Problem 1

1.1 As we know from class, two identical waves adding together in phase <u>quadruples</u> the power, so (a) is wrong. Also, two waves ½ cycle out of phase will cancel, irrespective of which is ahead and which is behind in phase, so (b), (c) are wrong

$\boxed{\text{(d)}}$ is correct.

1.2 A difference of 10 dB is a factor of 10.

The difference between 5×10^{-5} and 2×10^{-5} is only a factor of 2½. Spherical wave intensity falls off with the square of distance, so doubling the distance reduces the intensity by a factor of 4. So $\boxed{\text{(a)}}$ is the most attenuation ($\times10$ rather than $\times2.5$ or $\times4$).

1.3 For (a) and (c), the fundamental wavelength is $2L$, while for (b) it's $4L$. Larger wavelength means lower frequency, so $\boxed{(b)}$ is correct.

1.4 When a train coming towards you, from N, S, E, W, or any compass point in between, blows its whistle, you will hear a higher frequency than if you were moving with the train. So, $\boxed{(a)}$ is correct.

1.5 For a standing wave, the wave doesn't travel; its amplitude changes as a function of time and that's all. We can write (b) as

$$\sin(k_1x)\cos(k_2x)\sin(w_1t) = A(t)\sin(k_1x)\cos(k_2x)$$

So this is a wave of the form $\sin(k_1x)\cos(k_2x)$ that is stationary, with an amplitude $A(t) = \sin(w_1t)$ that varies in time.

So, $\boxed{(b)}$ is the correct answer (this one was meant to be challenging).

Midterm II - Problem 2 Solution

A light beam of intensity 20 W/m^2 emerges from a polarizer with its polarization axis oriented at 90° (vertically). It then passes through a second polarizer with axis oriented at 60°, and then a third polarizer with an axis oriented at 30°.

(a) What is the intensity of the light emerging from the third polarizer?

(b) What is the attenuation, in dB, of the light emerging from the third polarizer relative to that emerging from the first polarizer?

Solution:

Initially, the beam of light emerges from the first polarizer with an intensity $I_1 = 20$ W/m^2 and its polarization aligned along the vertical axis. Armed with this knowledge, the intensity of light which emerges from the second polarizer is

$$I_2 = I_1 \cos^2 \theta_{21}. \tag{1}$$

Here, $\theta_{21} = 30°$ is the angle of the second polarizer relative to the polarization axis of the light beam after it emerges from the first polarizer.

The light that is incident on the third polarizer will be polarized along an axis oriented 60° from the horizontal with an intensity given by I_2 above. Since the third polarizer is oriented at 30° from the horizontal, its angle relative to the polarization axis of the light beam after it has emerged from the second polarizer is $\theta_{32} = 30°$. The intensity of the light which emerges from the third polarizer is then

$$\begin{aligned} I_3 &= I_2 \cos^2 \theta_{32} \\ &= I_1 \cos^2 \theta_{32} \cos^2 \theta_{21} \\ &= 20 \text{ W/m}^2 \cos^4 30° = 11.25 \text{ W/m}^2. \end{aligned} \tag{2}$$

Now that the intensity of the light emerging from the third polarizer is known, its attenuation relative to the attenuation of the light emerging from the first is

$$\begin{aligned} \beta &= 10 \log_{10} \left(\frac{\mathrm{I}_1}{\mathrm{I}_3} \right) \\ &= 10 \log_{10} \left(\frac{20 \text{ W/m}^2}{11.25 \text{ W/m}^2} \right) = 2.499 \text{ dB}. \end{aligned} \tag{3}$$

Physics 6B-Winter 2010 Mid Term 2 (28 Feb 2011, 2:00 p.m.)
SOLUTIONS

Q3

$$y(x,t) = 0.1\sin(\pi x - 15t)$$

a) The wave equation has a <u>negative</u> ωt term and hence is moving toward the right (positive x direction).

b)

$$General\ equation\ of\ a\ wave: y(x,t) = A\sin(kx - \omega t) \qquad where\ k = \frac{2\pi}{\lambda}\ and\ \omega = 2\pi f$$

$$Wave\ velocity\ v = f\lambda = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k} \qquad where\ f = wave\ frequency\ and\ \lambda = wavelength$$

Comparing the given wave equation and the general form, we have k = π and ω=15

$$\therefore v = \frac{\omega}{k} = \frac{15}{\pi} = 4.7746\ m/s$$

c)

$$P\ \alpha\ A^2 \rightarrow \frac{P}{50}\alpha\frac{A^2}{50} \rightarrow \frac{P}{50}\alpha\left(\frac{A}{\sqrt{50}}\right)^2, \quad Hence\ if\ P' = \frac{P}{50} \rightarrow A' = \frac{A}{\sqrt{50}} = \frac{0.1}{7.07} = 0.014$$

$$y'(x,t) = 0.014\sin(\pi x - 15t)$$

d) $Time\ dependent\ equation\ at\ x = 1m\!:\ y(t) = 0.1\sin(\pi.1 - 15t) = 0.1\ sin(\pi - 15t)$

e)
$At\ x = 1.5m\!:\quad y(x,t) = 0.1\sin(1.5\pi - 15t)$
$Since\ -1 \le sin\theta \le 1, \quad y(x,t) is\ maximum\ when \sin(1.5\pi - 15t) = 1$
$This\ ococurs\ when\ \theta = 90^\circ = \frac{\pi}{2} \quad \rightarrow 1.5\pi - 15t = \frac{\pi}{2} \rightarrow \quad 15t = \pi \quad \rightarrow \quad t = \frac{\pi}{15} = 0.2094\ s$

NOTE: do not mix angles in degrees and radians in calculations. Always covert all angles to 1 standard.

f)
A 2nd waves with the <u>same amplitude</u>, <u>wavelength</u> and <u>frequency</u> as the original is to be introduced on to the string to achieve complete destructive interference (string is perfectly straight **<u>at all times</u>**)

Same amplitude: A_2=A_1=0.1; same wave length: λ_2= λ_1 and since $k = \frac{2\pi}{\lambda} \rightarrow k_2 = k_1 = \pi$
$Same\ frequency\!: f_2 = f_2\ \ and\ since\ \omega = 2\pi f \ \rightarrow \omega_2 = \omega_1 = 15$

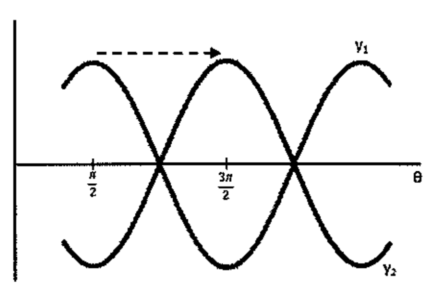
Destructive interference is achieved by having the 2nd wave phase shifted by $180°$ *or* π *(or any odd multiple of* π*)*

$2nd\ wave\!: A_2 sin(k_2 x - \omega_2 t \pm n\pi)\ \ where\ n = any\ \boldsymbol{ODD}\ integer$

$y_2(x,t) = 0.1\ sin(\pi x - \omega t \pm n\pi)$

$or\ \ y_2(x,t) = -0.1\ sin(\pi x - \omega t)\ \ because \sin(\theta \pm \pi) = -\sin(\theta)$

Note: $y_2(x,t) = 0.1\ sin(\pi x + \omega t)$ gives a standing wave that that yields nodes and anti-nodes and is **<u>not zero at all times</u>**.

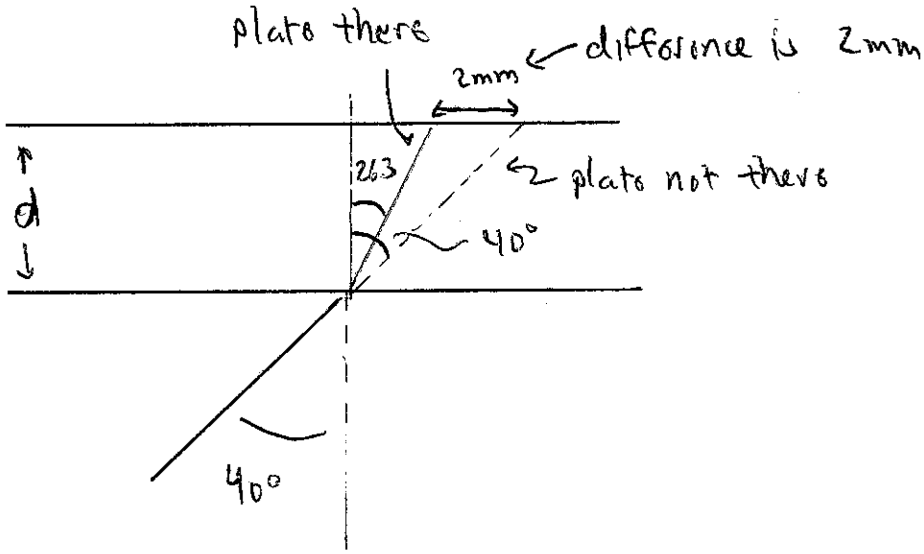


Problem 4

a) This is a direct application of Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ with $n_1 = 1$ $\theta_1 = 40°$ $n_2 = 1.45$, so

$$\sin\theta_2 = \frac{1}{1.45}\sin 40° = \boxed{26.3°}$$

b)

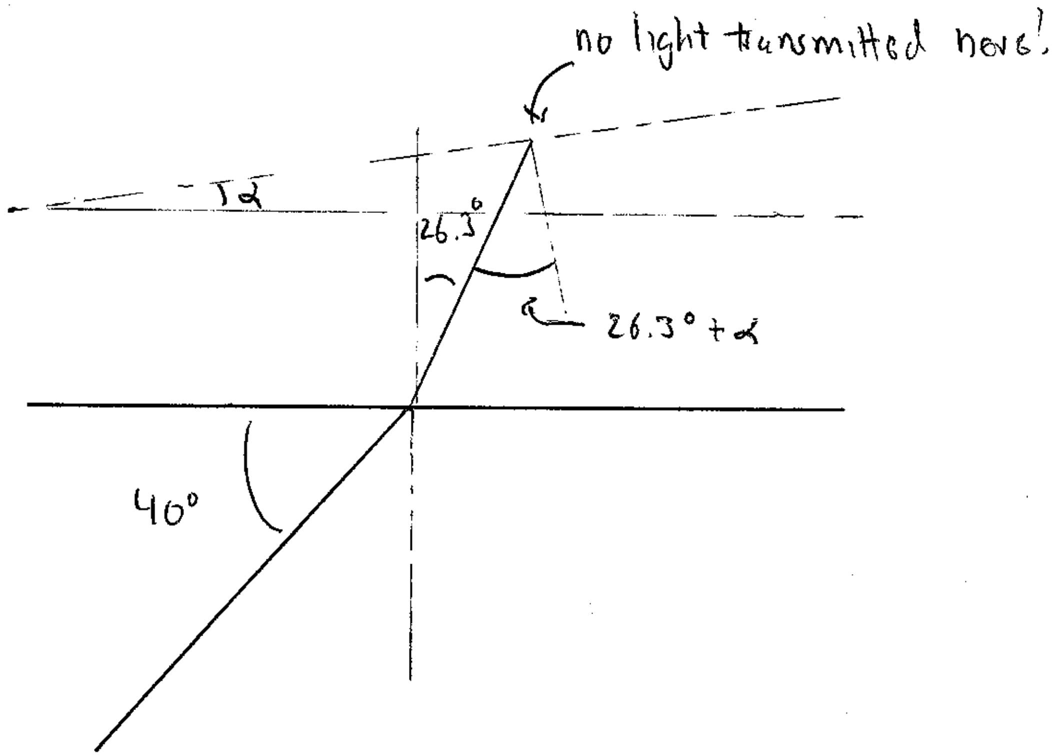


From diagram,

$$d\tan 40° - d\tan 26.3° = 2\,\text{mm}$$

$$d = \frac{2\,\text{mm}}{\tan 40° - \tan 26.3°} = \boxed{5.80\,\text{mm}}$$

c)



For no light to be transmitted, $26.3° + \alpha = \theta_c$

where θ_c is the critical angle, given by

$$\sin\theta_c = \frac{1}{1.45} \Rightarrow \theta_c = 43.6°$$

Thus, $\boxed{\alpha = \theta_c - 26.3° = 17.3°}$