PHYSICS 5A FALL 2001
FINAL EXAM

$v = \frac{\Delta x}{\Delta t}$

$a = \frac{\Delta v}{\Delta t}$

$x = \frac{1}{2}at^2 + v_0 t + x_0$

$v^2 = v_0^2 + 2a(x - x_0)$

$a = \frac{v^2}{r}$

$\vec{F}_{tot} = m\vec{a}$

$w = mg$

$f_{\rm k} = \mu_{\rm k}\eta$

$W = \vec{F} \cdot \vec{s}$

$KE = \frac{1}{2}mv^2$

$P = \vec{F} \cdot \vec{v}$

F = kx

$KE_1 + U_1 = KE_2 + U_2$

$\vec{p} = m\vec{v}$

$\vec{p}_{tot} = \sum_{i=1}^{n} m_i \vec{v}_i$

$\Sigma \vec{F}_{ext} = M\vec{a}_{cm}$

$\alpha = \Delta\omega/\Delta\phi$

$a_{tan} = r\alpha$

$I = \Sigma m_i r_i^2$

$I_p = I_{cm} + Md^2$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{L} = \vec{r} \times \vec{p}$

$\Delta\vec{L} = \Sigma\vec{\tau} \cdot \Delta t$

$\Delta PE_G = Gm_1m_2(\frac{1}{r_1} - \frac{1}{r_2})$

$\vec{v} = \frac{\Delta x}{\Delta t}\hat{x} + \frac{\Delta y}{\Delta t}\hat{y} + \frac{\Delta z}{\Delta t}\hat{z}$

$\vec{a} = \frac{\Delta v_x}{\Delta t}\hat{x} + \frac{\Delta v_y}{\Delta t}\hat{y} + \frac{\Delta v_z}{\Delta t}\hat{z}$

$v = v_0 + at$

$x - x_0 = \frac{1}{2}(v_0 + v)t$

$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$

$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$

$F = \frac{mv^2}{r}$

$f_{\rm s} \le \mu_{\rm s}\eta$

$W_{tot} = \Delta KE$

$P_{av} = \Delta W/\Delta t$

$W_{grav} = mgy_1 - mgy_2 = mg\Delta h = -\Delta U_{grav}$

$W_{elas} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = -\Delta U_{elas}$

$F(x) = -\Delta U/\Delta x$

$\vec{J} = \Delta\vec{p} = \Sigma\vec{F}\Delta t$

$\vec{r}_{cm} = (\Sigma m_i \vec{r}_i)/(\Sigma m_i)$

$\omega = \Delta\phi/\Delta t \qquad \omega = v/r$

$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$

$a_{rad} = v^2/r = \omega^2 r$

$K = \frac{1}{2}I\omega^2$

$\tau = I\alpha$

$\tau = rF\sin\theta_{rF}$

$\vec{L} = I\vec{\omega}$

$F = (Gm_1m_2)/(r^2)$

$G = 6.7 \times 10^{-11}$ N-m^2/kg^2

$PE_G = -(Gm_1m_2)/r$

$F = -kx$

$\omega = \sqrt{k/m}$

$x(t) = A\sin(\omega t + \phi)$

$\omega = (2\pi)/T = 2\pi f$

$\omega = \sqrt{g/L}$

Circle: $A = \pi r^2$; $C = 2\pi r$

Sphere: $A = 4\pi r^2$; $V = 4/3\pi r^3$

Cylinder: $A = 2\pi r^2 + 2\pi rh$; $V = \pi r^2 h$

$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$

$\vec{a}\cdot\vec{b} = ab\cos\theta_{ab}$

$|\vec{a}\times\vec{b}| = ab\sin\theta_{ab}$

$\sin 60^o = \cos 30^o = \sqrt{3}/2$

$\sin 45^o = \cos 45^o = 1/\sqrt{2}$

$\sin 30^o = \cos 60^o = 1/2$

$t = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

PROBLEM 1 [25 POINTS]

In what follows, you may assume that the acceleration due to gravity is exactly 10 m/s^2. You may also ignore the effects of friction. Remember to specify whether the work done is positive or negative.

An object of mass 20 kg is dropped (released at rest) from a tower of height 125 m, at time t = 0.

a) At what time does the object strike the ground, 125 m below?

b) With what speed does it strike the ground?

c) What is the total work done by gravity in this period of time?

Instead of being dropped from a tower of height 125 m, the object is thrown upward from the ground at time t = 0. The upward velocity of the object, immediately after it is thrown, is 50 m/s. The horizontal velocity, immediately after it is thrown, is also 50 m/s.

d) At what time does the object return to strike the ground? You may ignore the height of the person throwing the object.

e) With what speed does it strike the ground?

f) What is the total work done by gravity in this period of time?

PROBLEM 2 [20 POINTS]

In the following, assume that the acceleration due to gravity is 10 m/s^2.

A 30 kg board of length 10m rests on two blocks. One block is at the left end of the board, while the other is 4 meters from the right end of the board (see diagram). A 50 kg person steps onto the left end of the board.

a) With the person standing on the left end of the board, what is the magnitude of the force exerted on the board by the right-hand support?

b) If the person begins to walk slowly towards the right, how far does he get before the board tips?

PROBLEM 3 [20 POINTS]

A 10 kg mass lies at rest on the end of a relaxed massless spring with spring constant of k = 5 N/m (see diagram). A 2 kg blob of putty, moving at a speed of 3 m/s, hits and sticks to the 10 kg mass, causing the spring to compress. All effects of friction may be ignored.

a) What is the maximum amount of compression (in meters) of the spring?

b) How much time elapses between the time of impact of the putty and the time at which the spring is at its point of maximum compression?

PROBLEM 4 [25 POINTS]

Two asteroids, one of mass 5×10^{12} kg and the other of mass 2×10^{12} kilograms, are observed to be flying apart from each other (see diagram). The heavier asteroid, on the left, is traveling left at 1 m/s. The lighter asteroid is traveling right at 2 m/s. The asteroids are separated by 100 meters. The value of the gravitational constant is $G = 6.7 \times 10^{-11}$.

(a) How far apart do the asteroids get before being pulled back towards each other by their mutual gravitational attraction?

(b) At the point in time at which the two asteroids are at their maximum separation, a space-based lab station is half-way between the two asteroids. If the spacelab has a mass of 10^5 kg, what is the magnitude and direction of the gravitational force exerted by the asteroids on the spacelab? Assume that the lab is too light to affect the answer to part a).

PROBLEM 5 [25 POINTS]

A frictionless pulley of mass 12 kg and radius 0.5 meter has a massless string draped over it. On one end of the string is a 10kg mass, while on the other end is a 6kg mass (see diagram). Assume that the acceleration due to gravity is 10 m/s^2. The two masses are held at rest and then released without introducing any initial velocity.

a) How fast are the masses moving at the point at which the heavy mass has descended by exactly 2 meters?

b) Through what angle has the pulley turned 2 seconds after the masses are released?

PROBLEM 6 [20 POINTS]

A plywood disk of mass 7 kg and radius 2 m rotates on frictionless bearings about a vertical axis through its center. On the disk is a circular railroad track of radius 1.5 m, on which a battery-operated train of mass 1.2 kg runs. Initially, both the train and the disk are at rest. The train then accelerates for some amount of time, until its speed *relative to the tracks* (*not* relative to the ground) is 0.6 m/s. What is angular speed does the disk develop?

PROBLEM 7 [25 POINTS]

A small ball of mass 4 kg is suspended by two 2m-long strings which are separated by 2m at their point of attachment (see diagram I).

a) Find the tension in each string.

The (massless) pole from which the strings are suspended is now secured vertically to a device which can be used to rotate the pole about its axis (see diagram II).

b) What is the minimum amount of work that must be done on the pole (in getting it to rotate) so that the bottom string is fully extended (i.e., is tensioned by pull of the rotating ball)?