Nuclear and Particle Physics

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EXAMPLES 8

8.1 The time evolution of a state vector is formally written as $|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$. Using the time-dependent Schrödinger equation show that $U(t, t_0) = \exp[-iH(t - t_0)/\hbar]$.

8.2 Verify equation (8.5).

8.3 Show that $R(\epsilon) = 1 - i\epsilon L_z$, where L_z is the operator corresponding to the z component of angular momentum ($\hbar = 1$), generates an infinitesimal rotation ϵ about the z axis. A finite rotation θ can be generated by repeated application of $R(\epsilon)$ such that $R(\theta) = \text{Lt}(1 - i\epsilon L_z)^n$, where $n\epsilon \to \theta$ as $n \to \infty$ and $\epsilon \to 0$. Show that $R(\theta) = \exp(-i\theta L_z)$.

8.4 Using the angular momentum states $|j, m\rangle$ as a basis determine the matrix representations of the operators J^2, J_x, J_y and J_z for the cases $j = \frac{1}{2}$ and $j = 1$.

8.5 The orbital angular momentum l and the spin s of an electron in a hydrogen atom (say) can couple to give two values of the total angular momentum $j = l \pm \frac{1}{2}$. Thus, the P states can have $j = \frac{3}{2}$ (a quartet) or $j = \frac{1}{2}$ (a doublet). The state $|j, m\rangle = |\frac{3}{2}, \frac{3}{2}\rangle$ is uniquely defined in terms of the states

$|l, m_l\rangle|s, m_s\rangle$ as $|\frac{3}{2}, \frac{3}{2}\rangle = |1, 1\rangle|\frac{1}{2}, \frac{1}{2}\rangle$. Use the angular momentum lowering operators $J_- = L_- + S_-$ to obtain the other three states in the quartet. Check your answers by looking up the appropriate Clebsch–Gordan coefficients (appendix G).

8.6 The rotation matrices have elements

$$d^{(j)}_{mm'}(\beta) = \langle j, m| \exp(-i\beta J_y)|j, m'\rangle.$$

Calculate these elements for $j = \frac{1}{2}$.

8.7 The spherical harmonics $Y_l^m(\theta, \phi)$ are defined in terms of the associated Legendre polynomials $P_l^m(\cos\theta)$ through the equation

$$Y_l^m(\theta, \phi) = \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)\exp(im\phi).$$

Given that the associated Legendre polynomials are polynomials in even (odd) powers of $\cos\theta$ if $l - |m|$ is even (odd), show that the parities of the spherical harmonics are $(-1)^l$.

8.8 Given that the K and π mesons have spin 0 show that one of the weak decay processes $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\pi^+\pi^-$ must violate parity conservation.

8.9 The matrices

$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are, apart from the factor $\frac{1}{2}$, the Pauli spin matrices. Show by explicit matrix multiplication that they satisfy the usual commutation relations for angular momenta. Construct raising and lowering operators $\tau_\pm = \tau_1 \pm i\tau_2$ and show that they have appropriate properties by examining their effect on the I spin states $\binom{1}{0}$ and $\binom{0}{1}$, representing the proton and neutron respectively.

8.10 The $\Delta(1232)$ is a resonance with $I = \frac{3}{2}$. What is the predicted branching ratio for $(\Delta^0 \to p\pi^-)/(\Delta^0 \to n\pi^0)$? What would this ratio be for a resonance with $I = \frac{1}{2}$?

8.11 Express the ratio of the cross-sections for the reactions $K^-p \to \pi^-\Sigma^+$ and $K^-p \to \pi^+\Sigma^-$ in terms of the two possible I spin amplitudes.

8.12 Determine the ratio of the cross-sections for the reactions $\pi^-p \to \pi^-p$ and $\pi^-p \to \pi^0 n$ on the assumption that the two I spin amplitudes are equal in magnitude but differ in phase by 45°.

8.13 Show that the C-parity of a $\pi^+\pi^-$ system with relative orbital angular momentum l is $(-1)^l$. Use the conservation of angular momentum and parity to determine the possible angular momentum states involved in the annihilation process $p\bar{p} \to \pi^+\pi^-$. Express your answer in spectroscopic notation. Does charge conjugation invariance impose any further restrictions on the allowed angular momentum states?

8.14 The Maxwell equations are (i) $\nabla \cdot \boldsymbol{E} = \rho$, (ii) $\nabla \cdot \boldsymbol{B} = 0$, (iii) $\nabla \wedge \boldsymbol{E} + \partial \boldsymbol{B}/\partial t = 0$ and (iv) $\nabla \wedge \boldsymbol{B} - \partial \boldsymbol{E}/\partial t = \boldsymbol{j}$.

(a) Use them to derive the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = 0.$$

(b) Justify the introduction of *arbitrary* potentials $\boldsymbol{A}$ and ϕ such that

$$\boldsymbol{B} = \nabla \wedge \boldsymbol{A} \qquad \text{and} \qquad \boldsymbol{E} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t}$$

and show that $\boldsymbol{E}$ and $\boldsymbol{B}$ are invariant under the gauge transformation

$$\boldsymbol{A} \rightarrow \boldsymbol{A}' = \boldsymbol{A} + \nabla\psi \qquad \phi \rightarrow \phi' = \phi - \frac{\partial \psi}{\partial t}$$

where ψ is an arbitrary scalar function.

(c) Using the Lorentz gauge $\nabla \cdot \boldsymbol{A} = -\partial\phi/\partial t$, show that $\boldsymbol{A}$ and ϕ satisfy the wave equations

$$\nabla^2 \boldsymbol{A} - \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\boldsymbol{j}$$

$$\nabla^2 \phi - \frac{\partial^2 \phi}{\partial t^2} = -\rho.$$

You may find the identity

$$\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c} = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$$

which is valid for arbitrary vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$, helpful.

8.15 State which of the following processes are allowed and which are forbidden. If allowed, state which interaction is responsible; if forbidden give reasons.

(a) $\Xi^0 \rightarrow \Sigma^0 \gamma$
(b) $\mu^- + p \rightarrow \Lambda^0 + \nu_\mu$
(c) $\Sigma^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$
(d) $K^- + d \rightarrow \pi^+ + \Sigma^-$
(e) $K^+ \rightarrow \pi^0 e^+ \nu_e$.

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EXAMPLES 9

9.1 A resonance has a mass of 1232 MeV/c^2 and is produced in a πp formation experiment. Determine the kinetic energy of the incident pion (in the laboratory system) at which resonance occurs.

9.2 Calculate the minimum kinetic energy of a K^+ meson in the laboratory system required to produce a ϕ meson in the reaction $K^+p \to K^+p\phi$. Take the masses of the K^+, p and ϕ to be 0.493, 0.938 and 1.020 GeV/c^2 respectively.

9.3 Use the time-dependent Schrödinger equation for a free particle to derive the continuity equation $\partial\rho/\partial t + \nabla\cdot\boldsymbol{j} = 0$, where $\rho = \psi\psi^*$ is the probability density and $\boldsymbol{j} = (\hbar/2im)(\psi^*\nabla\psi - \psi\nabla\psi^*)$ is the probability current density. What is the probability current density for a plane wave?

9.4 Show that the energy distribution of protons projected from a thin hydrogenous target by a homogeneous beam of neutrons is uniform up to the maximum available energy T_0 provided that the interaction only involves S waves.

9.5 Calculate the S wave phase shift for the elastic scattering of a particle of mass m and kinetic energy T ($\ll mc^2$) scattered by a square potential well of depth U and radius R.

9.6 Consider the scattering of a spinless particle when no inelastic reactions are possible. Assuming that only S and P waves contribute to the scattering, write down an explicit expression for the scattering amplitude $f(\theta)$ in terms of the partial wave amplitudes T_0 and T_1. Hence derive the corresponding expression for the differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$. Express this in terms of the phase shifts δ_0 and δ_1 and discuss the possibility of determining δ_0 and δ_1 from measurements of $\mathrm{d}\sigma/\mathrm{d}\Omega$.

9.7 Use the properties of the Dirac δ function (appendix I) to show that the n-body density of states factors

$$\frac{\mathrm{d}}{\mathrm{d}E}\int\prod_{i=1}^{n-1}\mathrm{d}^3p_i \qquad \text{and} \qquad \int\prod_{i=1}^{n}\mathrm{d}^3p_i\,\delta\left(\sum_{i=1}^{n}\boldsymbol{p}_i-\boldsymbol{P}\right)\delta\left(\sum_{i=1}^{n}E_i-E\right)$$

are equivalent.

9.8 Show that the Lorentz invariant two-body phase space for particles with masses m_1 and m_2 and momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ in the centre-of-mass system is

$$R_2(E) = \frac{\pi p_1}{E}$$

where

$$p_1 = \frac{\{[E^2 - (m_2 - m_1)^2][E^2 - (m_2 + m_1)^2]\}^{1/2}}{2E}$$

9.9 Using equation (9.64) and the identity

$$\frac{dR_3}{dM_{12}} = \frac{dR_3}{dp_3}\frac{dp_3}{dM_{12}}$$

obtain an expression for the invariant mass distribution dR_3/dM_{12} in terms of M_{12}, E, m_1 and m_2 only. Write a computer program to evaluate dR_3/dM_{12} as a function of M_{12} for two pions produced in a $\pi\pi N$ final state in which the total centre-of-mass energy is 2.0 GeV.

9.10 (a) Show that a pseudoscalar meson ($J^P = 0^-$) cannot decay to two pseudoscalar mesons without violating parity conservation.
(b) Show that a scalar meson ($J^P = 0^+$) cannot decay via a parity-conserving strong interaction into three pseudoscalar mesons.

9.11 The ϕ meson has $I = 0$ and can decay to K^+K^-. Show that it has G-parity $(-1)^J$ where J is its spin. If the decay modes $\phi \to \pi^+\pi^-\pi^0$ and $\phi \to \pi^+\pi^-$ have branching fractions of 2.4×10^{-2} and 10^{-4} respectively show that the spin-parity must belong to the series 1^-, 3^- etc.

9.12 I spin conservation predicts that the ratio of the decay rates

$$R = \frac{\Gamma(\phi \to K^0\bar{K}^0)}{\Gamma(\phi \to K^+K^-)}$$

is unity, whereas the experimental ratio is $R_J \approx 0.7$. The difference can be accounted for by a centrifugal barrier factor such that

$$R_J = R\left(\frac{p_0}{p_+}\right)^{2J+1}$$

where $p_0(p_+)$ is the momentum of the neutral (charged) kaon in the ϕ rest system and J is the spin of the ϕ. Taking the masses of the ϕ, K^+ and K^0 to be 1020, 493.6 and 497.7 MeV/c^2 respectively show that $J = 1$ is preferred to $J = 3$.

9.13 In certain regions of phase space the reaction $\pi^-p \to \pi^-p\pi^+\pi^-$ proceeds via double resonance production, $\pi^-p \to \Delta^0(1232)\rho^0(770)$. The resonance decays are parity-conserving strong decays, $\Delta^0 \to p\pi^-$ and $\rho^0 \to \pi^+\pi^-$. Assuming that the production mechanism is single pion exchange determine the angular distributions of the decay products in the resonance rest systems. The spin-parities of the π, p, Δ and ρ are 0^-, $\frac{1}{2}^+$, $\frac{3}{2}^+$ and 1^- respectively.

9.14 Repeat the analysis for the production and decay of the $\Delta(1232)$ in example 9.13 assuming the Δ has $J^P = \frac{3}{2}^-$. What conclusions can be drawn concerning the possibility of determining the parity of a resonance by measuring the angular distribution of its decay products?

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EXAMPLES 10

10.1 A general SU(2) transformation, i.e. a rotation of the axes through an angle α in isospin space about the direction $\hat{\boldsymbol{n}}$, specified by the angles (θ, ϕ) in spherical polar coordinates, is given by

$$|\psi\rangle \rightarrow |\psi'\rangle = \exp\left(-\mathrm{i}\frac{\alpha}{2}\hat{\boldsymbol{n}}\cdot\boldsymbol{\tau}|\rangle\right)|\psi\rangle$$

where the components of $\boldsymbol{\tau}$ are the Pauli matrices. By expanding the exponential, collecting terms in even and odd powers of α and using the fact that $(\hat{\boldsymbol{n}}\cdot\boldsymbol{\tau})^2 = \mathbf{1}$, the unit matrix, show that

$$\exp\left(-\mathrm{i}\frac{\alpha}{2}\hat{\boldsymbol{n}}\cdot\boldsymbol{\tau}\right) = \mathbf{1}\cos\left(\frac{\alpha}{2}\right) - \mathrm{i}\,\hat{\boldsymbol{n}}\cdot\boldsymbol{\tau}\sin\left(\frac{\alpha}{2}\right).$$

Hence show that, under a rotation α about the 2 axis in isospin space, the nucleon doublet $|\psi\rangle \equiv \binom{p}{n}$ transforms as

$$p \to p' = p\cos\left(\frac{\alpha}{2}\right) - n\sin\left(\frac{\alpha}{2}\right)$$

$$n \to n' = p\sin\left(\frac{\alpha}{2}\right) + n\cos\left(\frac{\alpha}{2}\right).$$

Apply charge conjugation to these equations to show that the doublet $|\bar{\psi}\rangle \equiv \binom{\bar{n}}{\bar{p}}$ does *not* transform in the same way as the nucleon doublet (**2**). Show, however, that if we define the conjugate doublet $\bar{\mathbf{2}}$ as $|\bar{\psi}\rangle \equiv \binom{-\bar{n}}{\bar{p}}$, this does transform identically to the **2**.

10.2 Defining the conjugate antinucleon doublet as in example 10.1 has the advantage that the N and $\bar{\text{N}}$ doublets can be combined using conventional Clebsch–Gordan coefficients. By coupling the N and $\bar{\text{N}}$ doublets, and assuming the N$\bar{\text{N}}$ pairs are in singlet spin states with relative orbital angular momentum 0, determine the *G*-parity of the isospin triplet and singlet states. Write down the quantum numbers $I^G J^P$ of these states and identify them with physical particles.

10.3 Write down the matrix representations of the SU(3) operators $I_\pm$, I_3, $V_\pm$, $U_\pm$ and Y and verify the commutation relations given in equations (10.8). Hence, show that the step operators $U_\pm$ and $V_\pm$ induce the changes $\Delta Y = \pm 1$, $\Delta I_3 = \mp\frac{1}{2}$ and $\Delta Y = \pm 1$, $\Delta I_3 = \pm\frac{1}{2}$ respectively.

10.4 The fundamental quark triplet may be regarded as a V spin doublet $\binom{u}{s}$ and a V spin singlet d, or as a U spin doublet $\binom{d}{s}$ and a U spin singlet u. Show that $V_-|u\rangle = |s\rangle$ and $U_+|s\rangle = d$.

10.5 The direct product of two quark triplets gives rise to a sextet and an antitriplet (see figure 10.5). Determine the quark *wavefunctions* of these nine states.

10.6 The decomposition of the product of three quark triplets is $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3} = (\mathbf{6}\oplus\bar{\mathbf{3}})\otimes\mathbf{3} = \mathbf{10}_\text{S}\oplus\mathbf{8}_{\text{M}_\text{S}}\oplus\mathbf{8}_{\text{M}_\text{A}}\oplus\mathbf{1}_\text{A}$. Determine the normalized quark wavefunctions of (i) the *uud* and (ii) the *uds* states in the decuplet, the M_A octet and the M_S octet. (iii) What is the wavefunction of the unitary singlet state?

10.7 Show that the direct product of three spin doublets decomposes according to $\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2} = (\mathbf{1}_\text{A}\oplus\mathbf{3}_\text{S})\otimes\mathbf{2} = \mathbf{4}_\text{S} + \mathbf{2}_{\text{M}_\text{S}} + \mathbf{2}_{\text{M}_\text{A}}$ and write down explicit wavefunctions for the quartet and mixed-symmetry doublets.

10.8 Apply equation (10.24) to the U spin quartet of negatively charged $\frac{3}{2}^+$ baryons and show that

$$m_\Sigma - m_\Delta \approx m_\Xi - m_\Sigma \approx m_\Omega - m_\Xi.$$

10.9 The neutral members of the $J^P = \frac{1}{2}^+$ baryon octet form a U spin triplet, n, $\frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda^0)$, Ξ^0 with $U_3 = +1$, 0 and -1 respectively. Use equation

(10.24) to obtain the mass relation

$$\tfrac{1}{2}m_n + \tfrac{1}{2}m_{\Xi^0} = \tfrac{1}{4}m_{\Xi^0} + \tfrac{3}{4}m_{\Lambda^0}.$$

10.10 The negatively charged $\frac{3}{2}^+$ baryons Δ^-, $\Sigma^-(1385)$, $\Xi^-(1530)$ and Ω^- form a U-spin quartet, the neutral $\frac{1}{2}^+$ baryons n, Λ^0, Σ^0 and Ξ^0 a U-spin triplet (see example 10.9) and the negatively charged 0^- mesons π^- and K^- a U-spin doublet. Assuming U-spin conservation determine the amplitudes for the decays

$$\begin{array}{ll} \Delta^- \to n\pi^- & \Xi^-(1530) \to \Xi^0\pi^- \\ \Sigma^-(1385) \to nK^- & \Xi^-(1530) \to \Sigma^0K^- \\ \Sigma^-(1385) \to \Sigma^0\pi^- & \Xi^-(1530) \to \Lambda^0K^- \\ \Sigma^-(1385) \to \Lambda^0\pi^- & \Omega^- \to \Xi^0K^- \end{array}$$

10.11 Use the fact that the photon is a U-spin scalar to establish a relationship between the amplitudes for the electromagnetic decays $\pi^0 \to \gamma\gamma$ and $\eta^0 \to \gamma\gamma$.

10.12 Use U-spin conservation to obtain the amplitudes for the processes

$$\begin{array}{l} \pi^-p \to K^+\Sigma^-(1385) \\ \pi^-p \to \pi^+\Delta^- \\ K^-p \to K^+\Xi^-(1530) \\ K^-p \to \pi^+\Sigma^-(1385). \end{array}$$

10.13 Octet-singlet mixing in the pseudoscalar nonet can be expressed as

$$\begin{pmatrix} \eta' \\ \eta \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_8 \end{pmatrix}$$

where η' and η denote the physical states, η_1 and η_8 the singlet and octet states and R is the rotation matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

With respect to the η_1 and η_8 base states the mass matrix (see equation *10.30) is*

$$\begin{pmatrix} M_{11}^2 & M_{18}^2 \\ M_{18}^2 & M_{88}^2 \end{pmatrix}$$

By diagonalizing this matrix show that the mixing angle is given by

$$\tan^2\theta = \frac{M_{88}^2 - m_\eta^2}{m_{\eta'}^2 - M_{88}^2}.$$

10.14 Meson masses in the quark model are predicted to be the sum of constituent quark masses and a hyperfine interaction:

$$m(q_1\bar{q}_2) = m_1 + m_2 + a\frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{m_1 m_2}$$

where s_1 and s_2 are the spins of the constituent quarks with masses m_1 and m_2. Assuming $m_u = m_d = 0.310\ \mathrm{GeV}/c^2$, $m_s = 0.483\ \mathrm{GeV}/c^2$ and $a/m_u^2 = 0.64\ \mathrm{GeV}/c^2$ obtain the masses of the pseudoscalar and vector mesons and compare them with the measured values.

10.15 Obtain an explicit quark wavefunction for the Λ^0 with spin component $s_z = +\frac{1}{2}$. Hence, show that the quark model predicts that the magnetic moment of the Λ^0 is μ_s, the intrinsic magnetic moment of the s quark.

10.16 The J/ψ resonance is produced in an electron–positron collider in which the energy in each circulating beam is 1.5 GeV. The integrated cross-section for J/ψ production followed by decay into the e^+e^- channel may be written

$$\int \sigma(E)\, \mathrm{d}E = 4\pi ƛ^2 \frac{2J+1}{(2s_1+1)(2s_2+1)} \int_0^\infty \frac{\Gamma_e^2/4}{(E-E_R)^2 + \Gamma^2/4}\, \mathrm{d}E$$

where E is the centre-of-mass energy, $ƛ = \hbar/p$ where p is the centre-of-mass momentum, J is the spin of the resonance and s_1 and s_2 are the spins of the colliding particles. E_R is the energy at the resonance peak, Γ_e is the partial width for decay into the e^+e^- channel and Γ is the total width of the resonance. Assuming that the J/ψ can be produced in all possible helicity states show that

$$\int \sigma(E)\, \mathrm{d}E = \frac{3\pi^2}{2} ƛ^2 \left(\frac{\Gamma_e}{\Gamma}\right)^2 \Gamma.$$

(*Hint*: The integration is most easily performed by making the substitution $\tan\theta = 2(E - E_R)/\Gamma$.)

If the integrated cross-section is 870 nb MeV and the branching fraction Γ_e/Γ is 0.07, determine the total width.