

Physics 6B-Winter 2010 Assignment 3
SOLUTIONS

Ideal Gas Law

Part A

The ideal gas law $pV = nRT$ gives the relationship between the pressure and volume of a gas to its temperature and mass of an ideal gas.

$p = \text{pressure}, V = \text{volume}, T = \text{temperature in Kelvin} \text{ \& } n = \text{no. of moles (mass)}$

$R = \text{universal gas constant} = 8.3145 \text{ J/(mol.K)}$

We can rearrange the equation to give $\frac{pV}{nT} = R$

Since R is a constant, then $\frac{p_1 V_1}{n_1 T_1} = \frac{p_2 V_2}{n_2 T_2}$. We can now relate changes in the same gas to each other

In this case parameters unchanged: Volume, **V**, (rigid container), Number of moles, **n**, (enclosed gas)
Temperature has clearly risen so the unknown parameter here is pressure.

$$\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2} \rightarrow p_2 = \frac{T_2}{T_1} p_1 \rightarrow p_2 = \frac{40 + 273}{20 + 273} p_1$$

Part B

Pressure = 1 atm, Temperature = 20 °C = 20 + 273 °K, Volume = 22.4 L

$R = 0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1}$

$$n = \frac{pV}{RT} = \frac{1 \times 22.4}{0.08206(20 + 273)} = 0.932 \text{ mol}$$

Part C

Similar to Part A. Here, we are asked to assume no gas escapes and the temperature remains constant. We are also given the initial and final pressure as well as the initial temperature

$n_1 = n_2$ and $T_1 = T_2$, $P_1 = 1.5 \times 10^6 \text{ Pa}$, $P_2 = 0.95 \times 10^6 \text{ Pa}$, $V_1 = 0.025 \text{ m}^3$

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} \rightarrow P_1 V_1 = P_2 V_2 \rightarrow V_2 = \frac{P_1}{P_2} V_1 = \frac{1.5}{0.95} (0.025 \text{ m}^3) = 3.95 \times 10^{-2} \text{ m}^3$$

Part D

Similar to Parts A and C. Volume and number of moles are held constant

$$\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2} \rightarrow T_2 = \frac{P_2}{P_1} T_1 \rightarrow T_2 = \frac{0.95}{1.5} (200 + 273) = 299.6 \text{ } ^\circ\text{K} = 26.6 \text{ } ^\circ\text{C}$$

Dust Equipartitions

Part A

Given: $k_B = 1.38 \times 10^{-23} \text{ J/K}$, $T = 290 \text{ K}$

The Equipartition Theorem states that, for a gas at temperature T : $\frac{1}{2}m\langle v_x^2 \rangle = \frac{1}{2}m\langle v_y^2 \rangle = \frac{1}{2}m\langle v_z^2 \rangle = \frac{1}{2}k_B T$

Also we have the average velocity squared of the dust particles: $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

Thus the mean translational kinetic energy,

$$K = \frac{1}{2}m\langle v^2 \rangle = \frac{1}{2}m \left(\frac{3k_B T}{m} \right) = \frac{3k_B T}{2} = \frac{3(1.38 \times 10^{-23})(290)}{2} = 6 \times 10^{-21} \text{ J}$$

Part B

The root-mean-squared velocity of a particle is the square root of the average velocity squared of a particle.

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_B T}{m}}$$

The mass is not given directly but can be expressed in terms of density (ρ) and volume of the dust particle.

Assuming each particle to be spheres we have: $m = \rho \left(\frac{4}{3}\pi r^3 \right) = \frac{4}{3}\rho\pi \left(\frac{d}{2} \right)^3$

Substituting this into the rms velocity equation above give:

$$v_{rms} = \sqrt{\frac{3k_B T}{\frac{4}{3}\rho\pi \left(\frac{d}{2} \right)^3}}$$

Part C

Given: $d=5\mu\text{m}$, $\rho=2000 \text{ kg/m}^3$, mass (m) = $1.31 \times 10^{-12} \text{ kg}$

Substitute into the either v_{rms} equation above. Only mass or d and ρ are required

$$\text{Using mass (m): } v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(290)}{1.31 \times 10^{-12}}} = 3 \times 10^{-4} \text{ m/s} = 0.3 \text{ mm/s}$$

Problem 17.41

This problem requires the use of the ideal gas law equation $PV = nRT$ and the relationship $\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$

Part A

The flask is closed (n remains constant) then immersed in boiling water (100°C). We assume here that the flask cannot expand and hence the volume is constant.

$$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow P_2 = \frac{T_2}{T_1} P_1$$

Given: $P_1 = 1 \text{ atm}$, $T_1 = 20^\circ\text{C} = 293 \text{ K}$, $T_2 = 100^\circ\text{C} = 373 \text{ K}$

Note: above equation uses temperature in Kelvin

$$P_2 = \frac{373}{293} (1 \text{ atm}) = 1.27 \text{ atm}$$

Part B

At thermodynamic equilibrium with the water, the flask is opened. When air is allowed to escape, the number of molecules inside has now changed. Also, the pressure inside the flask equalizes with the atmospheric pressure. The volume (v) remains unchanged from closed to open state.

$$\text{Using } n = \frac{PV}{RT}, \text{ we have no. of moles escape} = n_{\text{initial}} - n_{\text{open}} = \frac{P_1 V_1}{RT_1} - \frac{P_1 V_1}{RT_1}$$

$$\text{Initial moles in the enclosed flask are the same before and after heating: } n_{\text{initial}} = \frac{(1 \text{ atm})(4\text{L})}{(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})(293 \text{ K})}$$

$$\therefore \text{no. moles escape} = \frac{(1 \text{ atm})(4\text{L})}{(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})(293 \text{ K})} - \frac{(1 \text{ atm})(4\text{L})}{(0.08206 \text{ L atm K}^{-1} \text{ mol}^{-1})(373 \text{ K})} = 0.0357$$

Part C

After allowing the pressures to equalize with atmospheric pressure, the flask is then closed and cooled back to 20 °C. Thus $P_1 = 1 \text{ atm}$ and the number of moles in the container remains unchanged during cooling.

$$\text{Using } \frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} \text{ together with } n_1 = n_2 \text{ and } V_1 = V_2$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{293}{373} (1 \text{ atm}) = 0.786 \text{ atm}$$

Problem 17.48

Part A

Compare the total heat required to raise the temperature of 50g of -10 °C ice to its melting temperature (0 °C) and melt it with the amount of heat taken away from 1kg of water to cool it from 15 °C to 0 °C

$$\text{Specific heat capacity: } c_{\text{ice}} = 2.108 \text{ kJ kg}^{-1} \text{ K}^{-1}, c_{\text{water}} = 4.187 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

$$\text{Latent heat of fusion: } L_{\text{ice}} = 330 \text{ kJ kg}^{-1}$$

$$\text{Heat needed to melt ice, } H_{\text{ice}} = (c_{\text{ice}} \times 0.05 \text{ kg} \times 10 \text{ K}) + (L_{\text{ice}} \times 0.05 \text{ kg}) = 17554 \text{ J}$$

$$\text{Heat removed to cool water, } H_{\text{water}} = (c_{\text{water}} \times 1 \text{ kg} \times 15) = 62805 \text{ J}$$

More heat is required to cool the water to 0 °C than needed to melt the ice.

Part B

To find the final temperature, the added ice must be in thermal equilibrium with the water.

$$\text{Energy to (Raise } T_{\text{ice}} + \text{Melt ice} + \text{Raise } T \text{ of 50g water)} = \text{Energy removed to cool 1 kg of water}$$

$$H_{\text{ice}} + (c_{\text{water}} \times 0.05 \text{ kg} \times (T_{\text{final}} - 0)) = (c_{\text{water}} \times 1 \text{ kg} \times (15 - T_{\text{final}}))$$

$$c_{\text{water}}(1.05 \times T_{\text{final}} - 1 \times 15) = -17554 \text{ J}$$

$$T_{\text{final}} = \left(15 - \frac{17554}{4187}\right) \div 1.05 = 10.29 \text{ °C}$$

Problem 17.57

$$\text{Rate of energy delivered by the Microwave} = 500 \text{ W} = 500 \text{ J/s,}$$

It is assumed that the amount of heat received by the water in 20 min will cause some to vaporize (we can check this by comparing the heat output of the microwave with the amount of heat needed to raise water to 100 °C). In order to do that, water has to be raised to its boiling temperature (100 °C) and then undergo a phase change to steam.

Latent heat of vaporization of water $L_{\text{steam}} = 2260 \text{ kJ/kg}$

Specific heat capacity of water, $c_{\text{water}} = 4.187 \text{ kJ kg}^{-1} \text{ K}^{-1}$

$(c_{\text{water}} \times m_{\text{water}} \times (100 - T_{\text{ini}})) + L_{\text{steam}} \times m_{\text{vaporized}} = \text{Energy delivered by microwave in 20 mins}$

$(c_{\text{water}} \times m_{\text{water}} \times (100 - T_{\text{ini}})) + L_{\text{steam}} \times m_{\text{vaporized}} = (500 \text{ J/s})(20 \times 60 \text{ s})$

$$m_{\text{vaporized}} = \frac{600000 - (c_{\text{water}} \times m_{\text{water}} \times (100 - T_{\text{ini}}))}{L_{\text{steam}}}$$

Amount of water left = $m_{\text{water}} - m_{\text{vaporized}}$

Problem 17.65

Thermal resistance of refrigerator walls $R_{\text{walls}} = 0.12 \text{ K/W}$

Interior temperature of fridge $T_{\text{int}} = 0^\circ \text{C}$

Exterior room temperature $T_{\text{ext}} = 20^\circ \text{C}$

Latent heat of fusion (to change water to ice at 0°C): $L_{\text{ice}} = 330 \text{ kJ kg}^{-1}$

To calculate how long the ice block will last, first determine the rate of heat transferred into the fridge

$$H = \frac{T_{\text{ext}} - T_{\text{int}}}{R_{\text{walls}}} = \frac{20}{0.12} \text{ W} = 166.67 \text{ J/s}$$

Since the ice is already at 0°C , no additional energy is required to cool it and all the heat transferred is used for melting the ice.

$$\therefore T_{\text{ice}} = \frac{\text{total heat needed to melt ice}}{\text{Rate of heat transferred}} = \frac{L_{\text{ice}} \times m_{\text{ice}}}{H} = \frac{(330 \times 10^3 \text{ J/kg}) \times m_{\text{ice}}}{166.67 \text{ J/s}}$$

Coefficient of Expansion Conceptual Question

Part A

The thermometer markings are calibrated to the volume expansion of mercury with temperature. Alcohol has a volume expansion 5.6 times greater than mercury.

Volume of the liquid in the tube = $\pi r^2 h$ where r = radius of tube and h = height of the liquid column

For the same temperature change and the same tube radius, alcohol level in the tube changes more than mercury. Hence, given that the quantity of alcohol in the thermometer accurately records 0°C , at 20°C , alcohol would have undergone a greater expansion and reached a higher level on the tube compared to mercury.

Part B

Similar to Part A, alcohol also shrinks more than mercury when cooled and thus would records a lower temperature with the thermometer at -10°C .

Part C

$$\text{Change in volume, } \Delta V = \pi r^2 \Delta h = \pi \left(\frac{d}{2}\right)^2 \Delta h$$

To maintain the same spacing between temperature markings, Δh remains unchanged

$$\frac{\Delta V_{\text{Alcohol}}}{\Delta V_{\text{mercury}}} = 5.6 = \frac{\pi \left(\frac{d_{\text{alcohol}}}{2}\right)^2 \Delta h}{\pi \left(\frac{d_{\text{mercury}}}{2}\right)^2 \Delta h} \rightarrow \frac{d_{\text{alcohol}}}{d_{\text{mercury}}} = \sqrt{5.6} \text{ wider}$$

Problem 17.34

Coefficient of Linear expansion for Pyrex glass = $32.5 \times 10^{-7} / ^\circ\text{C}$

$$\text{Expansion in diameter} = (32.5 \times 10^{-7})(\Delta T)(\text{initial diameter}) = (32.5 \times 10^{-7})(85 - 20)(1\text{cm}) \\ \approx 0.00021 \text{ cm}$$

$$\text{Final diameter} = d_{\text{initial}} + \text{expansion} = 1\text{cm} + 0.00021 \text{ cm} = 1.00021 \text{ cm}$$

Problem 17.71

Part A

Coefficient of linear expansion for brass = $19 \times 10^{-6} / ^\circ\text{C}$

Initial brass pendulum length at 20°C = 20 cm

$$\text{Thus at } 18^\circ\text{C, pendulum length} = L_{\text{initial}} - \text{Shrinkage in length} = 20 - (19 \times 10^{-6})(20-18)(20) = 19.99924 \text{ cm}$$

To find the error induced into the clock due to a shrinkage of the length, we must find the difference in the swing period between the 2 different pendulum lengths. Let T_{20} = Period at 20°C and T_{18} = Period at 18°C

$$\text{period (T) of pendulum} = 2\pi \sqrt{\frac{\text{Length}}{\text{gravity}}}$$

$$\text{No. of periods for an error of 1 min to develop} = \frac{60\text{s}}{T_{20} - T_{18}} = \frac{60}{\left(2\pi \sqrt{\frac{0.2}{9.8}}\right) - \left(2\pi \sqrt{\frac{0.1999924}{9.8}}\right)} = 3518128$$

$$\text{Length of time for the error to develop} = 3518128 \times T_{18} = 3157805 \text{ s} = 36.54 \text{ days}$$

Part B

A shorter pendulum results in a shorter period as seen in T_{20} and T_{18} from part A. Hence the faster swing will result in the clock hands moving faster.

The First Law of Thermodynamics Reviewed

$$1^{\text{st}} \text{ law of thermodynamics: } dQ = dW + dU$$

dQ = heat supplied to the system

dW = work done by the system

dU = change in internal energy of the system

Part A

Looking at the above equation, if energy (heat in this case) is provided to the system, it can be stored in the system or applied by the system to perform some work. Hence all the forms of energy must balance out and are conserved.

Part B-D: see definitions above

Part E:

If $dQ > 0$ and $dU = 0$. In order for the equation to balance, then $dW > 0$

Since dW is defined as the work done by the system, it must do positive work on the outside world

The First Law of Thermodynamics Derived

Part A

1st law of thermodynamics: $dQ = dW + dU$

dQ = heat supplied to the system

dW = work done by the system

dU = change in internal energy of the system

The choices are a combination of adding or removing heat from a system and the compression or expansion of a system.

With the above definition, when we add heat to the system, dQ is positive. Conversely dQ is negative when heat is removed from the system.

When we compress a system, we expend energy in act of doing so and this energy gets stored in the system or gets converted to heat or both. Examples would be compressing a spring or a pocket of air. Since dW is defined as work done by the system, when work is done on the system, such as compressing it, dW is negative and positive when the system expands.

Part B

Conservation of energy states that energy cannot be created or destroyed. If we add heat to a system ($dQ > 0$), then this energy is either stored in the system ($dU > 0$) or used by the system to carry out some other form of work ($dW > 0$). Hence we get:

$$dQ = dW + dU$$

Understanding pV Diagrams

A pV diagram relates the pressure of a fixed quantity of ideal gas to its volume. From the ideal gas law equation, $pV = nRT$, with quantity held constant, the equation can be rearranged to give:

$$\frac{pV}{T} = nR = \text{constant}$$

A single parameter may not be varied independently from the others.

pV-diagrams can be used to calculate work done as given by the area under the graph corresponding to that process on the pV-diagram.

Part A

Work done moving from point 1 to 2: $W_{1 \rightarrow 2} = p \times \Delta V = (3p_0)(3V_0 - V_0) = 6p_0V_0$

Part B

$$W_{2 \rightarrow 1} = p \times \Delta V = (3p_0)(V_0 - 3V_0) = -6p_0V_0$$

Part C

$$W_{5 \rightarrow 6} = p \times \Delta V = (p_0)(3V_0 - V_0) = 2p_0V_0$$

Part D

Can be done 2 ways

$$W_{1 \rightarrow 3 \rightarrow 6} = \text{area of trapezoid} = \frac{\text{sum of // sides}}{2} \times \text{height} = \left(\frac{p_0 + 3p_0}{2} \right) (2V_0) = 4p_0V_0$$

Or simply count the number of squares under the curve since we know each square is p_0V_0 .

Part E

Work done going from 2 \rightarrow 6

Since there is no change in volume, there is no work being done by or on the system. Note that this does not mean that energy is not conserved. The system's internal energy can still be changed.

Part F

Work done going from 1 \rightarrow 2 \rightarrow 6 \rightarrow 5 \rightarrow 1 can be approached in 2 ways.

1st way:

$$W_{1 \rightarrow 2 \rightarrow 6 \rightarrow 5 \rightarrow 1} = W_{1 \rightarrow 2} + W_{2 \rightarrow 6} + W_{6 \rightarrow 5} + W_{5 \rightarrow 1} = W_{1 \rightarrow 2} + 0 - W_{5 \rightarrow 6} + 0 = 6p_0V_0 - 2p_0V_0 = 4p_0V_0$$

$$W_{6 \rightarrow 5} = -W_{5 \rightarrow 6} \text{ (comparing parts A and B)}$$

$$W_{2 \rightarrow 6} = W_{5 \rightarrow 1} = 0 \text{ because there is no volume change}$$

2nd way:

Count the area bounded by the path 1 \rightarrow 2 \rightarrow 6 \rightarrow 5 \rightarrow 1. This turns out to be 4 squares and hence $4p_0V_0$.

Part G

Work done going from 1 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 1 can again be done both ways.

$$W_{1 \rightarrow 2 \rightarrow 6 \rightarrow 3 \rightarrow 1} = W_{1 \rightarrow 2} + W_{2 \rightarrow 6} + W_{6 \rightarrow 3 \rightarrow 1} = W_{1 \rightarrow 2} + 0 - W_{1 \rightarrow 3 \rightarrow 6} = 6p_0V_0 - 4p_0V_0 = 2p_0V_0$$

Or

Counting the number of squares enclosed: 2 squares = $2 p_0V_0$.

Isothermal Expansion: Work and Heat

An isothermal process is one where the temperature of the system remains unchanged. Applied to a fixed quantity of ideal gas results in $pV = nRT \rightarrow pV = \text{constant}$.

Part A

Of the 4 curves, C is the only one where the product of the ratios before and after the change remains constant.

Product before = $1 \times 1 = 1$

Product after = $0.25 \times 4 = 1$

Part B

The total internal energy of an ideal gas is equal to the kinetic energy of all its individual gas atoms

From the equipartition theorem we have:

$$\frac{1}{2}m\langle v_x^2 \rangle = \frac{1}{2}m\langle v_y^2 \rangle = \frac{1}{2}m\langle v_z^2 \rangle = \frac{1}{2}k_B T \quad \text{where } k_B = \text{Boltzman constant and } T = \text{temperature}$$

Average velocity squared of each gas atom: $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

Thus for N atoms, the kinetic energy, $K = N \left(\frac{1}{2}m\langle v^2 \rangle \right) = \frac{N}{2}m \left(\frac{3k_B T}{m} \right) = \frac{3Nk_B T}{2}$

Since there is no change in temperature, the internal energy of the gas remains unchanged.

Part C

Isothermal expansion: temperature is constant hence $pV = nRT \rightarrow pV = \text{constant}$

Since the gas is expanding (V increasing), thus pressure has to decrease correspondingly to maintain the constant.

Part D

Work done by the gas, W = area under the curve of a pV-diagram.

Thus when the gas expands from V_0 to $R_V V_0$:

$$W = \int_{V_0}^{R_V V_0} p dV$$

To find pressure as a function of volume:

$$pV = \text{constant} \rightarrow p_1 V_1 = p_2 V_2 \rightarrow p_2 = p_1 \frac{V_1}{V_2}$$

$$p(V) = p_0 \frac{V_0}{V}$$

Substituting into work done:

$$W = p_0 V_0 \int_{V_0}^{R_V V_0} \frac{1}{V} dV = p_0 V_0 [\ln V]_{V_0}^{R_V V_0} = p_0 V_0 [\ln R_V V_0 - \ln V_0] = p_0 V_0 \ln \left(\frac{R_V V_0}{V_0} \right) = p_0 V_0 \ln(R_V)$$

Part E

From the 1st law of thermodynamics: $dQ = dW + dU$

For an isothermal process, internal energy (dU) = 0 (from part B)
Hence, $dQ = dW = p_0 V_0 \ln(R_V)$

Part F

For an isothermal expansion, $dU = 0$ giving $dQ=dW$, hence heat is completely converted into work

We can calculate work done by finding the area under a curve of a pV-diagram. Hence the work done in an isobaric process (constant pressure) is the area under the curve O-A. Comparing this with the area under the isothermal expansion curve O-C, it is immediately clear that the isobaric expansion does more work.