Constructive and Destructive Interference Conceptual Question

The idea here is the following. If the two sources are in phase, then the only way they can be out of phase at a point in space is if there's a difference in path length from each source to the point in question. If s is this path length difference, then

$$s = n\lambda + \tfrac{1}{2}\lambda \Rightarrow \text{destructive interference}$$

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(A) The point is 3m away from either source, so

$$s = 3m - 3m = 0 = 0 \cdot \lambda \quad (n=0)$$

$\Rightarrow$ constructive

(B) The point is ~~1.5m~~ from the left source and 4.5m from the right source

$$s = 1.5m - 4.5m = -3m = -2\lambda + \tfrac{1}{2}\lambda \quad (n=-2)$$

since $\lambda = 2m$. $\Rightarrow$ destructive

(C) $S = 3.5m - 2.5m = 1m = 0 \cdot \lambda + \frac{1}{2}\lambda \ (n=0)$

$\Rightarrow$ destructive

(D) $S = 5m - 1m = 4m = 2\lambda \quad (n=2)$

$\Rightarrow$ constructive

Introduction to Two-Source Interference

(A) The order is A (largest), B & D tied for next largest, and C smallest.

For A the peaks align, so you add the maximum of both waves together. This is the biggest the sum could be.

For C, the waves always cancel one another. This is the smallest the sum can be

For B and D, the maximum of the first (second) wave happens when the second (first) wave is 0, so the amplitude is somewhere between that of A or C.

(B) Constructive interference happens when the phase difference is an integer times the wavelength, ($n\lambda$), destructive when it's $(n+\frac{1}{2})\lambda$. B, C, E, G are 2λ, -5λ, 0λ and 3λ, respectively $\Rightarrow$ these are constructive. The others are destructive:

BCEG, ADF

(C) For this one, we again need to look at the path length difference, as for the previous problem. If the difference is $s = n\lambda$, then the waves are in phase and it's constructive. $s = (n+\frac{1}{2})\lambda$ is destructive.

A is $(3\frac{1}{2})\lambda$ from either source

$$s_A = (3\tfrac{1}{2})\lambda - (3\tfrac{1}{2})\lambda = 0\lambda \Rightarrow \underline{\text{constructive}}$$

B is 2λ from left source, 3λ from right:

$$s_B = 2\lambda - 3\lambda = (-1)\lambda \Rightarrow \underline{\text{constructive}}$$

Then, $s_C = 4\lambda - 2\frac{1}{2}\lambda = (1\frac{1}{2})\lambda \Rightarrow \underline{\text{destructive}}$

$$s_D = 2\tfrac{1}{2}\lambda - 2\lambda = \tfrac{1}{2}\lambda \Rightarrow \underline{\text{destructive}}$$

$\Rightarrow$ <u>ccdd</u> [3]

(D) We already answer this:

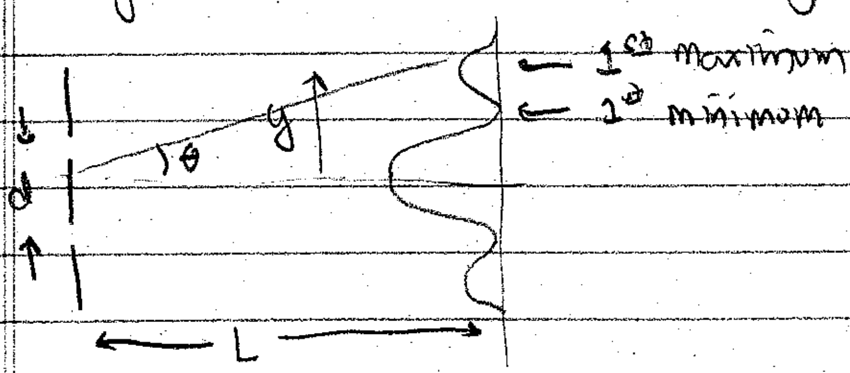
$\Delta s_A, \Delta s_C, \Delta s_D = 0, 1.5, 0.5$

(E) We now know how to do this, The answer is

$\boxed{1, 1, 1, 1, 1}$

The path formed by the 5 points is a locus of constructive interference.

Fringes from Different Interfering Wavelengths



For two-slit interference, the angle θ to the first maximum is given by

$d \sin\theta = \lambda$ or $\theta \doteq \lambda/d$

using the small angle approximation.

For the first minimum, the angle is only half of this

$$\theta \doteq \frac{\lambda}{2d}$$

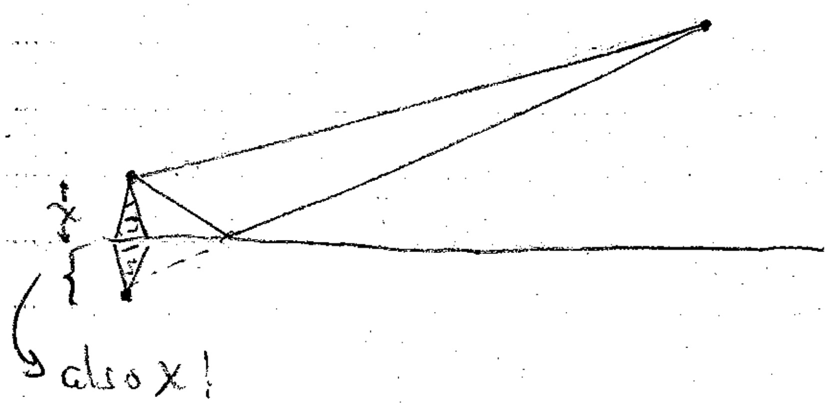
In this problem, we have two different wavelengths λ_1 and λ_2. The first produces a light fring at the same point (the same angle θ) that the second produces a dark fring. Thus,

$$\frac{\lambda_1}{d} = \theta = \frac{\lambda_2}{2d}$$

So simply

$$\lambda_2 = 2\lambda_1 = 1{,}198 \text{ nm} = \boxed{1.198\,\mu m}$$

Double Slit With Reflections



This is a basic two-slit interference problem, but with two things that must also be recognized:

① The extra path length for the reflected beam is the same as for a beam emitted a distance x below the surface (see diagram), so the distance between the two points of emission (the "slit separation) can be thought of as

$$d = 2x$$

② There's a 180° phase shift at the surface when the beam reflects.

Finally, we need to not be confused by the fact that the distance d along the water to the balloon is **not** the "slit separation" d referred to above.

DESTRUCTIVE INTERFERENCE

Using d for the slit separation, we would normally write for destructive interference $d \sin\theta = (n+\frac{1}{2})\lambda$. However, because of the extra 180° (one-half λ) phase shift, this becomes

$d \sin\theta = n\lambda$ $\begin{cases} \text{destructive int. when there's} \\ \text{an additional } \tfrac{1}{2}\lambda \text{ phase shift} \end{cases}$

The balloon is at some given angle up in the air, so the maximum λ for which this will hold is for the smallest n, ie, $n=1$. So, our condition for the maximum wavelength for destructive interference is

$$d \sin\theta = \lambda_{max}$$

$$\Rightarrow \lambda_{max} = d \sin\theta \doteq \boxed{2x \left(\frac{h}{d}\right)}$$

using $d = 2x$ for the slit separation, and

$\sin\theta \doteq \theta \doteq h/d$ ← here d is the distance across the water.

CONSTRUCTIVE INTERFERENCE

The 180° phase shift leads to

$d \sin\theta = (n + \tfrac{1}{2})\lambda$ $\begin{cases} \text{const. interference when there's} \\ \text{addition } \tfrac{1}{2}\lambda \text{ phase shift} \end{cases}$

and the maximum λ will be that for which n is smallest; ie, $n=0$

$$d\sin\theta = \frac{1}{2}\lambda_{max}$$

$$\Rightarrow \lambda_{max} = 2d\sin\theta = \boxed{\frac{4xh}{d}}$$

Single-Slit Diffraction

This is just a basic single-slit problem. We know that

$$\sin(\theta) = \frac{m\lambda}{a}$$

where a is the slit width, λ the wavelength, and m is the number of dark fringes you are away from the central bright line. The angle θ is the angle from the slit to the m^{th} fringe, relative to the central bright fringe which we call $\theta = 0$.

From the picture, it looks like it's $\frac{1}{2}(17.9)$ mm from the central fringe to the $m=3$ dark fringe. Using the small-angle approximation, then,

$$\sin(\theta) \approx \theta = \frac{\frac{1}{2}(17.9)\,\text{mm}}{80\,\text{cm}} = \frac{17.9\,\text{mm}}{2(800)\,\text{mm}} = 0.0112\ \text{rad}$$

Thus, $0.0112 = \frac{3\lambda}{a} \Rightarrow$

$$a = \frac{3(633\times10^{-9}\,\text{m})}{0.0112} = 1.7\times10^{-4}\,\text{m} = \boxed{170\,\mu\text{m}}$$

For part B, we note again that

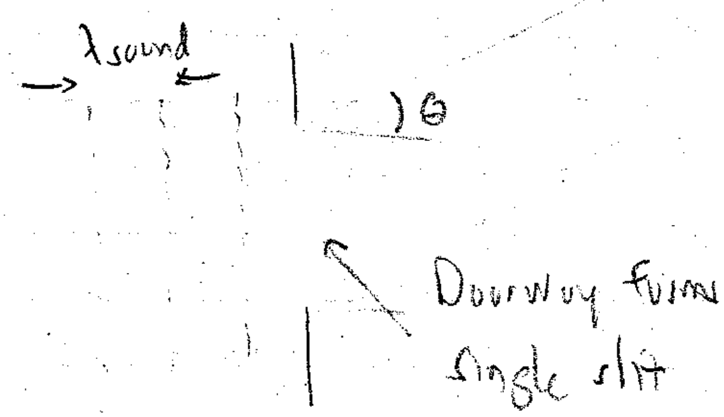
$$\sin\theta = \frac{m\lambda}{a}$$

and that, immersed in water, λ will shrink ($\lambda = \frac{\lambda_0}{n}$ when $n > 1$ is index of refraction) and a stay the same. So all angles lessen, and the width of the central peak $\boxed{\text{lessens}}$.

Doorway Diffraction

The doorway forms a single slit; again we use

$$\sin\theta = \frac{m\lambda}{a}$$



θ will be a minimum when $m = 1$. Then, we need to figure out λ, the wavelength of sound:

$$\lambda = \frac{V_s\ (m/s)}{f\ (1/s)} = \frac{V_s}{f}\ m \quad \text{where } V_s = \text{sound speed}, \quad f = \text{sound frequency}$$

$$\text{So, } \sin\theta = \lambda/a = \frac{V_s}{fa} \Rightarrow \theta = \sin^{-1}\left(\frac{V_s}{fa}\right) = 0.254 \text{ rad}$$

Using $a = 1.12\text{m}$ and noting that the small-angle approx. doesn't really apply.

Understanding Circular Aperture Diffraction

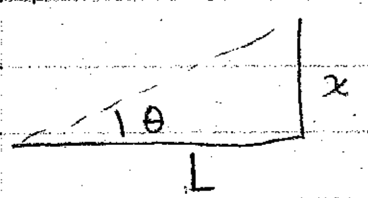
(A) The relationship between the angle and successive dark rings, for a circular aperture, is not as easily written as for slit diffraction, but for the first dark ring

$$\sin\theta = 1.22\frac{\lambda}{D}$$

so in radians $\theta = a\sin\left(\frac{1.22\lambda}{D}\right)$ or in degrees

$$\theta = \frac{180}{\pi}\, a\sin\left[\frac{(1.22)(0.0006328)}{.240}\right) = \boxed{0.184}$$

(B)



$$\frac{x}{L} = \tan\theta \approx \sin\theta = 1.22\frac{\lambda}{D}$$

$$\Rightarrow x = 1.22\frac{\lambda L}{D} = 1.22\left[\frac{(0.0006328)(3000)}{.240}\right]$$

$$= \boxed{9.65\text{ mm}}$$

(C) Quite simply, x from the previous part is the radius of the disk, so

$$A = \pi x^2 = \boxed{293\ mm^2}$$

(D) This is the same as part A, up to the substitution of different values for the wavelength and aperture diameter

$$\theta = \frac{180}{\pi} \arcsin\left[\frac{(1.22)(0.00000055)}{0.910}\right]$$ ← now working in m, not mm.

$$= \boxed{4.22\times10^{-5}} \text{ degrees}$$

(E) ~~A~~ second star separated from the first one by 4.22×10^{-5} will have its central bright spot in the middle of the first dark fringe, and thus is at the minimum angular separation to be resolved from the first star. So a separation of 5×10^{-5} degrees will be even more resolved

$$\Rightarrow \boxed{\text{yes}}$$

That's not much separation! This amount, for example, would be a separation of about 1000 feet on the surface of the moon.

Resolving Pixels on a Computer Screen

(a) For this issue of resolution, we apply Rayleigh's Criterion

$$\sin(\theta_1) = 1.22\,\lambda/d$$

Here, θ_1 is the angle subtended by two distinct pixels separated by $281\,\mu m = 2.81\times10^{-4}\,m$, when viewed by the eye at a distance of 1.3 m.
Since θ_1 is small,

$$\sin(\theta_1) \doteq \theta_1 = \frac{2.81\times10^{-4}}{1.3} = 2.162\times10^{-4}$$

Now, the diameter d of the eye is thus given by

$$\frac{1}{d} = \frac{\sin(\theta_1)}{1.22\,\lambda} = \frac{2.162\times10^{-4}}{(1.22)(5.5\times10^{-7})} = 322.1$$

$$\Rightarrow d = 3.10\times10^{-3}\,m = \boxed{3.10\,mm}$$

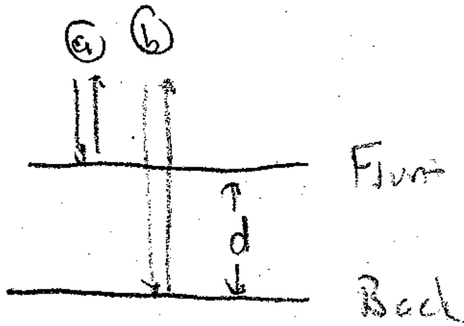
(b) Similarly, if x is viewing distance

$$\sin\theta_1 \simeq \theta_1 \simeq \frac{.36}{x} = \frac{1.22\lambda}{d} \Rightarrow x = \frac{(.36)(d)}{1.22\,\lambda}$$

$$= \frac{(.36)(3.10\times10^{-3})}{(1.22)(5.5\times10^{-7})} = \boxed{1663\,m}$$ or 1660 to 3 sig figs

Why Butterfly Wings Shimmer

(A)



The difference between paths (a) and (b) is $\boxed{2d}$

(B) For a full wavelength of path difference, $\boxed{\lambda = 2d.}$ This is also the longest possible wavelength for constructive interference

(C) In this case, we need to fit 3 full wavelengths into the path difference $2d$, so

$$2d = 3\lambda \quad \text{or} \quad \lambda = \frac{2}{3} d = \frac{2}{3}\, 300\,\text{nm} = \boxed{200\,\text{nm}}$$

(D) In this case, the problem specifies

$$\lambda_1 = \frac{2d}{1+\frac{1}{2}} = \frac{2d}{3/2} = \frac{4}{3} d = \boxed{400\,\text{nm}}$$

(E) Here, we just use the result of (B)

$$\lambda = 2d = \boxed{400\,\text{nm}}$$

(F) In a medium, the vacuum wavelength λ_0 shortens to

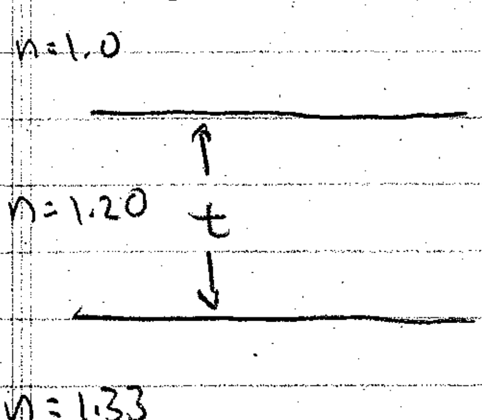
$$\lambda = \frac{\lambda_0}{n} = \frac{\lambda_0}{1.33}$$

By part (B) then,

$$\frac{\lambda_0}{1.33} = 2d \Rightarrow \lambda_0 = 2.66d = \boxed{532\text{ nm}}$$

(G) More 400-nm light would be reflected, for the reasons stated.

Thin Film (Oil Slick)



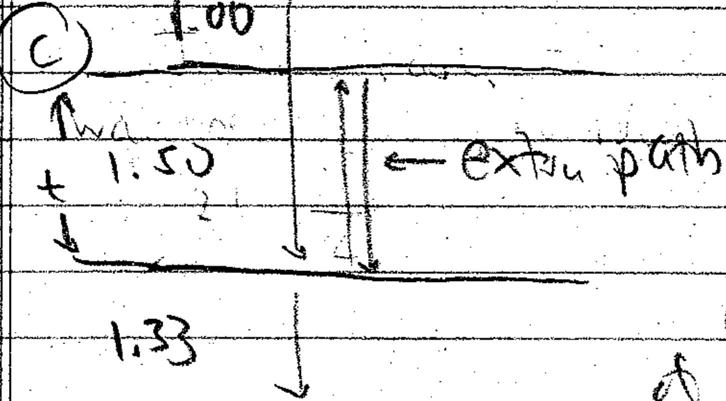
(A) Light reflecting of both the front and back surfaces will have a phase shift of π, so that phase shift cancels out when the two waves interfere. Thus we simply ask for one wavelength of extra path length:

$$2t = \lambda = \frac{\lambda_0}{1.20} \Rightarrow t = \frac{\lambda_0}{(2)(1.20)} = \boxed{313\text{ nm}}$$

(B) In this case there's no phase shift of π off the oil/water interface, so we get π from the air/oil interface, and only need π from the extra path length

$$\Rightarrow 2t = \frac{\lambda}{2} = \frac{\lambda_0}{2n} \Rightarrow t = \frac{\lambda_0}{(4)(1.5)} = \boxed{125\text{ nm}}$$

(C)



This one is a little tricky. Note that for the extra path, both reflections are off a lower index of refraction; so the full phase shift must come from the extra path:

$$\lambda_{oil} = 2t = 400\text{ nm}$$

But, $\lambda_{oil} = \dfrac{\lambda_{air}}{1.5} = \dfrac{1.33\,\lambda_{water}}{1.5}$

$$\Rightarrow \lambda_{water} = \frac{1.5}{1.33}\lambda_{oil} = \frac{600\text{ nm}}{1.33} = 451\text{ nm}$$

Using X-Ray Diffraction

(A) From the diagram shown in the problem,

$$n\lambda = 2d\sin\theta$$

For the first maximum, $n=1$, whence

$$d = \frac{\lambda}{2\sin\theta} = \frac{.220}{2\sin(21.5^\circ)}\ \text{nm} = \boxed{0.3001\ \text{nm}}$$

(B) Here, we use $n=2$ and d from above

$$\sin\theta_2 = \frac{2\lambda}{2d} = \frac{\lambda}{d} = 0.7331 \Rightarrow \boxed{\theta_2 = 47.1^\circ}$$

(C) **No** - as stated, were we to plug $n=3$ in above, we would find $\sin\theta_3 > 1$, which is a no-no.

Diffraction Grating Spectrometer

(A) For a diffraction grating, maxima are given by

$m\lambda = d\sin\theta$ so for $m=1$

$$\sin\theta = \frac{\lambda}{d} \Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

For a spacing of 800 lines/mm (8×10^5 lines/m)

$$d = \frac{1}{8\times10^5} = 1.25\times10^{-6}.$$

[i.e.]

For the lines given (4.98×10^{-7} m and 5.69×10^{-7} m) a difference

$$\Delta\theta = 3.6°$$ is found.

(B) The resolving power R is given by mN, where m is the order being viewed (2 in this case), and N the number of illuminated lines. This must be at least as big as the relative separation between the two lines you're trying to resolve, i.e.,

$$R \geq \frac{\overset{\text{wavelength of line}}{\lambda}}{\underset{\text{wavelength difference}}{\Delta\lambda}} = \frac{589}{589.59-589.00} = 998$$

So, $2N \geq 998$, or $N > 499$

At 1.25×10^{-6} m/line, this is 6.24×10^{-4} m or

$$\boxed{0.624 \text{ mm}}$$

Light of Differing Wavelength on a Grating

For the grating, again,

$m\lambda = d\sin\theta$. Using the small angle approxima. $\sin\theta \doteq \theta$, for $m=1$,

$\lambda = d\theta$. In terms of the separation y

observed a distance L away, $y = L\theta \Rightarrow \theta = y/L$

$$\lambda = \frac{dy}{L} \quad \text{or} \quad \Delta\lambda = \frac{d}{L}\Delta y$$

Using (working in m) $d = 1/90{,}000$ $L = 2.42$

and $\Delta y = 0.0034$,

$$\boxed{\Delta\lambda = 1.56 \times 10^{-8} \text{ m}}$$