the relevant information on the attentiation of gamma rays by matter.

**Problem 4** [10 pts]
Calculate the RMS resolution for an MWPC with a wire spacing of 2mm, assuming that each passing particle produces a signal in one and only one wire of the chamber. If there is a region between the wires for which a passing particle generates a signal on both of the two adjacent wires, does the resolution improve or worsen?

**Problem 5** [20 pts]
In class, we considered a p-type semiconductor detector, i.e., a slab of p-type silicon mated on one side to a thin n$^+$ implant held at a bias +V$_B$, and on the other side to a thin p$^+$ implant held at ground. We derived the time-dependence of the signal development for both electrons and holes, as a function of the electron and hole mobilities $\mu_e$ and $\mu_h$, the width $d$ of the p-type region, its resistivity $\rho = 1/\sigma$, and the distance $x_0$ of the electron-hole creation point from the p$^+$ implant. Derive the time-dependence of the signal development, for both electrons and holes, for an n-type detector, i.e. an n-type slab of width $d$ mated to an n$^+$ implant held at +V$_B$, and a p$^+$ implant held at ground, as a function of the distance $x_0$ of the electron-hole creation point from the $n^+$ (not $p^+$!) implant. Given the parameters (roughly) of silicon: $\mu_e = 1500$ cm$^2$/Vs, $\mu_h = 500$ cm$^2$/Vs, $\epsilon\rho \simeq 10^{-12}\rho\ s$ ($\rho$ in $\Omega$-cm), plot both the n-ype and p-type signal development vs. time for a single electron-hole pair created halfway through the sensitive region of the detector. Assume that both the n-type and p-type materials are doped to a resistivity of 1000 $\Omega$-cm.

**Problem 6** [15 pts]
Consider a colliding beam vertex detector consisting of two approximately massless silicon strip detectors concentric with the beampipe, at distances of 2.5 and 5 cm from the beamline, respectively. Assume that the strips are oriented parallel to the beamline, and achieve a resolution of 5 $\mu$m in the azimuthal ($\phi$) direction. The inner layer of this detector sits just outside of a cylidrical aluminum beampipe of 1 mm thickness and 2.5 cm radius. For cylindrical coordinates with the z-axis lying along the beamline, estimate the impact parameter resolution in the $r - \phi$ plane for a charged pion of infinite momentum, where the impact parameter is the distance of closest approach of the extrapolated vertex detector track to the beamline. Also estimate the $r - \phi$ impact parameter resolution for a charged pion of momentum $p$ = 0.5 GeV/c exiting the beampipe perpendicular to the beamline, assuming that the rest of the detector (the central tracker) allows the curvature of this particle through the magnetic field to be measured with perfect precision.

**Problem 7** [10 pts]
You have set up an experiment to measure the energy of the $\alpha$ particle in $^{241}Am^{95}$ decay. Every 10 seconds you read out the total energy deposited in your detector (if any), and reset the detector. After collecting data for a large number of 10 second intervals, you observe that for 60.7% of the intervals, no energy is deposited in your detector, while for the remaining 39.3% of intervals, a mean of 6.97 MeV is deposited in your detector. What is the energy of the $\alpha$ particle? (Hint: Make use of the Poisson distribution.)

**Problem 8** [15 pts]
NOTE: I found it helpful to writing a small computer program to implement the closed-form solution you will arrive at for the error propagation.
You do two experiments, A and B, that independently mesaure the mass (in GeV) and width (in MeV) of a new state of matter. Experiment A does a simultaneous fit to the data, to yield a mass of 4.1 GeV and a width of 1.7 MeV. Experiment A's error on the mass is 0.5 GeV and on the width 0.3 MeV; however, the fit that A does to extract the mass and width correlates the two measurements with a correlation cofficient of $\rho_A = 0.52$.
On the other hand, the *combination* of experiments A and B yields values of mass and width of 4.4 GeV and 1.55 MeV, respectively. The combined errors on the mass and width are 0.35 GeV and 0.22 MeV, respectively, with a correlation coefficient of $\rho_T = 0.41$
Find the value of and errors on the mass and width of experiment B that were combined with those of experiment A to yield the combined values just above. What was the value of the correlation coefficient for experiment B? Are experiments A and B consistent with one another? Why or why not?
Some answers: mass = 4.83 GeV; width = 1.24 MeV; width error = 0.33 MeV.