Homework # 8

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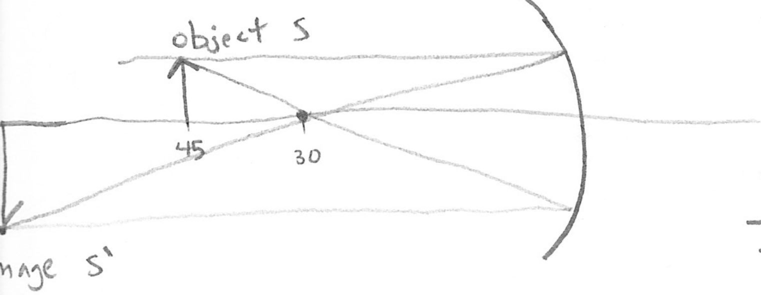
Concave mirror, with radius of curvature 60 cm. It asks for $f = \frac{R}{2} = 30$ cm.



What's s' ?

$$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \Rightarrow s' = 90 \text{ cm}$$

object distance = 45 cm

Find magnification.

$$m = -\frac{s'}{s} = -\frac{90 \text{ cm}}{45 \text{ cm}} = -2$$

real and inverted.

Now it asks you to solve the mirror equation for s':

$$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} \rightarrow s' = \frac{f}{1 - \frac{f}{s}}$$

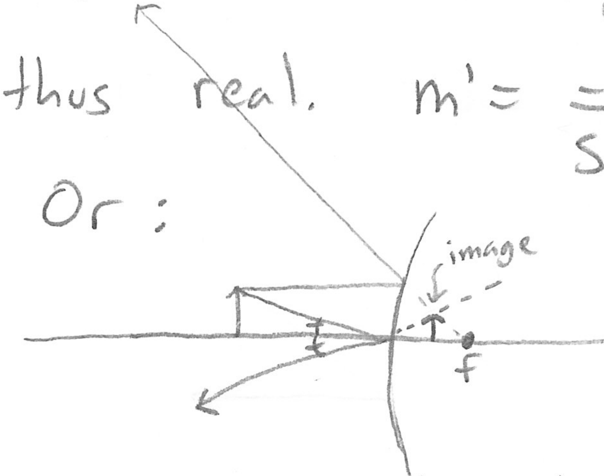
$$\boxed{s' = \frac{fs}{s-f}}$$

$$m = -\frac{s'}{s} = \boxed{\frac{-f}{s-f}}$$

... what's the image like for a similar mirror, but <u>convex</u>? ($F = -30\text{ cm}$)

Based on equations we just derived,

$s' = \frac{fs}{s-f}$, so if f is negative, s' is negative, thus real. $m' = \frac{-f}{s-f}$ will be positive. Or:



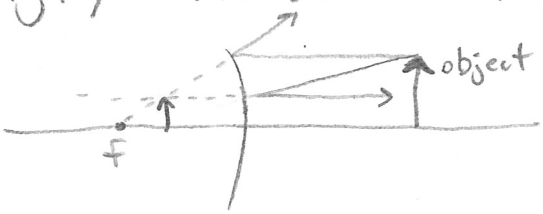
magnification is evidently <u>less</u> than one.

For a mirror with infinitely large radius of curvature (flat mirror),

$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s}$ yields $s' = -s$

Ray Tracing and Image Formation with Spherical Mirrors:

A) A convex mirror always forms an upright, reduced image:

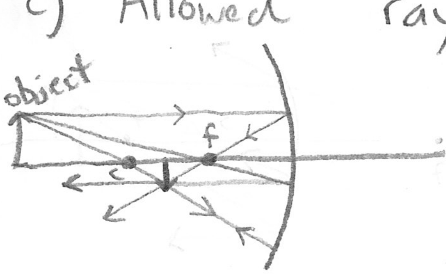


B) If the radius of curvature is 50 cm, how far away (d_o) should the object be placed to get an image 20 cm away?

$$\frac{1}{f} = \frac{1}{S} + \frac{1}{S'}$$

$$\frac{1}{25} = \frac{1}{d_o} + \frac{1}{20} \rightarrow d_o = -100 \text{ cm}$$

c) Allowed ray tracings: (concave mirror)



D) Now for concave mirror, if the object is still 100 cm away, where will the image be?

$$\frac{1}{f} = \frac{1}{S} + \frac{1}{S'} \qquad \frac{1}{25} = \frac{1}{100} + \frac{1}{S'} \rightarrow S' = 33.3 \text{ cm}$$

Spherical Mirror 1

You want to create an image 10 m from an object, inverted and half the height of object, using one spherical mirror.

We know $m = -\frac{1}{2} = \frac{-s'}{s}$, so $s = 2s'$

Also, $s - s' = 10$, so

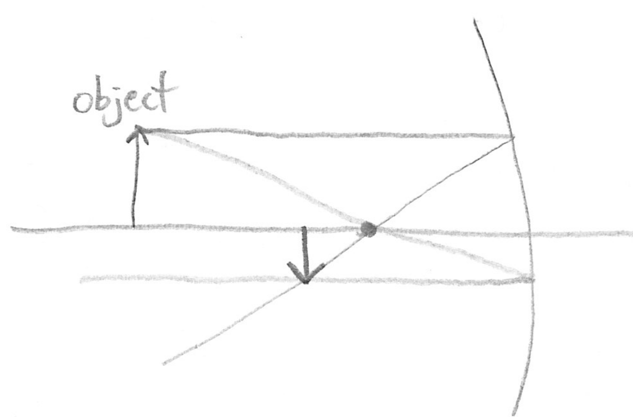
$2s' - s' = 10$

$s' = 10$, thus $s = 20$

$$f = \frac{1}{\frac{1}{s} + \frac{1}{s'}} = \frac{s}{1 + \frac{s}{s'}} = \frac{ss'}{s' + s} = \frac{10 \cdot 20}{10 + 20} =$$

$$\frac{200}{30} = \boxed{6.67\ m}$$

- must be concave, since f is positive ↑
- Radius of curvature must be $2 \cdot f = 13.3$ m
- Real image



Spherical Mirror 2

You want to create an image 10 m from an object, upright, half the size, using one spherical mirror.

What's f ? $m = \frac{1}{2} = -\frac{s'}{s}$ so $s = -2s'$

$$s' - s = 10$$
$$s' - (-2s') = 10$$
$$3s' = 10$$
$$s' = 10/3 \text{ so } s = -20/3$$

$$f = \frac{ss'}{s+s'} = -6.67 \text{ m}$$

Must be convex, since f is negative

radius = 2f = 13.3 m

Virtual, since negative image distance (opposite sign of s, anyways)

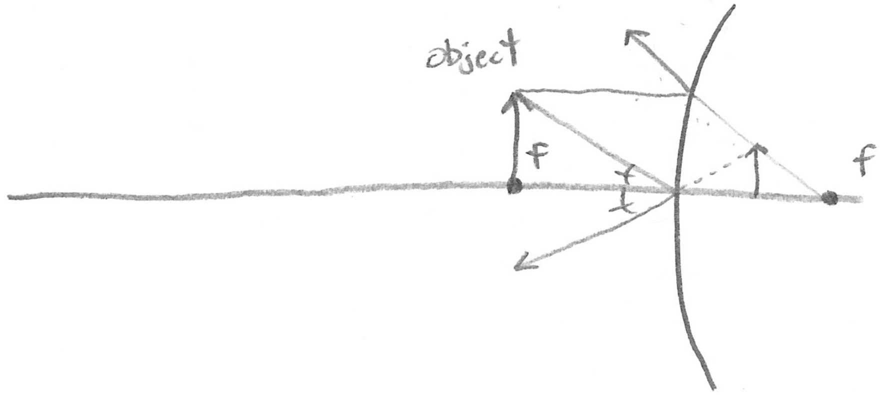
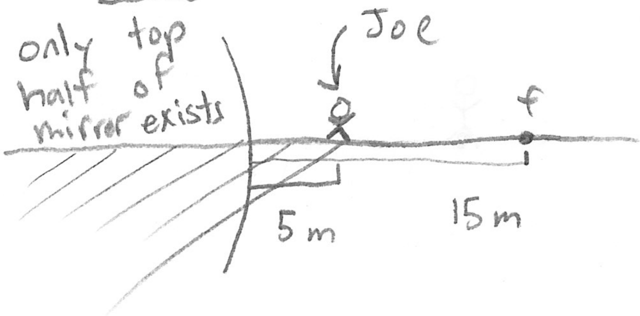


Image Size in a Mirror



f must = −15, since R is 30 m and it's convex.

How far away does his image appear?

$$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s}$$

$$\frac{1}{-15m} = \frac{1}{s'} + \frac{1}{5m} \rightarrow s' = -3.75m$$

add −5 m for answer: 8.75 m

What's his image's height?

$$m = -\frac{s'}{s} = -\frac{-3.75}{5} = .75,$$ multiply his actual height by this factor to get 1.2 m

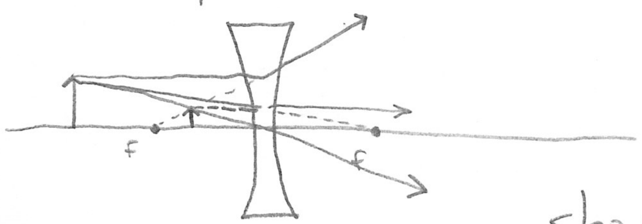
Now he falls on his face, with feet fixed. What's his image's length?

First calculate his head's image position:

$$\frac{-1}{15m} = \frac{1}{3.4m} + \frac{1}{s'}$$ (3.4m = 5 − his height) $s' = -2.772$ m, difference btwn this value and his feet's image (−3.75m) is the answer; .9783 m

Ray Tracing and Image Formation with a concave Lens:

allowed paths:



If the focal length is -7.5 cm, where should the object be placed so that its image is 3.7 cm from lens?

$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

$$-\frac{1}{7.5\text{cm}} = \frac{1}{s} + \frac{1}{-3.7} \rightarrow s = 7.3 \text{ cm}$$

for concave lens, s, s' on same side (negative)

What's m? $m = -\frac{s'}{s} = .507$

To increase m, move object closer to lens

The focal Length of a Lens:

An object is located 28 cm from a lens. The image is real and twice as high. What's f?

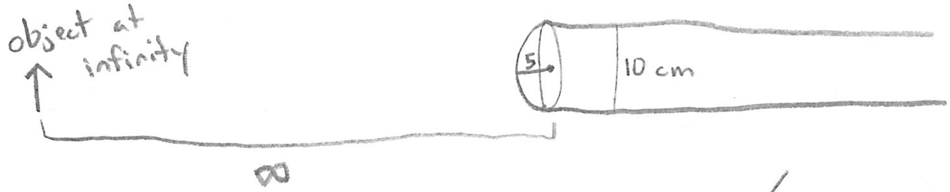
$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s}$ $f = \frac{ss'}{s+s'} = \frac{(28)(56)}{(28+56)} = 18.7$ cm

$s' = 2s$

B) Now if everything's the same, but concave lens, object 5 m high, how high is the image?

$\frac{1}{-18.7} = \frac{1}{s'} + \frac{1}{-28}$ so $s' = -11.2$ $m = -\frac{s'}{s} = .4$ $.4 \times 5\text{cm} = \boxed{2\text{cm}}$

Refraction at a Spherical Surface



long glass rod with convex end

Object image relation: (for refraction at a spherical surface)

$$\frac{n_a}{s_a} + \frac{n_b}{s_b} = \frac{n_b - n_a}{R}$$

- s_a → object dist.
- s_b → image dist.
- $n_b - n_a$ → refractive indices of corresponding material (n_a on left)
- R → radius of curvature (5 cm)

What's s_b, the image position?

$$\frac{n_a}{\infty} + \frac{1.6}{s_b} = \frac{n_b - n_a}{R}$$ ($\frac{n_a}{\infty}$ → 0)

$s_b = 13.3$ cm (on right, since positive)

What about if the object were, instead, placed 15 cm from left end?

$$\frac{1}{-15\text{ cm}} + \frac{1.6}{s_b} = \frac{1.6 - 1}{5\text{ cm}}$$

$s_b = 30$ cm (on right, since positive)

Again, for $s_a = |-3\text{ cm}|$ from left end:

s_b yields -7.5 cm ... (on left side, virtual, since negative)

Chromatic Aberration

Lenses typically have different focal lengths for different wavelengths.

Consider one with $f_{red} = 19.57$ cm, $f_{blue} = 18.87$ cm.

If an object 5 cm tall is 30 cm away, find the ratio: $\frac{\text{height of red image}}{\text{height of blue image}}$

→ to get this...

$\frac{1}{f_{red}} = \frac{1}{s} + \frac{1}{s'}$ ... $m = -\frac{s}{s'}$

height of red $= m * 5$

↙ follow same procedure as for red image...

answer: 1.11

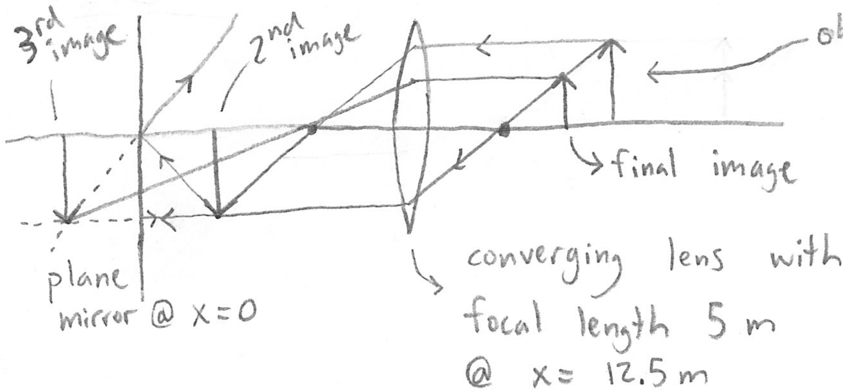
B) What would you expect to see if a circular piece of white paper with radius 5 cm were placed 30 cm from the lens, centered on axis?

→ it would have red edges.

Understanding Multiple Optics

"One lens' image is another lens' object"
- Confucius

In this system, we have the following:



First, find the location of the image from lens:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{5m} = \frac{1}{10m} + \frac{1}{s'}$$

$s' = 10$, so the image location is $12.5 - 10 = 2.5$ m

Second, find the image after it reflects on plane:

$$\frac{1}{\infty} = \frac{1}{s} + \frac{1}{s'} \rightarrow s' = -s = -2.5 \text{ m}$$

Next, this reflects through the lens again. Find s':

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{5m} = \frac{1}{(2.5+12.5)} = \frac{1}{s'}$$

$s' = 7.5$ so final location is $12.5 + 7.5 = 20$ m

real final image
(since the light actually reaches the location of the image)

Now, find $|m|$, after passing through just the 1st lens:

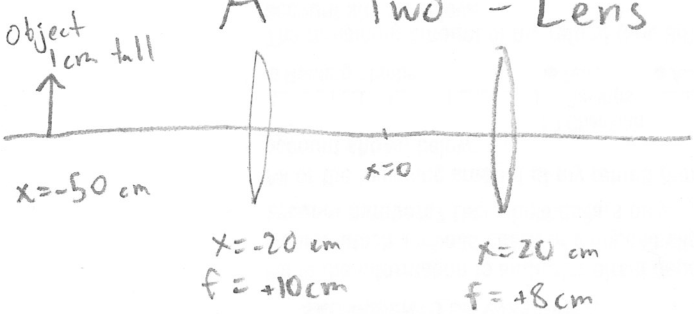
$$|m| = \left|-\frac{s'}{s}\right| = \frac{10m}{10m} = 1$$

No magnification occurs for plane mirror either, but for last pass through lens:

$$|m| = \left|\frac{-s'}{s}\right| = \frac{7.5}{15} = .5$$

Total magnification $= 1 * 1 * .5 = .5$
upright image

"A Two-Lens System"



Final image location?

After 1st lens: $\frac{1}{10\text{cm}} = \frac{1}{s'} + \frac{1}{30\text{cm}} \Rightarrow s' = 15\text{ cm}$ so position is $-20 + 15 = -5\text{ cm}$

After 2nd lens: $\frac{1}{8\text{cm}} = \frac{1}{s''} + \frac{1}{25\text{cm}} \Rightarrow s'' = 11.8\text{ cm}$, so position is $20 + 11.8 = 31.8\text{ cm}$

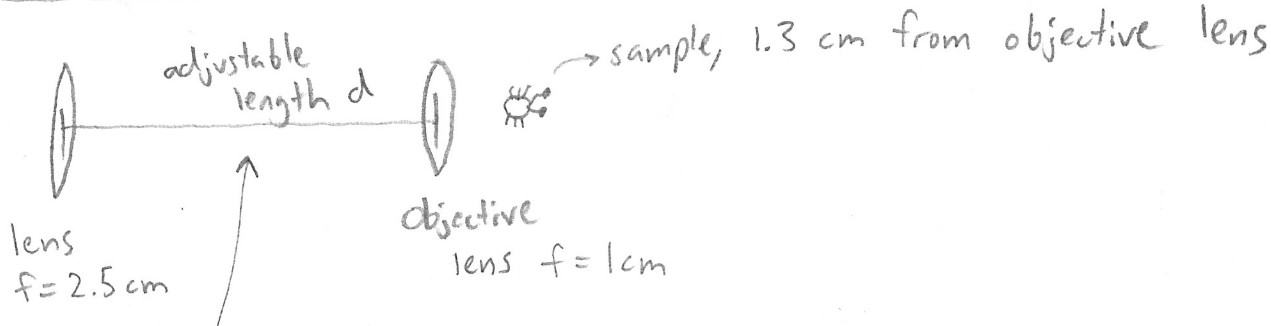
Magnification $= m_1 * m_2 = \left(\frac{-s'}{s_1}\right)\left(\frac{-s''}{s_2}\right) = \left(\frac{-15}{30}\right)\left(-\frac{11.8}{25}\right) = .236$

Final height $= (1\text{cm})(\text{mag}) = 1\text{cm} * .236 = .236\text{ cm}$

To achieve the same effect with 1 mirror at $x=0$

$$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \Rightarrow \frac{1}{f} = \frac{1}{31.8\text{cm}} + \frac{1}{50\text{cm}} \quad f = 19.4\text{ cm}$$

A microscope for Biology



What length should you choose so that the sample is in focus with completely relaxed eye?
(image must be at ∞ for this to occur.)

After passing through objective:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{1\,cm} = \frac{1}{1.3} + \frac{1}{s'} \rightarrow s' = 4.33\ cm \text{ to left}$$

After passing through second lens:

$$\frac{1}{2.5cm} = \frac{1}{(d - 4.33cm)} + \frac{1}{\infty} \rightarrow 0$$

d must = 6.83 cm