MIDTERM I SOLUTIONS

Problem 1

(a) The volume will expand by

$$\frac{\Delta V}{V} = \beta \Delta T = 10^{-4} \cdot 100 = 10^{-2} = 1\%$$

However, the length of any side expands as the cube root of the volume, and the area of any face as the square of the length, so the area expands more slowly (as the 2/3 root) than the volume

$\Rightarrow$ less than 1%

(b) The buoyant force is just the weight of the water that's displaced. As the material cools and shrinks, it displaces less water & the buoyant force lessens, so it

$\Rightarrow$ sinks faster

(c) Melt: $m L_{melt} = 334\ \mathrm{J/g} \cdot m$

Heat: $m c \Delta T = m \cdot 4.184\ \mathrm{J/gC} \cdot 100^\circ C = 418.4\ \mathrm{J/g} \cdot m$

Vaporize: $m L_{boil} = 2257\ \mathrm{J/g} \cdot m$ $\Leftarrow$ biggest one by far!

(d) $\Delta P / \Delta h = \rho g \Rightarrow \Delta P = \rho g \Delta h$

The change in pressure depends only on depth h, and not on the size of the pool (same pressure at 3m depth in swimming pool or in ocean)

$$\boxed{\Rightarrow 1:1}$$

(e) If the object has twice the density of water, then the buoyant force will be ½ the weight of the object (the weight of the displaced water will be half the weight of the object). So,

$$F_{buoy} - F_g = \frac{1}{2}mg - mg = -\frac{1}{2}mg$$

The string that suspends the mass must then have a tension that cancels this, ie,

$$\boxed{T = \frac{1}{2}mg}$$

Problem 2 [25 Points]

A jar is filled with 6kg of tap water and sealed. The sealed jar, which has an R value of 0.1 K/W, is placed outside on a day for which the temperature is 0^0C, and the water in the jar starts to cool.

(a) At the point at which the water in the jar has cooled to 10^0C, what is the rate of heat flow into the jar, expressed in J/s? [8 points]

Since the R value of a material is the ratio of the temperature difference (where the heat is flowing from and to) and the amount of heat flowing across that difference, we have:

$$R=\frac{\Delta T}{H}\rightarrow H=\frac{\Delta T}{R}=\frac{273\mathrm{K}-283\mathrm{K}}{0.1\mathrm{K}/W}=\frac{-10}{0.1}W=-100\mathrm{W}=-100\mathrm{J}/s$$

Where the negative sign tells us that 100 Joules per second are flowing *out* of the jar.

(b) At that same point, what is the rate of change of the temperature of the water in the jar, expressed in ^{0}C per second? [8 points]

Recognizing that the heat-flow, H, is the change in heat over the change in time and using the relation Q = mcT, we can write:

$$H=\frac{\Delta Q}{\Delta t}=mc_w\frac{\Delta T}{\Delta t}\rightarrow\frac{\Delta T}{\Delta t}=\frac{H}{mc_w}$$

Where, from part (a), H = -100J/s, m = 6kg and c_w = 4.184 J/gK. Putting this together we have:

$$\frac{\Delta T}{\Delta t}=\frac{-100\mathrm{J}/s}{6000\mathrm{g}\cdot 4.184\,J/g\cdot K}=\frac{-1}{251}K/s=-0.0040\,K/s$$

Finally, since the Celsius and Kelvin scales differ only by an additive constant we have:

$$T_K=T_C+273\mathrm{K}\rightarrow\frac{dT_K}{dt}=\frac{dT_C}{dt}$$

So we see that the rate of change of the temperature is the same for the Celsius and Kelvin scales. (How about the Fahrenheit scale?)

ALTERNATIVE SOLUTION:

If we assume (incorrectly) that the rate of change of the temperature remains constant as the water goes from, say, 10^0C to 0^0C we can divide this temperature change by the amount of time it would take as follows:

Find the amount of energy lost,

$$Q=mc_w\Delta T=6000\mathrm{g}\,4.184\,J/gK(273\mathrm{K}-283\mathrm{K})=-251000\mathrm{J}$$

find the time it would take to lose that energy with a constant heat-flow H,

$$t=\frac{Q}{H}=\frac{-251000\text{J}}{-100\text{J}/s}=2510\text{s}$$

The rate of change of the temperature is given by:

$$\frac{\Delta T}{\Delta t}=\frac{\Delta T_{loss}}{t}=\frac{-10\text{K}}{2510\text{s}}=-0.0040\text{K}/s$$

Where we recall that the negative sign is telling us that energy is flowing *out* of the jar.

(c) Some time later, the water in the jar has cooled to 5°C. Is the rate of cooling greater or less at that point that the rate for part (b)? Why? [8 points]

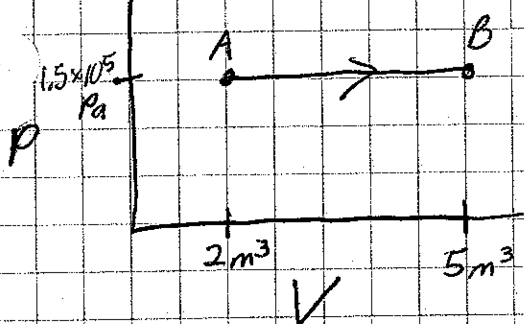
Now that the jar has cooled to 5°C the temperature difference in the formula for H has gotten smaller by a factor of two, making the absolute heat flow smaller by a factor of two.

$$H=\frac{\Delta T}{R}=\frac{-5\text{K}}{0.1\text{K}/W}=-50\text{W}$$

Since the rate of cooling is directly proportional to this temperature difference it also decreases by a factor of two.

$$\frac{\Delta T}{\Delta t}=\frac{H}{mc_w}=-0.0020\text{K}/s$$

Problem 3



A gas undergoes the change of state indicated on the PV diagram to the left. The gas temperature at A is 200 K. The gas is ideal, so the molar specific heats are given by $C_V = \left(\frac{3}{2}\right)R$ and $C_P = \left(\frac{5}{2}\right)R$, where R is the universal gas constant. Recall that the units of molar specific heat C_V, C_P are J/K·mol.

a. How much work is done by the gas on its surroundings in executing the change of state indicated by the path between points A and B?

The work done is just the area underneath the curve on the PV diagram.

$$W = \int_{V_1}^{V_2} P\,dV = P\int_{V_1}^{V_2} dV = P(V_2 - V_1)$$

$$= 1.5\times10^5\,\text{Pa}\,(5m^3 - 2m^3) = \boxed{4.5\times10^5\,J}$$

where I have used the fact that pressure is constant.

b. What is the temperature of the gas at point B?

The pressure and number of moles of gas are constant, so using the ideal gas law:

$$\frac{V}{T} = \frac{nR}{P} = \text{constant} \quad \text{so} \quad \frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$T_B = \frac{V_B}{V_A}T_A = \frac{5\,m^3}{2\,m^3}\,200K = \boxed{500K}$$

c. How much heat does the gas draw from the environment in going from point A to point B?

$$Q = n C_p \Delta T$$

← because process is at constant pressure

n is found via the ideal gas law

$$n = \frac{P_A V_A}{R T_A} = \frac{1.5 \times 10^5 \, Pa \cdot 2 m^3}{8.314 \, J/K \cdot mol \cdot 200K} = 180.4 \, mol$$

$$Q = 180 \, mol \times \frac{5}{2} \times 8.314 \, J/K \cdot mol \cdot (500K - 200K) = \boxed{1.1 \times 10^6 \, J}$$

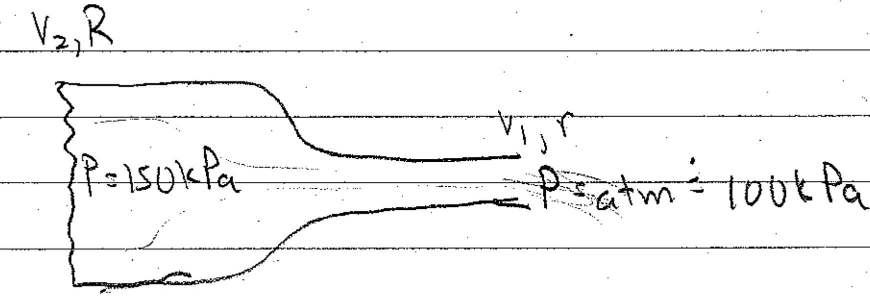
This can also be done by realising the heat drawn is the change in internal energy plus the work:

$$Q = \Delta U + W = n C_v \Delta T + W$$

$$= 180 mol \times \frac{3}{2} \times 8.314 \, J/K \cdot mol \, (500K - 200K) + 4.5 \times 10^5 J$$

$$= \boxed{1.1 \times 10^6 J}$$

Prob 4



ⓐ By continuity, $V_2 A_2 = V_1 A_1$, so

$$V_2 \pi R^2 = V_1 \pi r^2 \Rightarrow \boxed{V_2 = \frac{r^2}{R^2} V_1}$$

ⓑ By Pascal's law,

$$P_2 + \tfrac{1}{2}\rho V_2^2 = P_1 + \tfrac{1}{2}\rho v_1^2$$

Letting $\Delta P = P_2 - P_1$ and $\alpha = r^2/R^2$,

$$\Delta P = \frac{1}{2}\rho(v_1^2 - V_2^2) = \frac{1}{2}\rho v_1^2\left[1 - \alpha^2\right]$$

$$\alpha^2 = 1 - \frac{\Delta P}{\frac{1}{2}\rho v_1^2} = 1 - \frac{50\,kPa}{\frac{1}{2}(10^3)(11)^2} = 1 - .826 = 0.174$$

$$\left(\frac{r^2}{R^2}\right)^2 = 0.174 \Rightarrow R = \sqrt[4]{\frac{r^4}{0.174}} = \frac{r}{\sqrt[4]{0.174}} = \boxed{3.10\,m}$$

remembering that $50\,kPa = 50{,}000\,Pa$