## PHYSICS 221A FALL 2022 – FINAL PROJECT

Due at end of day Thursday, December 8, 2021. What you should turn in is the results of the various sections (confirming that you were successful in sections 1 and 2, and giving your answers for section three), a brief (1-2 paragraphs) description of your program, and anything else you mind find of relevance or interest. If you don't make it all the way through to the bitter end, then turn in a progress report of progress made. I hope everyone makes it through at least the first two sections.

For the following, you may use whatever programming language you would like.

## Project 1

Using the Inverse Transform Method discussed in class (see also section 33.2 of the Particle Data Book), develop an algorithm that generates mean-free-paths according to an exponential distribution with mean  $\lambda$ . Check the algorithm by counting the fraction of instances with mean-free-paths less than  $\lambda$  and comparing it with the analytical expectation.

Partial answer: The inverse transform function you should derive is  $-\lambda \log(1-u)$ .

## Project 2

Let y be the fractional energy carried off by the photon in an electron bremsstrahlung event. Approximate the y distribution in bremsstrahlung as

$$P(y) \propto 1 - y^{1/5}$$

(it's actually more like 1/y, but due to the pole at y=0, that distribution is somewhat more difficult to implement in a Monte Carlo simulation). Develop a Monte Carlo simulation of this energy distribution, and tune the effective mean-free-path  $\lambda_b$  of bremsstrahlung (to two significant figures) so that electrons lose all but 1/e of their energy after one radiation length  $X_0$ . [NOTE: I believe that you will need to use the Acceptance-Rejection technique to generate this distribution.]

Answer:  $\lambda_b = 0.27X_0$ .

## Project 3

Develop a longitudinal gamma-ray shower Monte Carlo (don't worry about the shower spread transverse to the direction of motion of the incoming gamma ray) using this model for bremsstrahlung. Use also the following assumptions:

- 1) The atmosphere is 28 radiation lengths deep, uniformly distributed over a vertical column of 20 km, and the gamma ray strikes it with normal incidence (purely vertically).
- 2) Conversions occur with a mean-free-path of 9/7  $X_0$ , with the electron (or positron) energy distributed uniformly between zero and the photon energy.
- 3) An electron or positron that falls below 100 MeV stops radiating, and loses its energy via ionization in exactly one radiation length. Don't model the ionization loss; just record the electron/positron and its one-radiation-length path length in your list of shower products. (Note that each positron will eventually find an electron to annihilate, but only after coming to rest).
- 4) A photon that is produced with an energy below 10 MeV doesn't convert, but instead immediately Compton scatters, producing an electron of the same energy that travels exactly one radiation length before coming to a stop. As for assumption 3), don't model the ionization process; just record the particle and its path in your list of shower products.

With this Monte Carlo in hand, estimate the average height above sea level of the shower max (height at which there are the most charged particles), the average number of charged particles at shower max, and the average number of charged particles reaching a mountain-top elevation of 4 km, for incoming gamma rays of energy 10, 30, 100, 300, and 1000 GeV.

Some answers I got (caveat emptor): 30 GeV (15 km,  $1.5 \times 10^2$ , 0.3); 1000 GeV (12.5 km,  $4.0 \times 10^3$ ,  $1.5 \times 10^2$ )