

Assignment 6

Problem 1: Creating a Standing Wave

Part A

The wave is traveling in the +x direction

Part B

$\text{Asin}(kx + wt) = \text{Asin}(k(x + vt))$ since vt is added to position, the wave is traveling in the negative direction.

Part C

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\text{Asin}(kx + wt) + \text{Asin}(kx - wt) = \text{Im}(Ae^{ikx+iwt} + Ae^{ikx-iwt})Ae^{ikx+iwt} + Ae^{ikx-iwt} = Ae^{ikx}(e^{iwt} + e^{-iwt})$$

the imaginary terms cancel out so

$$Ae^{ikx}2\cos(wt)$$

taking just the imaginary terms

$$2\text{Asin}(kx)\cos(wt)$$

Part D

The wave is $2\text{Asin}(kx)\cos(wt)$ as the t changes the nodes remain in place, the wave is oscillating, but not moving.

Part E

According to the formula, at $x = 0$, the displacement is zero

Part F

$$wt = \frac{\pi}{2}$$

Part G

There is energy stored in the string since it is oscillating

Energy is constant in the string

Although the string is straight at time

$$t = \frac{\pi}{2\omega}$$

parts of the string have nonzero velocity. Therefore, there is energy stored in the string. This is the answer.

Problem 2: Nodes of the standing waves

Part C

$$w = \frac{2\pi}{T}$$

$$w \frac{T}{4} = \frac{m\pi}{2}$$

$$y\left(x, \frac{T}{4}\right) = A \sin(kx)$$

Part D

Nodal points are where $\sin(wt) = 0$;

$$kx = m\pi$$

$$x = m\lambda/2$$

don't enter λ in the answer.

Problem 3 Normal Modes and Resonance Frequencies

Part C

Find the three longest wavelengths, (call them $\lambda_1, \lambda_2, \lambda_3$) that fit on the string, that is, those that satisfy the boundary conditions at $x=0$ and $x=L$. These longest wavelengths have the lowest frequencies.

The boundary conditions imply that $\sin(2\pi x/\lambda)$ must be zero at $x = 0$ and $x = L$. Thus $2\pi L/\lambda = m\pi$ for each of the λ

$$\lambda = L, 2L, 2L/3$$

Part D

The frequency of each normal mode depends on the spatial part of the wave function, which is characterized by its wavelength λ_i .

Find the frequency f_i of the i th normal mode.

$$f_i = \frac{v}{\lambda_i}$$

From the previous solution $\lambda = L, 2L, 2L/3$, plug this in and

$$f_i = \frac{v}{2L}, \frac{v}{L}, \frac{3v}{2L}$$

Problem 4

Problem 14.38

What is the longest-wavelength standing wave that can exist on this string?

Part A

The longest-wavelength on a string clamped at both ends is double the length of the string, thus $\lambda = 3 \times 2 = 6\text{m}$

Part B

If the wave speed is v , what is the lowest standing-wave frequency?

$$f = v/\lambda$$

Problem 5

Problem 14.39

When a stretched string is clamped at both ends, its fundamental standing-wave frequency is 145Hz.

Part A

What is the next higher frequency?

for a stretched string clamped at both ends $f = \frac{mv}{2L}$, $m = 1$ is the fundamental frequency, $f_2 = 2f_1$

Part B

If the same string, with the same tension, is now clamped at one end and free at the other, what is the fundamental frequency?

string clamped at only one end: $f = \frac{mv}{4L}$ but now $m = 1, 3, 5\ldots$

so just divide the fundamental frequency for the clamped string by 2.

Part C

What is the next higher frequency in case (b)?

The next mode $f = 3f_1$

Problem 6

Harmonics of a Piano Wire

Part A

A piano tuner stretches a steel piano wire with a tension of 765N. The steel wire has a length of 0.400 m and a mass of 3.00 g.

What is the frequency f_1 of the string's fundamental mode of vibration?

$$f = \frac{1}{2L} * \sqrt{\frac{T_s}{\mu}}$$

and $\mu = \text{Mass/Length}$ (don't forget to convert to kg/m

Part B

What is the number n of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to $f = x$ kHz?

use f from the previous answer

$$nf \leq x * 1000$$

$$n \leq x * 1000 / f$$

Problem 7

Introduction to Wind Instruments

Part A

Consider a pipe of length 80.0 cm open at both ends. What is the lowest frequency f of the sound wave produced when you blow into the pipe?

for the pipe with both ends open, the nodes are at the open ends. thus fundamental frequency $f = \frac{v}{2L}$

Part B

A hole is now drilled through the side of the pipe and air is blown again into the pipe through the same opening. The fundamental frequency of the sound wave generated in the pipe is now higher than before.

Part C

If you take the original pipe in Part A and drill a hole at a position half the length of the pipe, what is the fundamental frequency f' of the sound that can be produced in the pipe?

$f' = f/2$, where f is the old frequency;

Part D

What frequencies, in terms of the fundamental frequency of the original pipe in Part A, can you create when blowing air into the pipe that has a hole halfway down its length?

Only the even multiples of the fundamental frequency

Part E

What length of open-closed pipe would you need to achieve the same fundamental frequency f as the open-open pipe discussed in Part A?

Half the length of the open-open pipe

Part F

What is the frequency f'' of the first possible harmonic after the fundamental frequency in the open-closed pipe described in Part E?

The next frequency is 3*fundamental frequency.

Problem 8

Problem 14.77

Suppose you're operating a machine with two motors. If the two motors are not operating at the same rpm, you will hear a periodic variation in the intensity of the sound-that is, beats. Suppose the tachometer that indicates the rotation rate of one motor is malfunctioning. The operational one shows 3520 rpm. You hear a beat every 30s. You are certain the other motor is running fast.

Part A

What is its rotational speed in rpm?

The first motor is running at X rpm or X cycles per minute. The other motor will coincide with it every y seconds or convert to min. Thus there is a frequency difference between the two motors $\Delta f = 2 \text{ min}^{-1}$

second motor = first motor + Δf

Problem 9: Interference of Sound Waves

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172Hz. You are 8.00 m from speaker A. Take the speed of sound in air to be 344 m/s.

Part A

What is the closest you can be to speaker B and be at a point of destructive interference?

$$\lambda = v/f$$

The waves interfere destructively when they are π out of phase, that corresponds to half the wavelength, thus second speaker must be $\lambda/2$ from the humanoid.

Problem 10: Problem 14.69

You're standing by the roadside as a truck approaches, and you measure the dominant frequency in the truck noise at 1070 Hz. As the truck passes, the frequency drops to 910 Hz.

Part A

What is the truck's speed?

doppler shift formula $f = \frac{v+v_r}{v+v_s} * f_0$ where v_r is the speed of the receiver

with respect to the medium, v_s is the speed of the source with respect to the medium; positive if the source is moving away from the receiver and v is the speed of the wave in the medium. In the problem $v_r = 0$; $v = 342$ m/s and v_r we are looking for. Let f_1 be the frequency as the truck is moving toward the receiver.

$$f_1 = \frac{v}{v - v_s} * f_0$$

$$f_2 = \frac{v}{v + v_s} * f_0$$

$$\frac{f_1}{f_2} = \frac{v + v_s}{v - v_s}$$

$$v_s = \frac{f_1 + f_2}{f_1 - f_2}$$

Problem 11: The Doppler Effect on a Train

A train is traveling at 30.0 m/s relative to the ground in still air. The frequency of the note emitted by the train whistle is 262 Hz. The speed of sound in air should be taken as 344 m/s.

Part A

What frequency $f_{approach}$ is heard by a passenger on a train moving at a speed of 18.0 m/s relative to the ground in a direction opposite to the first train and approaching it?

doppler shift formula $f = \frac{v+v_r}{v+v_s} * f_0$ where v_r is the speed of the receiver with respect to the medium; positive if the receiver is moving toward the source, v_s is the speed of the source with respect to the medium; positive if the source is moving away from the receiver and v is the speed of the wave in the medium.

plugging the numbers into the formula will yield the answer.

Part B

What frequency $f_{receding}$ is heard by a passenger on a train moving at a speed of x m/s relative to the ground in a direction opposite to the first train and receding from it?

Using the previous formula, plug in the numbers.

Problem 12: Problem 29.43

Vertically polarized light passes through two polarizers, the first at x° to the vertical and the second at y° to the vertical.

Part A

What fraction of the light gets through?

$$\frac{S}{S_0} = \cos(x)^2 \cos(y - x)^2$$

Problem 13: Problem 29.67

In a stack of polarizing sheets, each sheet has its polarization axis rotated x° with respect to the preceding sheet.

If the stack passes P percent of the incident, unpolarized light, how many sheets does it contain?

The first sheet lets through 50 percent of the light and polarizes it along its axis, the rest of the sheets let through $\cos(x)^2$ each, thus

$$0.5 * \cos(x)^{2(n-1)} = P$$

the answer must be an integer.