

Physics 6B - Winter 2011
Homework 4 Solutions

Expansion and Compression of a Gas

Part A

In an adiabatic process, there is no heat transferred to or from the system i.e. $dQ = 0$. The first law of thermodynamics then reduces to

$$dQ = dW + dU \Rightarrow dU = -dW \quad (1)$$

where dW is the amount of work done by the gas and dU is the change in internal energy. Since the gas is expanding, the work being done by the gas is positive. Eqn. (1) then implies that the change in internal energy of the system must be negative, that is, the internal energy is decreasing. The only true statements are then:

- The work done by the gas is positive, and no heat exchange occurs.
- The internal energy of the system decreases.

Part B

Since the system undergoes a cyclic process, i.e. a process in which the initial and final states are the same, the change in internal energy must be zero. This allows for the reduction of the first law of thermodynamics to

$$dQ = dW. \quad (2)$$

The only true statements are then

- The total change in internal energy of the system after the entire process of expansion and compression must be zero.
- The total work done by the system must equal the amount of heat exchanged during the entire process of expansion and compression.

Isobaric, Isochoric, Isothermal and Adiabatic Processes

Part A

The equation of state of an ideal gas relates its pressure, p and the volume, V , to its temperature, T as

$$pV = nRT \Rightarrow T = cpV \quad (3)$$

where $c = 1/nR$ is a constant. From the graph, the temperature of the system at states A and B is

$$T = 4c \text{ K}$$

which means the process is isothermal. In accordance with the Equipartition Theorem, the internal energy of the system is proportional to the temperature. It must be the case that

- The states A and B have the same internal energy, so ΔU is zero.

Part B

As the gas expands, the internal energy of the system begins to decrease. It was found in part A, however, that the internal energy of the system must remain constant through an isothermal process. This can only occur if the amount of energy lost while the gas expands is restored to the system in the form of heat. It is then the case that

- W provides energy output, while Q provides energy input. They are equal in magnitude.

Part C

The work done by an ideal gas is defined to be

$$W = \int p dV. \quad (4)$$

Since the process is isochoric, $dV = 0$, which implies that the work done by the gas is zero. The first law of thermodynamics then reduces to

$$dU = dQ. \quad (5)$$

Because the temperature of the system decreases from $T_A = 4c$ K to $T_B = c$ K, the internal energy of the system must decrease. This implies that the system is losing energy in the form of heat. It is then the case that

- Q provides energy output, while W equals to zero.

Part D

It was found in Part C, that in going from state A to state D the internal energy of the system decreases. However, in going from state A to state B, the internal energy of the system must remain constant. It must be the case that during the isobaric portion of the process, the system must gain internal energy. Since the system is expanding during the isobaric process, the internal energy of the system will be further reduced due to the work done by the gas. The system must then absorb enough energy in the form of heat that will compensate for the energy lost due to the expansion of the gas and for that lost during the isochoric portion of the process. Q and W are then related as

- W provides energy output, while Q provides energy input; Q is larger.

Part E

As was discussed above, no work is done by the gas during an isochoric process. Since the temperature increases in going from state E to B, the internal energy must also increase. Eqn. (5) then implies that Q must provide an energy input.

Part F

Using the equation of state for an ideal gas, it can be seen that the internal energy of the system increases in going from state A to state F i.e. $p_A V_A < p_F V_F \rightarrow T_A < T_F$. Since the gas is expanding in going from A to F, the first law suggests that the internal energy of the system is being decreased. It is then the case that in order to have a net increase in the internal energy the amount of heat energy provided to the system must exceed the amount of energy lost due to work.

Simple Ways of Expanding

Part A

Since the curve OA is a horizontal line, this implies that the pressure is constant meaning that the process is **isobaric**.

Part B

Using the ideal gas law, a relationship between p and V is found to be

$$p = \frac{nRT}{V}.$$

From the graph, it follows that the pressure is inversely proportional to the volume of the gas, e.g. $p/p_0 = 1/4$ when $V/V_0 = 4$. It can be seen from the equation above that this can only be true if the temperature of the gas remains constant. The curve OC is then representative of a **isothermal** process.

Part C

The pressure of the system in state B is **less than** the pressure of the system in state O.

Part D

The work done by a system during a process is equal to the area under the curve describing the process on a pV diagram. The graph then reveals that the work done by the system is **greater than zero**.

Part E

Even though the equation of state will not reveal the exact temperature of a system in a given state in this case, it can still provide a value that is proportional to the temperature. At state O, the temperature is proportional to

$$T_O \propto \frac{p}{p_0} \bigg|_O \frac{V}{V_0} \bigg|_O = 1$$

while the temperature at state B is

$$T_B \propto 2.5.$$

The temperature of the system in state B is then **greater than** the temperature of system in state O.

Part F

It is known that the internal energy of a system is proportional to the temperature of the system. It follows that the internal energy of the system in state B is **greater than** the internal energy of the system in state O.

Piston in Water Bath Conceptual Work-Energy Problem

Part A

Because the piston is held in place throughout the process, the volume of the system remains unchanged. It follows from eqn. (4) that the work done by the gas is equal to zero since $dV = 0$. Furthermore, because the piston is held above a flame, energy is being transferred into the system implying that the heat transferred is positive. Finally, the first law of thermodynamics with $dW = 0$ yields

$$dU = dQ. \quad (6)$$

The change in internal energy is then positive since the heat transferred is positive. This makes sense because as you heat the system the temperature of the system will rise in turn raising the internal energy ($U \propto T$). **(0, +, +)**.

Part B

If the compression of the piston occurs very slowly, the system will continue to remain in thermal equilibrium with the water bath throughout the process. Since the temperature of the system does not change, the internal energy of the system will also remain unchanged. Furthermore, since the gas is being compressed, i.e. work is being done on the gas, the work is negative. It follows from the first law that the amount of heat transferred to the system must be negative. **(-, -, 0)**

Part C

Once again, because the piston is held in place, the work done by the gas is zero. Furthermore, since the piston is plunged into cold water, energy will leave the system in the form of heat in accordance with the zeroth law of thermodynamics. It follows from eqn. (6) that the change in the internal energy of the system must be negative. **(0, -, -)**

Part D

Because the piston is wrapped in insulation, no heat can enter or exit the system, i.e. $dQ = 0$. Furthermore, as the piston is pulled up, the gas will expand so that the work done by the gas is positive. It then follows from the first law of thermodynamics in the form of eqn. (1) that the change in internal energy must be negative. **(+, 0, -)**

pV Diagram for a Piston

Part A

The piston being locked in place implies that the process will be isochoric and will be described by a vertical line on the pV diagram. Furthermore, the piston is being held above a hot flame meaning that energy is being transferred into the system in the form of heat. It can be seen from eqn. (6) that this will raise the internal energy of the system in turn increasing the temperature. The only point on the pV diagram that satisfies these two conditions is **point 1**.

Part B

Since the piston is in contact with a water bath, as the piston is slowly compressed it will remain in thermal equilibrium with the water bath. This implies that the temperature of the gas will remain constant throughout the process. Furthermore, since the piston is being compressed, the volume of the system will decrease. The only point on the pV diagram that satisfies these two conditions is **point 7**.

Part C

This is similar to Part A, except that heat is now leaving the system. The point on the pV diagram which satisfies these conditions is then **point 5**.

Part D

Now that the piston has been isolated, then no heat energy is allowed to enter or exit the system. As the piston is pulled out of the container, the gas will expand decreasing the internal energy of the system (see eqn. (1)). In turn, the temperature of the system will decrease. The point on the pV diagram which satisfies these conditions is then **point 4**.

Two Closed Thermodynamic Cycles Conceptual Questions

Part A

Cycle 1 is a cyclic cycle so that the change in the internal energy of the gas is zero.

Part B

It can be seen from the pV diagram that 1 and 3 are isochoric processes meaning that their contribution to the total work is zero. If Cycle 1 is traversed clockwise, the gas does positive work by expanding during portion 2 and negative work when it is compressed during portion 4. Since the amount of work done during portion 2 is greater than that done during portion 4, the total work done must be **positive**.

Part C

As mentioned in Part A, Cycle 1 is a cyclic process which implies that $dU = 0$. The first law of thermodynamics then suggest that the amount of heat transferred to the gas must equal the amount of work done by the gas. Since the amount of work done by the gas was found to be positive in Part B, the amount of heat transferred must also be **positive**.

Part D

When Cycle 1 is traversed counterclockwise, the gas does positive work by expanding during portion 4 and negative work when it is compressed during portion 2. Once again, since the amount of work done during portion 2 is greater than the amount of work done during portion 4, the total work done by traversing Cycle 1 counterclockwise will be negative. It immediately follows that $W_{\text{clockwise}} > W_{\text{counterclockwise}}$.

Part E

Using the same argument as in Part C, $Q_{\text{clockwise}} > Q_{\text{counterclockwise}}$.

Part F

The net work done by a gas during a cyclic cycle is equal to the area enclosed by the lines describing the cycle on a pV diagram. Since the areas of Cycle 1 and Cycle 2 are the same, and the cycles are traversed clockwise in both cases, the work done by the gas in both cases is the same. $W_{\text{clockwise, Cycle 1}} = W_{\text{clockwise, Cycle 2}}$

Part G

Following the same argument made in Part C and Part F, the heat transferred in both processes is equal. $Q_{\text{clockwise, Cycle 1}} = Q_{\text{clockwise, Cycle 2}}$

A Flexible Ballon

Part A

Initially the gas expands isobarically meaning that the pressure of the gas remains constant throughout the process. In order for the pressure to remain constant, however, heat must flow into the system, otherwise the equation of state dictates that the pressure would decrease with increasing volume. The gas then expands adiabatically which, by definition, means that no heat is added or removed from the system. In order to calculate the amount of heat supplied to the system, it is only necessary to consider the first process.

Under constant pressure, the heat supplied to the system is given by

$$Q = nC_p(T_f - T_i).$$

It is then necessary to know the final temperature of the system once the expansion has taken place. This can be found by using the equation of state and noting that p/nR will

remain constant throughout the process. This leads to

$$\frac{T_i}{V_i} = \frac{p}{nR} = \text{constant} = \frac{T_f}{V_f}$$

$$T_f = \frac{V_f}{V_i} T_i$$

Since $V_f = 2V_i$ then $T_f = 2T_i$. The amount of heat supplied to the system is then

$$Q = nC_p(2T_i - T_i) = nC_p T_i.$$

With $n = .400 \text{ mol}$, $C_p = 33.26 \text{ J/mol} \cdot \text{K}$ and $T_i = 303 \text{ K}$

$$Q = 4031.112 \text{ J}.$$

Part B

It is known that the internal energy of the system is proportional to its temperature. Since the initial and final temperatures of the system remain unchanged, the change in the internal energy must also be zero

$$\Delta U = 0$$

Part C

It was found in part B that $dU = 0$ which simplifies the first law of thermodynamics to

$$dQ = dW.$$

Since the amount of heat supplied to the system is equal to the amount of work done by the gas it follows from the solution to part A that

$$W = 4031.112 \text{ J}.$$

Part D

Initially, the gas expands isobarically until its initial volume, $V_i = .00675 \text{ m}^3$, has doubled. Then it expands adiabatically until it reaches its final volume V_f . It must be noted that the adiabatic process brings the gas back to its original temperature meaning that $T_f = T_i$. Recall that for an adiabatic expansion the product $TV^{\gamma-1}$ remains constant. As a result, this allows the volume and pressure of a system once it has expanded adiabatically to be related to its initial volume and pressure as

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow V_2 = \left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}} V_1.$$

Taking V_1 and T_1 to be the volume and temperature once the gas has expanded isobarically, and T_2 to be the final temperature of the gas, the final volume of the gas is then

$$V_f = 2^{\frac{1}{\gamma-1}} (2V_i) = .108 \text{ m}^3$$

where the value $\gamma = 4/3$ was used.

An Expanding Monatomic Gas

As the gas expands, the amount of heat that is absorbed by the system is equal to $dQ = 1160$ J while the amount of work that is done by the gas is $dW = 2160$ J. The first law of thermodynamics can then be used to relate these quantities to the change in the internal energy of the system as

$$dQ = dU + dW \Rightarrow dU = U_f - U_i = dQ - dW. \quad (7)$$

The Equipartition Theorem states that for an ideal gas with three degrees of freedom the internal energy is given by

$$U = \frac{3}{2}nRT \quad (8)$$

where n is the number of moles, R is the ideal gas constant and T is the temperature of the system. Initially, the system is at a temperature $T_i = 400.15$ K so the internal energy is then

$$U_i = \frac{3}{2}nRT_i.$$

Similarly, the internal energy of the system once the gas has finished expanding is

$$U_f = \frac{3}{2}nRT_f.$$

Substituting U_i and U_f into eqn. (7), the final temperature of the gas is

$$\begin{aligned} \frac{3}{2}nRT_f - \frac{3}{2}nRT_i &= dQ - dW \\ T_f &= T_i + \frac{2}{3nR}(dQ - dW) \\ &= 384.2637 \text{ K} = 111.1137^\circ \text{ C} \end{aligned}$$

A Carnot Refrigerator

Part A

The Carnot cycle is a cyclic process which means that the internal energy of the system remains unchanged after the system has undergone the full cycle. Eqn. (3) then states that the amount of work done by the gas after completing the cycle will be equal to the net energy transferred into the system

$$W = |Q_h| - |Q_c|.$$

Using the relation

$$\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h},$$

and the expression for the work done by the gas, the coefficient of performance can be rewritten as

$$K = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{1}{\frac{|Q_h|}{|Q_c|} - 1}$$

$$\begin{aligned}
&= \frac{1}{\frac{T_h}{T_c} - 1} \\
&= \frac{T_c}{T_h - T_c}
\end{aligned}$$

Part B

The helium inside the refrigerator is absorbing 2.00 J of energy per hour. If the helium is to remain at constant temperature, the same amount of heat must be removed from the system, i.e. $Q_c = 2.00$ J. The coefficient of performance can then be used to find the work required to accomplish this task. Since the work is being supplied by the electrical energy, it must be the case that $W = E$. Knowing that the temperature of the heat reservoir is at $T_h = 294$ K and the temperature of the cold reservoir is $T_c = 4$ K, the energy per hour that must be used to maintain the helium at constant temperature is

$$K = \frac{Q_c}{E} \Rightarrow E = \frac{Q_c}{T_c}(T_h - T_c) = 145 \text{ J}$$

Part C

Following a similar procedure as in Part B with $Q_c = 5.00$ J, $T_c = 280$ K and $T_h = 300$ K, the amount of energy that is required is

$$K = \frac{Q_c}{E} \Rightarrow E = \frac{Q_c}{T_c}(T_h - T_c) = .357 \text{ J}$$

Melting Ice with a Carnot Engine

Let Q_h be the amount of energy absorbed by the gas from the hot reservoir and Q_c the amount of energy expelled by the gas to the cold reservoir. The work done by the engine is then

$$W = Q_h - Q_c. \tag{9}$$

It is known that in the allotted time, the heat expelled from the engine was enough to melt a mass of ice equal to $m_{ice} = 3.65 \times 10^{-2}$ kg. The amount of heat expelled is then

$$Q_c = -m_{ice}L_f = -12191 \text{ J}$$

where $L_f = 3.34 \times 10^5$ J/kg was used. The negative value of Q_c indicates that the energy is leaving the system. Noting that the hot reservoir is at boiling, i.e. $T_h = 373.15$ K and the cold reservoir is at $T_c = 272.15$ K and using the relation

$$\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}, \tag{10}$$

the amount of heat absorbed by the system is

$$|Q_h| = \frac{T_h}{T_c} |Q_c| = 16654.116 \text{ J}.$$

Substituting these values into eqn. (9), the amount of work done by the system during the allotted time is then

$$W = 4463.116 \text{ J}$$

From Hot to Cool: A Change in Entropy

As the ice melts, its temperature remains constant at $T_{ice} = 273.15$ K, i.e. the process is isothermal. For an isothermal process, the change in the entropy of the system is found as

$$\Delta S = \frac{Q}{T}. \quad (11)$$

Knowing that the amount of heat required to melt the ice is $Q_{ice} = m_{ice}L_f$, the change in the entropy is then

$$\begin{aligned} \Delta S_{ice} &= \frac{m_{ice}L_f}{T_{ice}} \\ &= \frac{(.001 \text{ kg})3.34 \times 10^5 \text{ J/kg}}{273.15 \text{ K}} \\ &= 1.2227 \text{ J/K}. \end{aligned}$$

The amount of heat that was required to melt the ice was provided by the ice water, i.e. $Q_{ice} = -Q_{water}$. Since the temperature of the ice water also remains constant, the change in entropy is calculated to be

$$\begin{aligned} \Delta S_{water} &= -\frac{Q_{ice}}{T_{water}} \\ &= -\frac{(.001 \text{ kg})3.34 \times 10^5 \text{ J/kg}}{293.15 \text{ K}} \\ &= -1.1393 \text{ J/K}. \end{aligned}$$

The net change in entropy of the system is then

$$\Delta S_{sys} = \Delta S_{ice} + \Delta S_{water} = .084 \text{ J/K}.$$

Carnot Heat Engine Pressure versus Volume Graph Conceptual Question

Part A

Let Q_h be the amount of energy absorbed by the Carnot engine in one cycle and Q_c the energy expelled. The total change in entropy is then

$$\Delta S = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c}.$$

Using eqn. (10), and substituting for T_h in the above equation, the change of entropy is then

$$\Delta S = \frac{|Q_h||Q_c|}{|Q_h|T_c} - \frac{|Q_c|}{T_c} = 0$$

Part B

In accordance with the zeroth law of thermodynamics, the temperature of the surroundings must be **greater than** the temperature of the gas.

Part C

In this particular case, the amount of heat lost by the surroundings is equal to the amount of heat gained by the gas. This means that the change in entropy is just going to depend on the temperature of the substances in question. Since the temperature of the surroundings is greater than the temperature of the gas, and the entropy is inversely proportional to temperature of the substance, the entropy change of the surroundings is less than the entropy change of the gas.

Part D

The total change in entropy of the system is equal to the sum of the change in entropy of the gas and the change in entropy of the surroundings

$$\Delta S_{system} = \Delta S_{gas} + \Delta S_{surroundings}.$$

Because energy is being provided by the surroundings to the gas, $Q_{gas} = -Q_{surroundings}$, the change in entropy of the surroundings will be negative. However, it was found in Part C that the change in entropy of the gas is greater than the change in entropy of the surroundings which means that the total change in entropy of the system will be **positive**.

Part E

In accordance with the zeroth law of thermodynamics, in order for heat to flow from the gas to the surroundings, the temperature of the surroundings must be **less than** the temperature of the gas.

Part F

Using a similar argument as that used in Part C, the magnitude of the entropy gained by the surroundings is **greater than** the magnitude of the entropy lost by the gas.

Part G

Using a similar argument as that used in Part D, the net entropy change of the entire system is **positive**.