PHYSICS 221B – HOMEWORK SET 4

Due in my office (slip it under the door) Friday March 24, 2017.

Background: Halzen and Martin, Chapter 7, and sections 14.1–14.4; Burcham and Jobes, sections 13.7–13.9.

Problem 1
Requiring that the free-particle Lagrangian be locally invariant under $SU(3)$ gauge transformations introduces an interaction term with gauge coupling strength g_s, yielding a Lagrangian of the form

$$L_{SU(3)} = \overline{\psi}(i\gamma^\mu d_\mu)\psi - g_s(\overline{\psi}\gamma^\mu T_a \psi)G^a_\mu,$$

where $T_a = \lambda_a/2$ is one-half the a^{th} $SU(3)$ generator, G^a_μ is the a^{th} gauge field introduced by the $SU(3)$ covariant derivative, and the index a runs from 1 to 8. Show that this Lagrangian is invariant under infinitessimal $SU(3)$ gauge transformations of the form

$$\psi \to (1 + i\alpha^a(x)T_a)\psi$$

provided that the gauge fields G^a_μ transform according to

$$G^a_\mu \to G^a_\mu - \frac{1}{g_s} d_\mu \alpha^a(x) - f^{abc}\alpha_b(x)G_{\mu,c}.$$

Problem 2
Consider the rates for the processes $g \to gg$ and $g \to q\overline{q}$. Show that for the first of these, the square of the $SU(3)$ gauge coupling g_s^2 is modified by a color factor

$$\delta_{mn}C_A = \sum_{j,k} f^{jkm} f^{jkn},$$

$j, k = 1, ..., 8$, where the f^{jkm} are the $SU(3)$ structure constants, while for the second of these, the appropriate color factor is

$$\delta_{kl}T_F = \sum_{a,b} T^k_{ab} T^l_{ba},$$

$a, b = 1, ..., 3$, where $T^k = \lambda^k/2$ is half the k^{th} $SU(3)$ generator in the fundamental representation.

Problem 3

Suppose you believed that the theory of the strong force should be constructed from a gauge theory based on the Lie group $SO(3)$, the group of rotations in three dimensions. Show that, in this case, all three of the qg (C_F), gg (C_A), and qq (T_F) color factors are equal to 2. (Recall that for $SU(3)$, C_F, C_A, and T_F are 4/3, 3, and 1/2, respectively).

Problem 4

In class, we listed the mixed gaugino/higgsino sparticle states as $\tilde{\chi}_1^{\pm}$, $\tilde{\chi}_2^{\pm}$, $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, and $\tilde{\chi}_4^0$ – a total of eight particle states. The corresponding gaugino and higgsino particle states, however, include the γ, Z^0, $W^{\pm}$, h^0, H^0, $H^{\pm}$ and A states – a total of nine particle states. Argue that, even though the number of physical particle states is different, the total number of field-theoretical degrees of freedom incorporated within the gaugino/higgsino particle states is, as required by supersymmetry, equal to that of the corresponding sparticle states.

Problem 5

In class, we mentioned the explicit mechanism for supersymmetry breaking known as gauge mediation. In Gauge-Mediated Supersymmetry (GMSB), the LSP is the essentially-massless gravitino, the spin-3/2 partner of the graviton. In most regions of GMSB parameter space, the next-to-LSP (NLSP) is the bino $\tilde{B}$, the SUSY partner of the SM U(1) gauge boson, which decays 100% of the time to a SM gauge boson and a gravitino. Write down all possible decay modes for this NLSP. What are the relative branching fractions for these modes in the limit that the bino mass goes to infinity? To zero?

Problem 6

Should SUSY be realized in nature, a process of great interest would be the study of stau production via electron-positron annihilation.

a) Draw all possible leading order disgrams for this process.

b) What is the angular distribution of the $\tilde{\tau}^-$ slepton in the electron-positron cms frame?

c) Using the Feynman rules for the couplings of spinor and scalar particles to intermediate vector bosons, make use of the known supersymmetric couplings to write down the Lorentz-invariant matrix element M_{fi} for the pair production of right-handed stau leptons via the annihilation of electrons and positrons for which the electron beam is purely left-handed.