

Assignment 1 SOLUTIONS

1. 18k Gold

Part A

The overall density is the total mass divided by the total volume, so let's think about the volume for 1 gram of 18k gold, which I'll call $V_{1g\ 18k}$.

The volume of 1 gm of 18k gold is equal to the sum of the volume of each component of the mixture (I'll call them V_g, V_s, V_c for gold, silver and copper respectively). You don't know those component volumes immediately, but you know their relative masses and their densities.

So looking at 1 gram of 18k gold...

$$\rho_{18} = \frac{1}{V_{1g\ 18k}} = \frac{1}{V_g + V_s + V_c}$$

$$\rho_g V_g = 0.75\text{ g} \implies V_g = 0.75/19.3 = 0.0389\text{ cm}^3$$

$$\rho_s V_s = 0.16\text{ g} \implies V_s = 0.16/10.5 = 0.0152\text{ cm}^3$$

$$\rho_c V_c = 0.09\text{ g} \implies V_c = 0.09/8.90 = 0.0101\text{ cm}^3$$

$$\rho_{18}(V_g + V_s + V_c) = \rho_g V_g + \rho_s V_s + \rho_c V_c = 1$$

$$\rho_{18}(0.0389 + 0.0152 + 0.0101) = 1$$

$$\rho_{18} = 15.57\text{ g cm}^{-3}$$

Part B

Same basic idea – you find the volume of each component of the mixture, except this time you use that information to find the "percent by volume" (what you're given with to start is the "percent by mass").

Looking at 1 gram of 14k gold...

$$\rho_g V_g = 0.585 \implies V_g = 0.585/19.3 = 0.0303\text{ cm}^3$$

$$\rho_s V_s = 0.04 \implies V_s = 0.04/10.5 = 0.0038\text{ cm}^3$$

$$\rho_c V_c = 0.312 \implies V_c = 0.312/8.90 = 0.0351\text{ cm}^3$$

$$\rho_z V_z = 0.063 \implies V_z = 0.063/7.14 = 0.0088\text{ cm}^3$$

$$V_{\text{total}} = V_g + V_s + V_c + V_z = 0.0780$$

Converting to percentages...

$$\text{Gold: } 38.8\%$$

$$\text{Silver: } 4.87\%$$

$$\text{Copper: } 45.0\%$$

$$\text{Zinc: } 11.3\%$$

2. U-Tube

Part A

The right cap is 4 times larger than the left cap. Although one inch of sand on the right cap is much heavier than one inch of sand on the left cap (4 times as heavy), the *pressures* exerted by the sand are the same on both caps, since pressure is force per unit area. Since the pressures exerted by the sand are equal, the pressures at the base of each arm due to the water must be equal. This requires equal heights of water in the two arms.

Part B

The same mass (1 kg) is on each arm, so the force on each arm is the same, but since the area of the right arm is larger, that force results in lower pressure than in the left arm. Since all parts of the liquid at the same depth are at the same pressure, the left cap must be lower and the right cap higher after the weights are added.

Part C

To bring them back to equal height, you need to equalize the pressure on the two arms. The weight on the larger arm must be $4\times$ the weight on the smaller arm, because the larger arm has $4\times$ the area of the smaller arm ($A = \pi r^2$). So since there's 1 kg on the smaller arm, there needs to be 4 kg on the larger arm.

Part D

Imagine an imaginary platform at the dotted line on right (large) side. Since its at the same height as the small platform the pressure must be the same at the dotted line as at the base of the small platform. Since the imaginary large platform has $4\times$ the area of the smaller platform, it is supporting $4\times$ the weight (and $4\times$ the mass). The total mass above it is 4 kg: the 1 kg mass and 3 kg of water.

3. Problem 15.27

Since the problem asks for *gauge pressure* that means the pressure over and above atmospheric pressure. Since the tube is open at the top, you know that the pressure at the top of the liquid is atmospheric pressure, so the gauge pressure there is 0.

You can then work your way down the tube, using $p = p_0 + g\rho h$ (where h is the depth below the surface) using $p_0 = 0$ Pa.

To get an answer in Pascals (N/m^2) you need to be a little careful about units, and work consistently in MKS.

$$\begin{aligned} p_{\text{bottom}} &= 0 + g\rho_{\text{oil}}h_{\text{oil}} + g\rho_{\text{water}}h_{\text{water}} \\ &= 0 + (9.8)(1000 \times 0.82)(5.2/100) + (9.8)(1000 \times 1.0)(5.2/100) \\ &= 927.5 \approx 930 \text{ Pa} \end{aligned}$$

4. Water Tank on Mars

Note: Different versions of the problem have different specific numbers: Here the height of the water in the tank is $h = 14.3\text{m}$, the atmospheric pressure in the tank is $p_0 = 110\text{kPa}$, the atmospheric pressure outside the tank is 89kPa , the area of the tank floor is 2.5m^2 , and the acceleration due to gravity is $g_m = 3.71\text{N kg}^{-1}\text{m}^{-1}$

Again, the pressure at the bottom of the tank is given by $p = p_0 + g_m\rho h$. Here we know that the pressure at the surface of the water is $p_0 = 110\text{kPa}$, and $g = 3.71\text{N kg}^{-1}\text{m}^{-1}$, and the density of water is 1000kg/m^3 just as on Earth.

$$\begin{aligned} p &= p_0 + g_m\rho h \\ &= 110,000 + 3.71(1000)(14.3) = 163,053\text{Pa} \end{aligned}$$

This is the downward pressure on the bottom of the tank, and, since the area of the bottom of the tank is 2.5m^2 it exerts a downward force of

$$F_{\text{down}} = p \times A = 163,053 \times 2.5 = 407,633\text{ N}.$$

But the air outside the tank is exerting an upward force...

$$F_{\text{up}} = p_{\text{rmair}} \times A = 89,000 \times 2.5 = 222,500\text{N}$$

The net force is the difference between these two forces, which (to 3 sig fig) is $185,000\text{N}$ downwards.

5. Problem 15.47

Again, exact variable values may vary...

Note: Since the fluid in this example is air, the pressure change due to changes in height is negligible ($g\rho \sim 9.8\text{N/m}^3 \sim 9.8\text{Pa/m}$, so changes in air pressure over even 10 meters would tiny compared to 500kPa .) This means you don't have to worry about whether the ends of the hydraulic line are at the same height.

This means that the pressure at the lift is the same as the pressure at the other end, *i.e.*, 500kPa , and it exerts an upward force of

Pressure at one end is equal to the pressure at the other, and the force it exerts on the 44cm platform is

$$F = p \times A = 500 \times 10^3 \times \pi(0.22^2) = 76,027\text{N}$$

The mass it can support is given by $F = mg \implies m = 7758\text{kg}$.

(I would quibble with the request to give your answer to 4 sig fig without specifying a value of g more precisely than $g = 9.8\text{N kg}^{-1}\text{m}^{-1}$).

6. Archimedes

Part A

The magnitude of the buoyant force is equal to the weight of fluid displaced by the object. This is true for every object submerged partially or completely in a fluid. The pressure from the surrounding fluid was just enough to support the weight of the water before it was displaced, this pressure produces the buoyancy force. If the buoyancy force is equal to or larger than the gravitational force on the object, the object floats; if not it sinks. But either way the buoyancy force is equal to the weight of the displaced fluid.

Part B

The magnitude of the buoyant force is equal to the weight of the amount of fluid that has the same total volume as the object.

This is true for an object completely submerged in a fluid. For a floating object the buoyant force equals the weight of the fluid displaced, and the volume of fluid displaced is smaller than the volume of the object.

Part C

The magnitude of the buoyant force equals the weight of the object.

This is true for an object that floats. For an object that sinks, the buoyant force is smaller than the weight of the object, and can't support it.

Part D

The magnitude of the buoyant force is less than the weight of the object. This is true for an object that sinks. The reason the object sinks is that the buoyant force cannot support its weight.

Part E

The object would float submerged less deeply than in the first container because in a denser liquid, a smaller volume of liquid needs to be displaced to give a buoyant force equal to the object's weight.

Part F

More than one of these outcomes is possible. In this case *more* liquid needs to be displaced to give a buoyant force equal to the object's weight. If that greater amount is still less than the object's volume, the object floats. If it's more than the object's volume, even when the object is fully submerged the buoyant force won't be large enough to support the object.

Part G

Both objects float, but B displaces more liquid, so its weight must be greater; since g doesn't vary, its mass also must be greater. Since both objects have the same size and shape, the one with more mass (B) must be the more dense.

7. Buoyant Force Conceptual Question

Part A

The buoyant force on the block is W ; since the block is floating in equilibrium, the buoyant force is equal to the block's weight.

Part B

The density of the block is 0.5 g/cm^3 . The weight of the water displaced is equal to the weight of the block, but the block has twice the volume, so it must have half the density of water.

Part C

The mass of the block is 20gm. When the block is fully submerged it's displacing twice its weight of water, and that amount is equal to the block plus the weights:

$$\begin{aligned}\text{Mass of block} + 20 \text{ gram weights} &= \text{Weight of water displaced} = 2(\text{Mass of block}) \\ M_{\text{block}} + 20 \text{ g} &= 2M_{\text{block}} \\ 20 \text{ g} &= M_{\text{block}}\end{aligned}$$

Part D

The liquid is less dense. A larger volume of liquid has to be displaced for weight of the displaced liquid to equal the weight of the block.

Part E

The buoyancy forces in the two cases have to be equal, so the mass of the displaced fluid in the two cases has to be equal.

$$\begin{aligned}\rho_{\text{unknown}} \times \frac{1}{3}V_{\text{block}} &= \rho_{\text{water}} \times \frac{1}{2}V_{\text{block}} \\ \rho_{\text{unknown}} \times \frac{1}{3} &= 1 \times \frac{1}{2} \\ \rho_{\text{unknown}} &= \frac{3}{2}\end{aligned}$$

8. Submerged block

We know that the total weight of the displaced liquid has to equal the weight of the cube. Part of the displaced liquid is oil, part is water.

$$\begin{aligned}\text{Weight of displaced liquid} &= \text{Weight of cube} \\ \text{Weight of displaced oil} + \text{Weight of displaced water} &= \text{Weight of cube} \\ g\rho_2 L^2 d + g\rho_3 L^2 (L - d) &= g\rho_1 L^3 \\ \rho_2 d + \rho_3 (L - d) &= \rho_1 L \\ d(\rho_2 - \rho_3) &= (\rho_1 - \rho_3)L \\ d &= \frac{(\rho_1 - \rho_3)}{(\rho_2 - \rho_3)}L\end{aligned}$$

9. Problem 15.50

Volume of water displaced when empty beaker floats...

$$V_{\text{empty}} = \pi r^2 h = \pi (2^2) \left(\frac{10}{3} \right) = 41.89 \text{ cm}^3$$

The weight of this displaced water is equal to the weight of the beaker, so the mass of the beaker is 41.89 g. The beaker will be fully submerged when it displaces 3 times that volume

or 125.66 cm^3 , so you could put up to 83.77 g in the beaker; that allows for five 15 g balls but not six.

10. Submerged ball

Tension + buoyancy = weight of ball. Buoyancy is $g\rho_f V$, so...

$$T = g(m_b - \rho_f V)$$

11. Bernoulli's Equation

Part A

The pressure at point 2 is lower than the pressure at point 1. The tube is narrower at point 2, so the flow speed there is faster. Since the tube is horizontal, pressure depends only on flow speed, and Bernoulli's equation shows that the faster the flow speed, the lower the pressure.

Part B

Physically, the pressure drop experienced by the fluid element between points 1 and 2 acts on the fluid element as a net force that causes the fluid to increase in speed.

Part C

The pressure drop is larger than the pressure drop occurring in a purely horizontal flow. Both the increase in height and the narrowing of the tube contribute to the pressure drop.

Part D

A greater amount of work is needed to balance the increase in potential energy from the elevation change. Pressure drops occur where work is being done.

12. Blood Pressure

Part A

The flow velocity as it passes through the blockage is $5v_0$, since the passage is $1/5$ its original size.

Part B

Kinetic energy is proportional to v^2 so when v increases by a factor of 5, kinetic energy increases by a factor of 25.

Part C

Neglecting any changes in height, we know

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \quad .$$

We're looking for the pressure difference $p_2 - p_1$:

$$\begin{aligned} p_2 - p_1 &= \frac{1}{2}\rho(v_1^2 - v_2^2) \\ &= \frac{1}{2}(1050)([0.14]^2 - [5 \times 0.14]^2) \\ &= -247 \text{ Pa} \\ &= -240 \text{ Pa} \quad (\text{at level of approximation as used in the intro to the problem}). \end{aligned}$$

Parts D,E

With a 90% blockage, the speed increases by a factor of 10, and the kinetic energy increases by a factor of 100.

Part F

This is similar to C, but we'll make the $\frac{1}{2}\rho v_1^2 \approx 10\text{Pa}$ approximation early on:

$$\begin{aligned} p_2 - p_1 &= \frac{1}{2}\rho(v_1^2 - v_2^2) \\ &= \frac{1}{2}\rho(v_1^2 - [10v_1]^2) \\ &= -99 \times \frac{1}{2}\rho(v_1^2) \\ &= -990\text{Pa} \end{aligned}$$

Water Flowing from a Tank

I did this problem first without making the "large tank, small hole" approximation...

Part A

The quantity dV/dt takes some thinking about, but you can see it both as the volume of water flowing out of the spout, and as the surface of the water in the tank dropping.

$$\frac{dV}{dt} = A_1 v_1 = A_3 v_3$$

Then look at Bernolli's equation and start putting in things you know...

$$\begin{aligned} p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 &= p_3 + \rho gh_3 + \frac{1}{2}\rho v_3^2 \\ p_1 = p_3 = p_{\text{atm}} \implies \rho gh_1 + \frac{1}{2}\rho v_1^2 &= \rho gh_3 + \frac{1}{2}\rho v_3^2 \\ v_1 = \frac{A_3}{A_1} v_3 \implies \rho gh_1 + \frac{1}{2}\rho \frac{A_3^2}{A_1^2} v_3^2 &= \rho gh_3 + \frac{1}{2}\rho v_3^2 \\ 2gh_1 + \frac{A_3^2}{A_1^2} v_3^2 &= 2gh_3 + v_3^2 \\ 2g(h_1 - h_3) &= v_3^2 \left(1 - \frac{A_3^2}{A_1^2}\right) \end{aligned}$$

When you make the "large tank, small hole" approximation ($A_3/A_1 \approx 0$) this becomes...

$$\begin{aligned} 2g(h_1 - h_3) &= v_3^2 \\ 156.8 &= v_3^2 \\ v_3 &= 12.52\text{ m/sec} \\ \frac{dV}{dt} = 0.016v_3 &= 0.2\text{ m}^3/\text{sec} \end{aligned}$$

If you assume that v_1 is not zero the answer is larger by $(9/8)^{1/2}$

Part B

Bernoulli's equation relates the pressure, flow velocity and height at different points in the fluid:

$$p + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

We know add these quantities at point 1: the pressure there is equal to atmospheric pressure, so since we're looking for gauge pressure we can say $p_1 = 0$. We'll continue to assume the flow velocity there is zero, so $v_1 = 0$.

Point 2 is 8m lower, and the flow velocity is non-zero (though not as large as at point 3). Before we can use Bernoulli's equation, we need to find v_2 , which we can do because we know how its related to dV/dt and v_3 which we found in Part A ($v_3 = 12.52 \text{ m/sec}$):

$$0.048v_2 = 0.016v_3 \implies v_2 = 12.52/3 = 4.17$$

This gives:

$$\begin{aligned} p_1 + \rho gy_1 &= p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2 \\ 0 + \rho g(10) &= p_2 + \rho g(2) + \frac{1}{2}\rho v_2^2 \\ 8g\rho &= p_2 + \frac{1}{2}\rho v_2^2 \\ 8(9.8)(1000) - 0.5(1000)(4.17)^2 &= p_2 \\ 69,705 &= p_2 \end{aligned}$$

14. Problem 15.56

Part A.

The small tube is open to the atmosphere, so the fluid at the top of the tube must be at atmospheric pressure. In a static system where no fluid is moving, pressure depends only on height, so the fluid in the small tube must reach the same height as the fluid in the other open tube: $h_2 = h_1$,

Part B.

First we will find the pressure (call it p_2) at the bottom of the small tube ($h = 0$). Once we know this we, we can find the h_2 by knowing that $\rho gh_2 = p_2 - p_{\text{atm}}$

To find p_2 , compare the large upper tube at h_1 to the smaller tube at $h = 0$, and use the fact that we know the speed of the fluid in the lower part of the tube must be twice the speed of the fluid in the large part, because the lower tube has half the cross-sectional area of the

upper tube.

$$p_{\text{atm}} + g\rho h_1 + \frac{1}{2}\rho v^2 = p_2 + \frac{1}{2}\rho(2v)^2$$

$$g\rho h_1 + \frac{1}{2}\rho v^2 - \frac{1}{2}\rho(2v)^2 = p_2 - p_{\text{atm}}$$

$$g\rho h_1 + \frac{1}{2}\rho(v^2 - 4v_1^2) = p_2 - p_{\text{atm}}$$

$$g\rho h_1 - \frac{3}{2}\rho v^2 = p_2 - p_{\text{atm}}$$

$$g\rho h_1 - \frac{3}{2}\rho v^2 = g\rho h_2$$

$$gh_1 - gh_2 = \frac{3}{2}v^2$$

$$h_2 - h_1 = -\frac{3}{2g}v^2$$

$$h_2 = h_1 - \frac{3}{2g}v^2$$