PHYSICS 221B – HOMEWORK SET 2

Due in class Tuesday, Febraury 14, 2017.

Background: Halzen and Martin, Chapter 12; Burcham and Jobes, Chapter 11.

Problem 1

Define the right- (left-) handed spinors

$$u_{R(L)} = (\frac{1 \pm \gamma_5}{2})u,$$

where '+' corresponds to right-handed, and '-' to left-handed. Show that the Dirac currents obey the following 'helicity decompositions':

$$\overline{u}O_\mu u = \overline{u}_R O_\mu u_R + \overline{u}_L O_\mu u_L$$

for vector and axial-vector currents O_μ,

$$\overline{u}Ou = \overline{u}_R O u_L + \overline{u}_L O u_R$$

for scalar and pseudoscalar currents O, and

$$\overline{u}O_{\mu\nu} u = \overline{u}_R O_{\mu\nu} u_L + \overline{u}_L O_{\mu\nu} u_R$$

for the tensor current $O_{\mu\nu}$. Thus, vector interactions are helicity-preserving, while scalar and tensor interactions flip helicity.

Problem 2

Consider the decay $\pi^- \to l^- \overline{\nu}_l$. Show that the square of the matrix element for this process contains the spinor product $u_L u_L^\dagger$, where u is the charged lepton spinor. As required by angular momentum conservation, the only contribution to the decay comes from the 'wrong-helicity' contribution, for which the left-handed lepton is polarized in its direction of motion. By considering the explicit forms of the spinors in the above spinor product for the case that the lepton in emitted in the $+\hat{z}$ direction, show that this introduces a factor

$$f_{hel} = \frac{1}{2}(1 - \beta)$$

into the decay width, where β is the velocity of the charged lepton. Use this to estimate the $\pi^- \to e^- \overline{\nu}_e$ branching fraction. Compare this estimate to the measured value in the PDG booklet. To what do you attribute the discrepancy?

Problem 3

(Halzen and Martin, problem 12.12): Given the rate Γ for the decay $\mu^- \to e^-\overline{\nu}_e\nu_\mu$, predict the rate for the decay $\tau^- \to e^-\overline{\nu}_e\nu_\tau$, assuming a mass of 1.8 GeV/c^2 for the mass of the τ lepton. The observed branching fraction for this decay is approximately 20%. Can you explain this branching ratio? Predict the τ lifetime, compare it to the experimental value, and comment.

Problem 4

As we'll soon see, the interpretation of the weak force as a gauge theory introduces a third weak boson, the Z^0. Assume that this neutral boson couples to the Dirac current

$$J^0_\mu = \frac{g}{\sqrt{2}}(\overline{\psi}_1\frac{1}{2}\gamma_\mu(1-\gamma_5)\tau_3\psi_1 + \overline{\psi}_2\frac{1}{2}\gamma_\mu(1-\gamma_5)\tau_3\psi_2)$$

where $\psi_1 = (\psi_u, \psi_{d'})$, $\psi_2 = (\psi_c, \psi_{s'})$, and τ_3 is the third pauli spin matrix. (The physical Z^0 obtains an additional pure vector piece when electroweak unification is considered; that won't introduce a substantive change in this problem and we'll ignore it for now).

Draw the leading order Feynman diagram for the Z^0-mediated decay $K^0_L \to \mu^+\ \mu^-$. Show that for this form of the weak neutral current, the amplitude for this process is identically 0, even though both the s and d quarks are admixtures of the weak eigenstates s' and d'.

Problem 5

Draw the lowest order Feynman diagrams for the two following decay modes of the D^0 meson:

$$D^0 \to K^- + e^+ + \nu_e$$

$$D^0 \to \pi^- + e^+ + \nu_e$$

Estimate the ratio of branching fractions for these two modes, and compare to the measured value. How might you account for the discrepancy between your prediction and the measured values?