PHYSICS 221B – HOMEWORK SET 1

Due in class Thursday January 26, 2017.

Background: Halzen and Martin, Chapters 8 and 9; Burcham and Jobes, Chapter 12.

Problem 1
Calculate the squared momentum transfer from a 10 GeV electron scattered elestically through an angle of 10 degrees by a nucleon at rest in the lab. What is the 'Compton' wavelength associated with the momentum transfer? How does this compare with what you know to be the size of the nucleon?

Problem 2
For a spherically symmetric charge distribution $\rho(\vec{x})$, show that the form factor

$$F(q) = \int_V \exp(i\vec{q}\cdot\vec{x})\rho(\vec{x})d\vec{x}$$

is approximately equal to

$$1 - \frac{1}{6}|q|^2\langle r^2\rangle,$$

where r^2 is the mean-square charge radius.

Problem 3
As outlined in class, for the case $m_e = 0$, show that the leading-order Feynman amplitude for $e^- - \mu^-$ scattering, in the frame in which the muon is at rest, takes the form

$$|\overline{M}_{fi}|^2 = \frac{8e^4}{q^4} 2m_\mu^2 E'E[\cos^2\frac{\theta}{2} - \frac{q^2}{2m_\mu^2}\sin^2\frac{\theta}{2}],$$

where

$$q^2 = -4E'E\sin^2\frac{\theta}{2},$$

with E (E') the energy of the initial (final) state electron, and θ its scattering angle.

Problem 4
Show that for *elestic* scattering of a projectile off of a target of mass M

$$\nu = \frac{Q^2}{2M}$$

in the frame in which the target is initially at rest. Here, Q^2 is the negative of the square of the four-momentum transfer from projectile to target, and ν is its 0^{th} (energy) compenent.

Problem 5
Derive the structure functions $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ for the elastic scattering of an electron probe off of a muon target via the exchange of a single virtual photon probe. What about the case of electron-proton elastic scattering, again via the exchange of a single virtual photon probe?

Problem 6
(Burcham and Jobes, 12.5-6). Consider the 'infinite momentum' frame in which the nucleon mass is negligible, so that the nucleon has four-momentum $P = (P, 0, 0, P)$. Now, visualize the nucleon as a stream of partons each with zero momentum transverse to the direction of motion of the nucleon, and each carrying some variable fraction x of the nucleon's four-momentum. Show that, when probed via the exchange of a vector boson with squared four-momentum $q^2 = -Q^2$ and energy ν, then x is precisely the Bjorken scaling variable

$$x = \frac{Q^2}{2M\nu},$$

where M is the nucleon mass. Also, show that for energies high enough that all masses may be neglected,

$$y = \frac{\nu}{E_{lab}} = \frac{1}{2}(1 - \cos\theta^*),$$

where y is the Lorentz scalar $(p \cdot q)/(p \cdot k)$, with p and k the target and probe four momenta, and θ^* the probe scattering angle in the cms frame.