PHYSICS 221B – HOMEWORK SET 3

Due in class Tuesday, February 28, 2017.

Background: Halzen and Martin, Chapters 13 and 14; Burcham and Jobes, Chapter 13. In addition, for further reading, an excellent reference on Gauge Theory is Aitchison and Hey, *Gauge Theories in Particle Physics*, Adam Hilger, 1982.

Problem 1
Show that the requirement that the solutions $\psi(\vec{x},t)$ of the Klein Gordon Equation

$$d^\mu d_\mu \psi(\vec{x},t) + m^2\psi(\vec{x},t) = 0$$

be invariant under *local* phase transformations

$$\psi(\vec{x},t) \rightarrow exp[iq\phi(\vec{x},t)]\psi(\vec{x},t),$$

where $\phi(\vec{x},t)$ is any continuous function of spacetime, can be satisfied by introducing an interaction term identical in form to the minimal electromagnetic interaction, i.e., by imposing the requirement that

$$d^\mu \rightarrow D^\mu = d^\mu + iqA^\mu,$$

with A^μ transforming according to

$$A^\mu \rightarrow A^\mu - d^\mu\phi(\vec{x},t).$$

Problem 2
Consider the electroweak Hamiltonian

$$H_{EW} = i\overline{\chi}\gamma^\mu D_\mu \chi$$

generated by the $SU(2)\otimes U(1)$ covariant derivative

$$D_\mu = d_\mu + \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu + \frac{ig'}{2}YB_\mu.$$

Show that if the physical neutral fields A_μ and Z_μ are given by the linear combinations of neutral gauge fields

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W,$$

where

$$g\sin\theta_W = g'\cos\theta_W = e,$$

then the A_μ interaction is precisely that of QED. Specifically, show that A_μ couples to charged leptons with the appropriate strength, and conserves parity. Hint: Calculate the appropriate term in the electroweak interaction hamiltonian separately for the left- and right-handed charged lepton currents.

Problem 3
The 'left-right asymmetry' A_{LR} is defined as the asymmetry between the total Z^0 production cross section in e^+e^- annihilation for left- and right-handed electron beam (and unpolarized positrons):

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}.$$

Derive the Born-level relation between A_{LR} and the square of the weak mixing angle $\sin^2\theta_W$.

Problem 4
Consider the Born level process $e^+e^- \to Z^0 \to f\overline{f}$, in the case that neither beam is polarized. Let $z \equiv \cos\theta$, where θ is the angle between the incoming electron beam and outgoing fermion (or incoming positron beam and outgoing antifermion). Using angular momentum and parity violation arguments, show that the 'forward-backward asymmetry' for this fermion species, defined by the relation

$$A^f_{FB}(z) = \frac{\sigma^f(z) - \sigma^f(-z)}{\sigma^f(z) + \sigma^f(-z)}$$

has the form

$$A^f_{FB}(z) = A_e A_f \frac{2z}{1+z^2},$$

where A_f is the quantitative extent of parity violation in the Z^0-fermion coupling:

$$A_f = \frac{(g^f_L)^2 - (g^f_R)^2}{(g^f_L)^2 + (g^f_R)^2}.$$

Hint: Consider separately the angular distributions for the decay of $m_j = +1$ and $m_j = -1$ Z^0 bosons, and then combine them with the appropriate weights dicatated by parity violation in the Z^0-electron coupling.

Problem 5

Neutrino beams for DIS experiments are produced by allowing a roughly mono-energetic pion beam to decay, and then absorbing all but the neutrinos in an iron-impregnated earthen berm. What flavor of neutrinos are produced in this process? Show that the spectrum of neutrino energies produced is flat within kinematic limits. What are these limits for a 100 GeV pion beam? Roughly how long would a pure iron berm have to be in order to yield a pure neutrino beam at the far end? What would be the transverse dimension of the neutrino detector that resides in the experimental hall at the end of the berm?

Problem 6

Consider a neutrino beam that is pure ν_μ at $t = 0$. Show that the probability that any given neutrino will be detected as a ν_e is given by

$$P(\nu_\mu \rightarrow \nu_e) = \frac{1}{2}\sin^2 2\phi(1 - \cos\frac{m_2^2 - m_1^2}{2p}t)$$

where ϕ is the mixing angle between the week and mass (m_1, m_2) eigenstates, and $p >> m_1, m_2$ is the momentum of the neutrino beam.