Supersymmetry, Axions and Cosmology

T. Banks, M. Dine

Santa Cruz Institute for Particle Physics, Santa Cruz CA 95064

M. Graesser

California Institute of Technology, 452-48, Pasadena, CA 91125

Abstract

Various authors have noted that in particular models, the upper bound on the axion decay constant may not hold. We point out that within supersymmetry, this is a generic issue. For large decay constants, the cosmological problems associated with the axion’s scalar partner are far more severe than those of the axion. We survey a variety of models, both for the axion multiplet and for cosmology, and find that in many cases where the cosmological problems of the saxion are solved, the usual upper bound on the axion is significantly relaxed. We discuss, more generally, the cosmological issues raised by the pseudoscalar members of moduli multiplets, and find that they are potentially quite severe.
1 Introduction

There are three known solutions to the strong CP problem. One is the possibility that the $u$ quark mass vanishes. While there are arguments and lattice calculations which cast doubt on this possibility, it is probably fair to say that it cannot be totally ruled out within our present level of understanding. The second is that CP is a good symmetry of the underlying theory, spontaneously broken in such a way that the observed $\theta$ is very small (we will refer to this as the Nelson-Barr mechanism [1]). The third possibility is that there exists an axion. The decay constant (mass) of this axion is tightly constrained by astrophysical and cosmological considerations.

From the point of view of conventional effective field theory, the axion idea seems at first sight implausible. It requires postulating a symmetry and then supposing that it is only broken by tiny QCD effects. Within the framework of supersymmetric field theories, a sufficiently light axion to solve the strong CP problem can be obtained with large discrete symmetry groups. For non-supersymmetric theories, the symmetries must be very large. But in string theory, axions which seem to have the correct properties abound. Indeed, the term “axion” is used rather generally for scalar fields with $2\pi$ periodicity (in states which conserve CP, these fields are pseudoscalars). Such axions arise both in supersymmetric and non-supersymmetric string backgrounds. String theory axions are generally related to components of antisymmetric tensor gauge fields, either in extra dimensions or the usual four. There are often arguments that in appropriate regions of string theory moduli space, continuous shifts of these fields are approximate symmetries which are broken primarily by the coupling to low energy gauge multiplets.

If we assume low energy supersymmetry, these axions are accompanied by (scalar) moduli. With our current understanding of string theory, we cannot determine whether, in the full non-perturbative theory, any of these axions can solve the strong CP problem, but it is quite plausible that they do. In this paper, we will distinguish generic axions by lower case, and reserve “Axion”, with a capital A, for the QCD axion.

In this paper we will focus on two issues connected with axions and their partners in supersymmetric theories.

- With very mild assumptions, axions are light relative to their scalar partners. If the axion decay constants are of order the Planck scale, the axions pose cosmological problems
significantly more serious than those associated with scalar moduli. For smaller decay constants, these problems may be ameliorated. We will discuss other possible solutions as well.

- The partner of the QCD Axion, the Saxion, will have a mass of order the weak scale or smaller. While the existence of this particle has been noted in a number of contexts, here we focus on a general point: The cosmological problems posed by this modulus are far more severe than those posed by the Axion. So it does not make sense to discuss cosmological limits on the Axion before considering a solution to the Saxion problem. Many solutions to the saxion problem solve the Axion problem automatically. In other words, the usual cosmological upper bound on the axion decay constant is not necessarily relevant to supersymmetric theories.

One of the principle suggestions to solve the cosmological moduli problem is that the (scalar) moduli are heavy, with masses of order 10’s of TeV, and that their decays reheat the universe above nucleosynthesis temperatures. In this case, we will see that there is a window in axion decay constant, $f_a$, around $10^{15}$ GeV in which Axions might well be the dark matter. A similar window, as we will see, exists for other possible axions.

In the next section, we investigate the problem of axion mass. We focus on models in which supersymmetry is broken at an intermediate scale, leaving low energy supersymmetry breaking for the conclusions. We explain why axions are parameterically light, and discuss conditions under which there is an axion light enough to solve the strong CP problem. Section 3 contains a review of the QCD Axion in the context of supersymmetry. Some simple field theory models which give variable $f_a$ are described. In section 4, we review aspects of the conventional moduli problem, and discuss the implications of light pseudoscalars. We enumerate possible solutions of these problems. In section 5, we turn to the cosmology of the QCD Axion; we explain why, for decay constant less than $M_p$, the cosmological problems of the Saxion are more severe than those of the Axion. Possible solutions of the Saxion problem, and their implications for the Axion, are considered. In general, we find that the Axion window extends up to $f_a = 10^{15}$ GeV, but the precise limits are model-dependent. In section 6, we discuss alternative solutions to the strong CP problem, and their possible implementations.

We also remark on some aspects of preheating in the context of moduli. While coherent production of scalars during inflation has been discussed as another aspect of the moduli problem, we explain why these are essentially the same thing. Coherent production of gravitinos has also been discussed in the literature. As we will see, the real issue is production of “mod-
ulinos.” This problem, like the problem of axions, is significantly less severe than the usual moduli problem. Solutions to the latter generally resolve the former. In our concluding section we discuss some remaining issues, including some thoughts on axions in non-supersymmetric models.

2 Are Axions Light?

In discussing the moduli problem in string theory, one usually focuses on the “scalar” component of the moduli, and ignores the pseudoscalar. The term pseudoscalar is, of course, a misnomer. It implies an unbroken CP symmetry. The more precise distinction refers to the fact that in string/M theory, in a modulus supermultiplet there is typically a field with a $2\pi$ periodicity. The QCD Axion would be one example of such a field. We will follow a standard usage in string theory, and refer to these fields as axions, or occasionally simply pseudoscalars. Both choices have their linguistic limitations. In general, we can ask how such fields can gain mass, and how massive they might be.

The first question is whether the pseudoscalar is lighter than the scalar field. In principle, once supersymmetry is broken, both fields can gain mass. One might expect this mass to be comparable to the mass of the scalar. But in many pictures for how supersymmetry might be broken, this is not the case.

String theory, if it describes nature, is presumably strongly coupled. On the other hand, it must also contain, effectively, small parameters, if it is to explain the hierarchy and the small values of the observed gauge couplings. We will discuss two suggestions which have been put forward to explain these facts shortly. But independent of specific models, if one assumes that supersymmetry is related to the solution of the hierarchy problem, one usually also assumes that the superpotential, at the very least, is hierarchically small. For example, if the unified gauge couplings are related to a modulus, $S$, the superpotential is assumed to be of order $e^{-S/b}$, for some rational number $b$.

The periodicity properties of the pseudoscalar can provide significant constraints on the superpotential: the superpotential must take the form:

$$W = e^{-aM} + e^{-bM} + \ldots$$

for some constants $a,b,\ldots$, $a < b < \ldots$ So if we ignore possible symmetry breaking from the Kahler potential, the full, supergravity potential will be of order $e^{-2aM}$, but the leading
terms which violate the Peccei-Quinn symmetry will be suppressed by a further exponential, $e^{-(b-a)M}$.

In making this argument, we are assuming that there are not large corrections to the Kahler potential which violate the Peccei-Quinn symmetry. In principle, terms like

$$e^{-(M-M^\dagger)} f(M + M^\dagger)$$

(2)

are consistent with the $2\pi$ shift symmetries. It is usually assumed that because such terms do not arise in string perturbation theory, they are very small. But we expect that the minimum of the potential lies in a regime where perturbation theory is not a useful guide. For the superpotential, with our assumption that $W$ is extremely small, holomorphy still potentially provides significant constraints, but for the Kahler potential, this is not the case. Arguments have been given that these terms might be small, but they are not based on reliable calculations\[5\]. If these corrections are $\mathcal{O}(1)$, they would solve the problem of pseudoscalar moduli. But this would also mean that the axions seen in string perturbation theory are not relevant to the solution of the strong CP problem.

We can examine proposals for supersymmetry breaking in order to get some sense of what might happen. Two suggestions for how moduli might be stabilized while generating small gauge couplings and large hierarchies are known as “Kahler stabilization”\[6\] and the “racetrack”\[7\].

In Kahler stabilization, one assumes that non-perturbative dynamics, such as gluino condensation, generates a superpotential for the moduli. For large values of the moduli, the form of this superpotential can be determined. For example, for the weak coupling dilaton, $S$, of the heterotic string, $W \sim e^{-aS}$, for some constant $a$. In the Horava-Witten limit, $W \sim e^{-aS-bT}$. The hypothesis is that the ground state lies at large values of the moduli, so the superpotential can be reliably calculated, but that the Kahler potential cannot be calculated, and that it is the Kahler potential which leads to stabilization of the moduli. The motivation behind these proposals is that holomorphy constrains the nonperturbative corrections to the superpotential to be exponential in the moduli, while the Kahler potential is not similarly constrained. In particular, in string perturbation theory we expect\[8\] corrections to the Kahler potential of order $e^{-b v Re S}$.

In these models, the Kahler potential is also responsible for cancelling the cosmological constant, so no field independent term is added to the superpotential. In the case of gaugino condensation, assuming that there is only a single modulus, $S$, for simplicity, the leading term in the superpotential has the form $e^{-3S/b_0}$. Requiring that this scale account for supersymmetry
breaking masses of order 1 TeV, gives a scale of gaugino condensation of order $10^{13}$ GeV. This means $e^{-S/b_o} \approx 10^{-5}$.

In the case of Kahler stabilization, corrections to $W$ will arise, for example, from operators such as $W_\alpha^2 W_\beta^2$ in the high scale effective lagrangian. These lead to a superpotential of the form:

$$W = e^{-3S/b_o} + e^{-6S/b_o}. \quad (3)$$

This form is dictated by holomorphy and the non-anomalous discrete $Z_2$ R-symmetry preserved by the higher-dimension operator. This superpotential gives rise to a mass-squared for the pseudoscalar of order $e^{-3S/b_o}$, smaller than the mass of the scalar, and perhaps more importantly, a mass smaller than the scale of supersymmetry breaking by $e^{-3S/2b_o} \sim 10^{-7.5}$. So the pseudoscalar, in this picture, is much lighter than the scalar. Assuming, for example, that the scalar moduli have masses of order TeV, the axion has a mass of order 10 KeV (if the mass is 100 TeV, the axion has mass of order 1 MeV)

The pseudoscalar in this model is light, but it is not light enough to play the role of the QCD Axion. The QCD Axion might simply lie in another multiplet. This multiplet might not couple to any gauge group stronger than QCD. Alternatively, there can be a further suppression of the mass due to discrete symmetries. A discrete $Z_{2n}$ R symmetry can suppress $W_{a}^{2n+2}$ type operators, up to some value of $n$. One needs roughly $n \geq 3$ to solve the strong CP problem. More generally, such symmetries could suppress the masses of other axions, perhaps mitigating some of the cosmological problems we will describe.

So far we have assumed there is only one modulus and one strong group. But there could be additional moduli that couple to other groups. We assume, as before, that there is no constant term in the superpotential. This is consistent with the assumption that there is only one scale in string theory. We also assume that for these moduli, supersymmetry breaking results in a local minimum even if only the leading term in the superpotential is retained. Further, the dominant contribution to the moduli masses is assumed to come from supersymmetry breaking. Then the previous results still follow, namely, that all the pseudoscalars will be extremely light.

For the case of the racetrack, the situation is somewhat different, and again model-dependent. Indeed, there are not, to our knowledge, entirely satisfactory models in which supersymmetry is broken in this fashion, so we have to speculate even more about what such a scheme might look like. Roughly, however, we would guess that the superpotential will have
the structure:

$$ W = \alpha e^{-aS} + \beta e^{-bS} $$

(4)

with $a$ and $b$ very nearly equal to each other. One might imagine $a, b \sim \frac{1}{N}$, $a - b \sim \frac{1}{N^2}$, and $S = \frac{1}{g^2} \sim N^2$, for some large integer $N$. If $\alpha$ and $\beta$ have the opposite sign there is a locally supersymmetric minimum. Here the mass of the supermultiplet can be large, unrelated to the size of supersymmetry breaking. If $\alpha$ and $\beta$ have the same sign there is no local minimum. Brustein and de Alwis have argued for a more general statement [10]. Assuming a dilaton Kahler potential similar to that at tree level, and that the superpotential is steep, a characteristic of racetrack models, they find that all local minima are also locally supersymmetric.

One may try to avoid this conclusion by adding in more fields:

$$ W = Xf(S) + Yg(S) $$

(5)

where $f$ and $g$ have the racetrack form (4), but the parameters appearing in these functions are different. For non-vanishing $S$ this model breaks supersymmetry. Depending on the choice of parameters, local minima can be found that break supersymmetry at large values of $S$. These minima, however, are approximately supersymmetric. One finds that the splitting cannot be increased too much, otherwise the local minimum is lost. If the (scalar and pseudoscalar) moduli have masses sufficiently far above the scale of supersymmetry breaking, there will be no moduli problem [8].

More generally, these last models, in the limit that the supersymmetry-splittings are small, are representative of a third possibility, that some moduli are stabilized above the scale of supersymmetry breaking, and that supersymmetry breaking is a low energy effect, as in gauge mediation. In this case, there need be no moduli problem at all. We will comment on this possibility later.

A few remarks about the Axion multiplet in such models are in order. In general, there is no reason why the Axino and Saxion masses shouldn’t be of order $m_{3/2}$. The operators:

$$ \int d^4\theta (A + A^\dagger)^2 Z^\dagger Z; \int d^4\theta (A + A^\dagger)^2 (Z + Z^\dagger), $$

(6)

where $F_Z \approx m_{3/2}M$, give mass to both fields. One might try to forbid the second operator by a symmetry, but such a symmetry is also likely to forbid gaugino masses.
3 The QCD Axion and Supersymmetry

In this section, we consider some aspects of the QCD Axion in the context of supersymmetry. We review the general couplings of the Axion and Saxion, and construct field theory models of axions with various decay constants. In effective field theory, we show how, with suitable discrete symmetries, one can obtain an Axion capable of solving the strong CP problem with a wide range of decay constants.

3.1 The QCD Axion and the Saxion

As we have said, in general, the Axion must be part of a superfield, \( \mathcal{A} \), whose scalar component is \( s + iA \). \( s \) is known as the saxion. The existence of the axion coupling to QCD implies a supersymmetric coupling:

\[
L_a = \frac{\mathcal{A}}{16\pi^2 f_a} W_a^2.
\]

(7)

If there are gauge groups more strongly coupled than QCD, \( \mathcal{A} \) can couple to them only if they possess accidental chiral symmetries which hold to a high degree of accuracy. The field \( s \) cannot gain mass larger than some characteristic scale of supersymmetry breaking, since otherwise the Axion would gain such a large mass as well. The axion multiplet also contains an axino, whose possible cosmological implications have been widely discussed. As we will remark shortly, the properties of the axino are model-dependent. In general, as we will explain, one expects that the axino is quite massive, so that the relevant cosmological question is the number of axinos produced subsequent to inflation.

3.2 Models for Axions and Expectations for Axion Decay Constants

While string theory may be the most robust context in which to consider axions, it is interesting to consider other frameworks. There are two reasons for this. First, as we have noted, short distance, non-perturbative effects in string theory could spoil the Peccei-Quinn symmetry needed to solve the strong CP problem. Second, most frameworks for thinking about supersymmetry in string theory constrain the Axion decay constant to be rather large, typically of order the Planck scale, give or take (possibly crucial) factors of \( 16\pi^2 \). In the Horava-Witten limit of the heterotic string, its decay constant can be still smaller. Certainly numbers like \( 10^{15} \) GeV don’t seem at all unreasonable. But scales like \( 10^{11} \) GeV would seem more likely associated with, say, the scale of supersymmetry breaking in the effective low energy theory.
We can construct, as have many authors\cite{12, 13}, various field theory models which give rise to accidental Peccei-Quinn symmetries. One can, in particular, construct models with superpotential:

\[ W = c + \frac{1}{M^n} S^{n+2} S' S \bar{q} q + \frac{1}{M^{m+n}} S^{m+n+2} S' + \ldots \]  

(8)

where \( q \) and \( \bar{q} \) carry color and perhaps weak isospin and hypercharge; \( c \) is a constant with dimensions of mass cubed, \( m \) is some large integer, and \( M \) is the Planck scale. It is important that \( m \) (and certain other integer powers) be large in order that the Peccei-Quinn symmetry (the approximate global symmetry which rotates \( S \) by a phase) hold to the required degree of approximation. This structure can be enforced by discrete symmetries. It is important, in this framework, that the symmetry not be an \( R \)-symmetry, since this will be violated by the constant in \( W \) required to cancel the cosmological constant.

Supersymmetry breaking will generate a potential for \( S \) and \( S' \) which can lead to a large expectation value for \( S \). Integrating out the fields \( q \) and \( \bar{q} \), leads to Axion-like couplings for \( A = \ln(S) \). In the case where supersymmetry is broken in a hidden sector at an intermediate scale, \( S \) and \( S' \) will acquire susy-breaking masses of order \( m_{3/2} \). With suitable signs for these terms, the \( S \) vev will be of order:

\[ S = f_a \approx (m_{3/2} M^n)^{\frac{1}{m+n+1}}. \]  

(9)

The Saxion mass is of order \( m_{3/2} \) for such models. Note that for \( n = 1 \), the Axion decay constant is about \( 10^{11} \) GeV, in the range of decay constants in which Axions are the dark matter according to the conventional analysis.

The Axion in this model gets a potential of the form

\[ L_{pq} = S^{2n+m+4} \]  

(10)

(Other terms, such as those arising from the \( A \) terms, are of comparable size. Also, the constant \( c \) added to cancel the cosmological constant leads to mass mixing between the Axion and \( S' \), but this effect is numerically smaller than the term we describe here.) This is suppressed by \( \frac{S^m}{M^m} \) relative to the saxion mass squared. While this is a very small number, in order that the QCD contributions dominate, it is necessary that \( m \) be very large, the precise value depending on the Axion decay constant (\(< S >\)).

As an aside, note that this potential also illustrates the general issue we discussed earlier of light pseudoscalars in moduli multiplets. On the one hand, one requires a discrete symmetry
to obtain a large expectation value for $S$. On the other hand, this symmetry tends to make the pseudoscalar and scalar masses very different.

An alternative possibility is that supersymmetry is broken at lower energies, as in gauge-mediated models. In this case, for large $S$, one expects that the supersymmetry-breaking part of the $S$ potential behaves as

$$V_s(S) = -\epsilon |F|^2 \left( \ln(|S|^2/M^2) \right)^2.$$  \hspace{1cm} (11)

Here $\epsilon$ typically includes various loop factors, and so may be rather small, and $M$ is the messenger scale. Assuming a superpotential of the form above, gives for $S$:

$$S \approx (\epsilon |F|^2 M^{2n})^{1/3n}.$$  \hspace{1cm} (12)

The mass for the scalar is $m_s \sim \sqrt{\epsilon} F/S$. But again, the pseudoscalar mass squared is suppressed by the amount $S^m/M^m$.

4 The Scalar and Pseudoscalar Moduli Problems

4.1 The Scalar Moduli Problem

This problem is most concisely phrased in the language of string theory (quantum gravity). One supposes that one has some fields which vary over scale $M$, with a potential whose characteristic size is $m_{3/2}^2 M^2$ ($M$ might be the Planck or string scale, or, as we will discuss later, $f_a$). At early times, the characteristic curvature of the modulus potential is of order $1/t \approx H$ \cite{15}. So one expects the minimum, if any, of the modulus potential to lie a distance of order $M$ from the flat space minimum. The modulus begins to oscillate about its true minimum when $H \approx m_{3/2}$, at which time it constitutes a fraction of order one of the total energy density of the universe. Even if the universe is radiation dominated at this time, it quickly becomes matter dominated and remains so until the modulus decays. A plausible expectation for the decay width is

$$\Gamma = \frac{1}{2\pi} \frac{m_{3/2}^3}{M^2}.$$  \hspace{1cm} (13)

Assuming $m_{3/2} \approx 1$ TeV and $M \sim M_{Pl}$ this gives a decay time long after nucleosynthesis, and a reheating temperature of order a few KeV. This totally spoils the successes of conventional big bang nucleosynthesis.

One possible solution to this problem is to suppose that the minimum of the potential in the very early universe coincides with the minimum now. This is natural if the minimum preserves
Another proposal to solve this problem is to assume that the modulus is much more massive, with a mass more like 100 TeV. In this case, the reheating temperature is about 7 MeV, high enough to restart nucleosynthesis. Assuming that reheating is the solution, there are a number of issues which one must address. One is production of baryons. The dilution of any pre-existing baryons in these decays is very substantial, a part in $10^7$. In order that one end up with a reasonable baryon density, one must suppose that baryons are produced in the decays (through R-parity violating operators), or that the pre-existing baryon density is very large (e.g. as a result of very efficient AD Baryogenesis).

Second, if there is a stable LSP, there is a potential issue with overproduction of these objects. But Moroi and Randall have pointed out that if the decaying modulus is so heavy, the decays to LSP’s can be suppressed by the ratio of the LSP mass to the heavy modulus mass (more generally the ratio of the masses of the MSSM fields, squarks, sleptons, gauginos, etc.). For example, couplings of a modulus through kinetic terms to fermions, for on shell fermions, are proportional to the mass of the fermion. Similar, couplings to scalars are proportional to the mass-squared of the scalars. Decays to gravitinos are potentially a problem, but one might expect that if the moduli are so heavy, so are gravitinos, and that decays to gravitinos might be kinematically forbidden. Moroi and Randall indeed argued that such a large modulus (and gravitino) mass might arise naturally in anomaly mediated supersymmetry breaking.

In thinking about axions, we will want to consider more general possibilities for the scalar “decay constant,” i.e. the scale over which the field varies, as well as its couplings to gauge fields and other fields. As the decay constant of the scalar becomes smaller, adequate reheating can be obtained with a smaller mass. In particular, a mass of order 1 TeV and decay constant of order $10^{15}$ gives reheating to 10 MeV. However, the lower mass raises again the specter of overproduction of LSP’s. In this case, one might require either breaking of R parity, or a wino LSP (see ).

4.2 Scalar Moduli Decays

The decays of the saxion have been discussed in the literature, and decay of the scalar moduli raise similar issues. Scalars can, in principle, decay to quarks and leptons, to gauge fields, and to axions. The decays of saxions to gauge bosons are model-independent, and can be
parameterized in terms of $f_a$. The decays to axions and matter fields are model-dependent. Among familiar string moduli, at weak coupling the heterotic string field $S$ couples at tree level to axions, but not to matter fields; the $T$ modulus couples both to axions and to matter fields. In the class of field theory models described above, in which $S$ couples to the vector-like pair $q, \bar{q}$, there are tree level couplings of the saxion to axions, of the form

$$\frac{a}{f_a} s (\partial_{\mu} a)^2$$

(14)

where $a$ is a constant of order unity. There are no tree level couplings to quark and lepton fields. Couplings to gauge bosons arise at one loop. As a result, the principle decay mode of the saxion in such models is to axions. As discussed subsequently, this leads to cosmological difficulties.

It is possible to modify this class of models in such a way that the saxion does couple to matter fields already at tree level. By suitably choosing discrete charges, one can arrange that the Higgs boson transforms under the Peccei-Quinn symmetry, giving rise to such tree level couplings, with strength proportional to the Higgs Yukawa coupling.

There are two sets of issues involved with decays to axions. First, in cases where the moduli don’t couple to matter fields at tree level, axions can easily end up carrying an order one fraction of the energy when the scalars decay. If the scalars couple to matter fields, this fraction can be suppressed by the number of matter fields. However, if the axions are not lighter than roughly a few hundred electron volts, this is still problematic, since they will come to dominate the energy density before recombination.

### 4.3 The Pseudoscalar Moduli Problem

We have seen that there are many possible values for the axion mass and decay constant. Consider, for example, the Kahler stabilization model discussed above. We indicated that for the pseudoscalar component of $M$, the mass is naturally a few orders of magnitude below that of the scalar. If the decay constant of this axion is of order the Planck mass, then when it begins to oscillate it carries an order one fraction of the energy density of the universe. Even if the decay of the scalar reheats the universe to temperatures of order nucleosynthesis temperatures, the axion quickly comes to dominate the energy density, leading to an unacceptable cosmology.

If the axion decay constant is smaller, than the situation is different. First, the scalar modulus lifetime is much shorter, and for a given scale of supersymmetry breaking, the reheating
temperature is higher. For example, if the scalar mass is $10^3$ GeV and the decay constant is $10^{15}$, the reheating temperature is of order a few MeV.

Second, when the axion begins to oscillate, it carries a fraction of the energy density of order $f^2_a/M_p^2$. Now, if the universe reheats to nucleosynthesis temperatures, this fraction will be small enough if $f_a < 10^{15}$ GeV or so. In this case, at 10 MeV, the energy fraction is less than $10^{-6}$, so it is at most of order one at recombination. So in this case, both moduli problems are potentially solved.

We do, however, have to worry about the problem of scalar decays to axions, discussed in the previous section. As we have indicated, in most models, the scalar already decays to axions at tree level. Scalar decay to axions leads to two problems. During nucleosynthesis axions increase the expansion rate of the universe. This limits the branching fraction of saxion decays into axions to be less than roughly $1/20$, so that effectively the axions do not contribute more than one neutrino species. This is easily satisfied in models where the saxion also decays to a large number of standard model species.

The second problem is that even if the universe reheats to temperatures above nucleosynthesis, once it has cooled to temperatures somewhat below the axion mass the axion will come to dominate the energy density. It is important, as we noted, that the axion not carry an order one fraction of the energy density when the moduli decay. It is also important that the axions not be heavy; otherwise, they will come to dominate the energy density long before recombination. One solution to this problem is to suppose that the axion is very light. The axion abundance today is found using the standard calculation relating it to the density at the time of decay. Including the above constraint on the branching fraction, the overclosure bound is satisfied with a mass of order, say, 50 – 100 eV. As we have indicated, such a light axion can arise, even with strong gauge groups, in the presence of discrete symmetries.

If there are additional axions, there are various possibilities for their masses and couplings. For example, if there is only one strong gauge group, a second axion may play the role of the QCD axion, provided that non-perturbative effects which break the PQ symmetry are small enough. A third axion might be much lighter still, so that it does not pose significant cosmological difficulties.

We can summarize, then, the solutions to the pseudoscalar moduli problem:

- The pseudoscalar is sufficiently heavy that its decays restart nucleosynthesis. This, how-
ever, requires that the scalar moduli have masses of order $10^5$ TeV or larger.

- The scalar moduli masses are sufficiently large to restart nucleosynthesis, while the decay constant is large ($f_a > 10^{15}$), and the pseudoscalars are extremely light, so that they do not begin to oscillate until times of order recombination time. This requires that the pseudoscalar mass be of order $10^{-36}$ GeV. This requires, in our discussion above, very large discrete symmetries.

- There are order one PQ breaking terms in the Kahler potential, so the pseudoscalar and scalar have comparable masses. The main difficulty with this proposal is that it leaves us without a candidate for the QCD axion, except perhaps from something generated at low energies as in our field theory model above.

- The decay constant of the axion supermultiplet is $10^{15}$ GeV or smaller. In this case, the scalar mass can be of order 1 TeV, yielding sufficient reheating to restart nucleosynthesis. The axion mass density is suppressed by $f_a^2/M_p^2$. However, it is also probably necessary that the axion be very light, with mass of order several tens of eV or less. Otherwise, as we noted above, non-relativistic axions, produced either by misalignment or from scalar decays will come to dominate the energy density well before recombination. Again, we have indicated above how discrete symmetries might lead to such a light axion.

- As we will discuss later, another possibility is that there are no axions.

### 4.4 Other Cosmological Problems: Axions, Gravitinos, Modulinos

In the conventional picture the universe just after inflation was described by a thermal gas of particles and superparticles. Gravitinos were produced in the collisions of these particles. If these gravitinos decayed during the era of nucleosynthesis, their abundance must have been less than $n_{3/2}/n_\gamma \lesssim 10^{-12}$ in order that not too much helium was destroyed or too much deuterium produced. Here and in the following $n_\gamma \equiv \xi(3)T^3/\pi^2$ does not include a factor of $g_*$, the number of relativistic degrees of freedom. This effect provides the strongest constraint on the initial reheat temperature of the universe [19, 20]. The gravitino abundance during the nucleosynthesis era is obtained by integrating the Boltzmann equation. Just after inflation, the gravitino to photon ratio was roughly $g_*T_{RH}(1)/M_{Pl} \sim T_{RH}(1)/M_{Pl}$. But since the number of degrees of freedom has changed between the end of inflation and the nucleosynthesis era, one approximately accounts for this effect by adding in a dilution factor of $\gamma = g_*(T_{RH}^{(1)})/g_*(keV) \approx$
The abundance at the time of nucleosynthesis is approximately \( n_3 \approx 2 \times n_\gamma \times \frac{T_{RH}^{(1)}}{M_{Pl}} \sim 10^{-2} \frac{T_{RH}^{(1)}}{M_{Pl}} \). 

(15)

This implies a limit of roughly \( 10^9 \) GeV on the reheat temperature.

The late decay of a Saxion or saxion produces entropy, which dilutes any preexisting gravitinos or other relics. The dilution factor is obtained in one of two equivalent ways. One may use the standard formula for the entropy production of a late-decaying particle \([21]\). This method involves following the gravitino to photon ratio through all the eras. Or instead one can note that after inflation, the gravitino to saxion number remains constant and equal to roughly \( \left( \frac{m_S}{M_{Pl}} \right) = \left( \frac{f_a^2}{M_{Pl}^2} \right) \). This lasts until the saxion decays, converting its energy to radiation at a temperature \( T_{RH}^{(2)} \). At this point the saxion to photon number at the second reheating is determined. Combining this with the previous saxion to gravitino number gives

\[
\frac{n_3}{n_\gamma} \approx \frac{M_{Pl}^2 T_{RH}^{(2)}}{f_a^2 T_{RH}^{(1)}} \left( \frac{T_{RH}^{(1)}}{M_{Pl}} \right).
\]

(16)

Note that the ratio of degrees of freedom between the nucleosynthesis and first reheating eras appearing in \((15)\) does not appear here. This is because the Saxion decays just before nucleosynthesis, long after decaying standard model particles and superparticles have transferred their entropy to electrons and photons. More generally, we define the total dilution factor \( \gamma \) to be \( Y(T_{RH}^{(2)}) \equiv Y(T_{RH}^{(1)})/\gamma \). It is given by

\[
\gamma \approx \frac{f_a^2 T_{RH}^{(1)}}{M_{Pl}^2 T_{RH}^{(2)}} = \frac{Y_{SM} m_S}{T_{RH}^{(2)}}.
\]

(17)

This can be substantial, allowing for a higher reheat temperature. With \( f = 10^{15} \) GeV, \( T_{RH}^{(2)} = 10 \) MeV, \( T_{RH}^{(1)} = 10^{11} \) GeV one finds \( \gamma \sim 10^7 \). This changes the bound on the reheating temperature to \( 10^{13} \) GeV.

There is another point worth emphasizing. The decay constant appearing in the dilution factor is that of the decaying saxion. As we will stress in subsequent sections, there are cosmological scenarios in which this particle is not the Saxion. For the more general saxions there is no cosmological upper bound on their decay constant (provided they decay before nucleosynthesis). The dilution factor for \( f_a \sim M_{Pl} \) is much larger.

One may also worry about the thermal production of axinos, or more generally, modulinos. An estimate for their abundance in the absence of a decaying modulus can be obtained by
comparing with gravitino production. The gravitino production rate is proportional to $1/M_{Pl}^2$; for modulinos it is proportional to $1/f_a^2$. If the modulus couples only to gauge fields, the rate is suppressed by an additional factor of $(\alpha/\pi)^2$. However, we have already seen that moduli which couple only in this way are problematic, and we have argued that we are principally interested in models in which moduli have large tree level couplings to most standard model matter fields. So, like gravitino production, modulino production processes is also enhanced by roughly the number of light matter fields. So these processes are quite dangerous. For $f = 10^{15}$ GeV and $T_{RH}^{(1)} = 10^{11}$ GeV, one finds that subsequent to the first reheating $n_A/n_\gamma \sim 10^{-1}$. A further dilution of only $10^7$ is three to four orders of magnitude too small. In this case the Saxion cannot provide enough dilution. But as mentioned above, if the decaying particle is a saxion with a larger decay constant $\sim M_{Pl}$, then the dilution factor might be large enough to provide sufficient dilution.

Non-thermal production of gravitinos and modulinos has been discussed recently in the literature [22]. The change in the inflaton potential near the end of its life can result in a rapid change in the fermion masses, leading to production of these particles. More generally, there will be particle production for particles that are not conformally coupled. Whether this leads to an overproduction of particles depends on the details of the inflationary model. But reheating temperatures as low as $10^2$ GeV may be required [22]. For example, the dangerous relic to photon ratio can be as large as $n_X/n_\gamma \sim \beta T_{RH}/M$, where $M \sim 10^{15}$ GeV characterises the change in mass scales at the end of inflation, and $\beta$ is a number that depends on the dependence of the modulino mass on the inflaton [22]. A large value of $n_X/n_\gamma \sim 10^{-5}$ ($T_{RH} = 10^{11}$ GeV and $M = 10^{15}$ GeV, $\beta \sim 10^{-1}$) is then not unreasonable. In the conventional scenario this is much too large. But we have seen that the late decay of either the Saxion or another saxion with a larger decay constant, can produce a dilution of $10^7$ or much larger. This amount of dilution might be sufficient.

In sum, modulino production is a significant constraint on the picture we have presented here. The decays of a massive modulus with a large decay constant provide the simplest way to sufficiently dilute these particles.

5 Cosmological Limits on the QCD Axion

In this section, we focus on the general question of the limits on the QCD Axion in supersymmetric theories. There are many possibilities we can consider, both for models of supersymmetry
breaking and for the behavior of the early universe. We can consider supersymmetry breaking at intermediate scales (as we have up to now for generic axions) or gauge mediation, for example. We will focus in this section, as we have up to now, on supersymmetry breaking at intermediate scales, saving the discussion of low energy breaking for the conclusions.

Again, we argue that it only makes senses to consider limits on the Axion in the context of acceptable Saxion cosmologies. We will see that there are cosmological windows on the Axion beyond the usual ones.

Of the many possible cosmic histories we might consider, we will focus on a limited set. First, there might be other moduli, with larger decay constants than \( f_A \), which dominate the energy density at least from the time that \( H = m_{3/2} \). Alternatively, we might imagine that there are no such moduli, and that the universe reheats after inflation to some high temperature. In that case we ask if and when the Saxion comes to dominate the energy density.

5.1 Conventional Axion Cosmology

In the conventional cosmology the Axion does not begin to oscillate until the temperature is around a few GeV. The actual value depends on the decay constant. A rough estimate for when the Axion begins to oscillate may be obtained using \( m_a(T) \approx 3H(T) \), where the temperature-dependent mass is

\[
   m_a(T) = 0.1m_a(T = 0) \times \left( \frac{\Lambda_{QCD}}{T} \right)^{3.7}. \tag{18}
\]

This expression is valid for \( \pi T \gg \Lambda_{QCD} \), where here \( \Lambda_{QCD} \equiv 200 \text{ MeV} \). For lower temperatures the Axion mass is to a good approximation given by its zero temperature value. Since the Axion potential is temperature dependent, the energy density in Axions does not redshift as \( R^{-3} \), but instead as

\[
   \rho_a(T) = \frac{m_a(T)}{m_a(T_0)} \frac{R_0^3}{R^3} \rho_a(T_0). \tag{19}
\]

In the conventional picture, the Axion begins to oscillate at a temperature much larger than the QCD scale. Requiring that the energy in Axions constitutes less than 1/3 of the current energy density gives the usual limit, \( f_a \lesssim 3 \times 10^{11} \text{ GeV} \).
5.2 Axion Evolution in the Presence of a Saxion

Suppose that supersymmetry is broken at an intermediate scale as in supergravity models. In this case, the Saxion mass will be of order TeV (perhaps tens of TeV). As a result, the conventional assumption of radiation domination at the QCD scale is not necessarily correct. We will first suppose that the Axion supermultiplet is the only moduli supermultiplet. This scenario has been discussed in [4], [24], so most of this subsection is review.

In the conventional inflationary scenario the Saxion begins to oscillate during the matter-dominated phase of the inflaton. Assume, as is conventional, that all of the energy of the inflaton is transferred to radiation during reheating. Then the fraction of energy stored in the Saxions immediately at the start of the first radiation-dominated era is \( \rho_s \simeq f_s^2 \rho_R / M_{Pl}^2 \). The Saxions will come to dominate the universe when the temperature has dropped to \( T_s \simeq (f_s^2 / M_{Pl}^2) T_{RH}^{(1)} \).

Here \( T_{RH}^{(1)} \) is the first reheating temperature. In order for the late decays of Saxions to be effective, this cross-over temperature must, first, occur before the saxion decays. Otherwise, saxion decays will not effect the Axion cosmology appreciably. This gives a lower bound:

\[
f_A > \frac{M_{Pl}^{5/6} m_s^{1/2}}{(T_{RH}^{(1)})^{1/3}} = 10^{13.5} \text{ GeV} \left( \frac{m_s}{\text{TeV}} \right)^{1/2} \left( \frac{10^9 \text{ GeV}}{T_{RH}^{(1)}} \right)^{1/3}
\]

(neglecting factors of order one). If this bound is not satisfied, the Axion will begin to oscillate when the universe is radiation dominated, and the conventional limits apply. This bound may instead be viewed as requiring that the initial reheat temperature must be larger than

\[
T_{RH}^{(1)} \gtrsim 3 \times 10^4 \text{ GeV} \left( \frac{10^{15} \text{ GeV}}{f_A} \right)^3 \left( \frac{m_s}{\text{TeV}} \right)^{3/2}.
\]

There is in addition another requirement, namely that the cross-over temperature must be above a GeV, so that the Axion begins to oscillate during the Saxion-dominated phase. This requires

\[
f_A \gtrsim 10^{13.5} \text{ GeV} \left( \frac{10^9 \text{GeV}}{T_{RH}^{(1)}} \right)^{1/2}.
\]

This bound is independent of the Saxion mass (assuming it begins oscillating during the end of inflation). Clearly a high reheat temperature is needed.

The reason for this second bound may be understood as follows. The relevant quantity is the Axion energy density at the time of Saxion decay. Due to the temperature dependence of the
potential, the energy density increases adiabatically relative to the $R^{-3}$ redshift by an amount $m_A(T)/m_A(T_{osc})$. Inspecting (18), this factor can be large. This is as in the conventional cosmology, precisely because the Axion begins to oscillate when the temperature is around a GeV.

But if the Saxion (or any other modulus) begins to dominate the energy before the universe cools to a GeV, then the story is different. The reason is that in a matter dominated universe the Hubble parameter must, by definition, be much larger than its value in a radiation dominated universe at the same temperature. When the temperature reaches a GeV in the Saxion-dominated universe, the Hubble parameter is still too large and the Axion hasn’t begun to oscillate. Instead it begins to oscillate when the temperature is roughly a 100 MeV. At these temperatures the Axion mass is approximately given by its zero temperature value and there is no large enhancement.

Assuming the Saxion decays solely into radiation, the ratio of energy in Axions to radiation at the start of the second radiation era is $f_A^2/M_P^2$, dropping factors of order one. Requiring that the current Axion energy density be less than about a third of the critical energy density gives the bound

$$f_A < 10^{14.5} \sqrt{\frac{m_a(T_{osc})}{m_a(0)}} \frac{h_0}{0.7} \frac{10\text{MeV}}{T_{RH}^{(2)}} \text{GeV}.$$  \hspace{1cm} (23)

But stated in this way the upper bound is a little misleading, since not all values of the decay constant up to $10^{15}$ GeV are allowed. As previously indicated, there are lower bounds. In fact as we can see, there is a small window between $\sim 10^{13.5}$ GeV and $\sim 10^{15}$ GeV in which there are no cosmological problems. For smaller values of the Axion decay constant there is no Saxion cosmological problem. But there is a conventional Axion abundance problem since the Saxion decays are not useful. For Saxion decay constants below about $\sim 3 \times 10^{11}$ GeV, the conventional analysis applies and there is neither a Saxion nor an Axion cosmological problem.

Finally, one also requires that the second reheating from the Saxion decay is above an Mev, but below 100 MeV. This does limit the parameter space. For instance, these lower and upper bounds imply $10^{16.5}$ GeV $\gtrsim f_A \gtrsim 10^{14.5}$ GeV for a Saxion mass of order TeV (neglecting factors of order 1), and scale as $m_S^{3/2}/\text{TeV}^{3/2}$. 

19
5.3 Axion Evolution in the Presence of Another Modulus

We saw in the previous section that an Axion decay constant in an intermediate range was not allowed if the Axion is the only modulus. If there is a second modulus with a different decay constant the limitations of the single-modulus scenario can be relaxed. By having a larger decay constant for the new modulus, two things happen, which both favor allowing the full window of Axion decay constants: first, it is easier for the new modulus to dominate the universe before it reaches a GeV, and second, the requirement that the second reheating temperature is not too high is easier to satisfy. We now show that this scenario allows the window $f_A < 10^{15}$ GeV with no lower cosmological bound.

Suppose that shortly after inflation, the universe is dominated by a massive modulus, other than the Saxion. This will occur if its decay constant is $F \sim M_p$. It is important that the decay constant of this other modulus is unrelated to the Saxion and Axion decay constant $f_A$. In this case, the Saxion also begins to oscillate essentially immediately. It carries a fraction of the energy density of order $f_A^2/M_p^2 \sim f_A^2/F^2$. If $f_A \ll F$, the Saxion carries only a small fraction of the energy density of the universe, and decays long before the massive modulus. The modulus is assumed sufficiently heavy (greater than 20 TeV) that it reheats the universe above nucleosynthesis temperatures, but below the QCD scale. At this time, the energy density in Axions is suppressed relative to that in moduli by a factor $f_A^2/M_p^2$, which is sufficiently small that Axions don’t dominate before recombination if $f_A < 10^{15}$ GeV. So this picture is self-consistent. For Axion decay constants less than $10^{15}$ GeV, there is neither a Saxion nor an Axion density problem. Note that the reheating in the modulus decay also dilutes gravitinos which may have been produced during the first reheating.

For smaller values of the modulus decay constant $F$ the subsequent cosmology and relic Axion abundance in this scenario is determined by several more parameters: the Axion decay constant $f_A$; the reheat temperature $T_{RH}^{(1)}$ due to the inflaton decay; and the reheat temperature $T_{RH}^{(2)}$ due to the decay of the scalar modulus. We have explored this parameter space sufficiently to establish that there is an additional window of allowed axion decay constants.

6 Gauge Mediation

So far, we have assumed that supersymmetry is broken at an intermediate scale, and the gravitino mass is of order 1 TeV. An alternative possibility is that supersymmetry is broken
at a lower scale, as in gauge mediation. Again, we can contemplate a variety of assumptions about cosmology.

The first point, however, is that in this case the moduli are relatively light. To get some feeling for the issues, suppose that the scale of supersymmetry breaking is of order \( \sqrt{F} \). As we have discussed, the saxion mass in this case is likely to be of order \( \epsilon^{1/2} \frac{F}{f_a} \), where \( \epsilon \) is typically some combination of loop factors. The leading saxion coupling to Standard Model matter is given by the axion-like interactions with gauge fields, induced by integrating out heavy vector-like fields. This has a suppression factor of \( \frac{\alpha}{2\pi} \), so the saxion lifetime is then of order:

\[
\Gamma \approx \frac{1}{2\pi} \left( \frac{\alpha_s}{2\pi} \right)^2 \frac{\epsilon^{3/2} F^3}{f_A^5}
\]  

(24)

Correspondingly, the reheat temperature is of order

\[
T_R \approx \frac{(10^{-2} \epsilon^{3/4} F^{3/2} M_{Pl}^{1/2})}{f_A^{5/2}} = 10 \text{ GeV} \left( \frac{\epsilon}{0.2} \right)^{3/4} \left( \frac{\sqrt{F}}{10^8} \right)^{3} \left( \frac{10^{12}}{f_A} \right)^{5/2}.
\]  

(25)

In other words, if \( f_A \) is \( 3 \times 10^{11} \) GeV, it is necessary that the scale of supersymmetry breaking be greater than about \( 10^{6.5} \) GeV in order to reheat to above 10 MeV. For \( f_a = 10^{15} \) GeV, the scale of supersymmetry breaking must be of order \( 10^{9.5} \) GeV to obtain sufficient reheating. From the point of view of gauge mediation, such a high scale for supersymmetry breaking might be problematic: Planck suppressed terms will be comparable to the gauge corrections. Note that the reheat temperature here scales as \( f^{-5/2} \), whereas in gravity mediation it is much softer, \( \sim f^{-3/2} \).

Now if \( f_A \lesssim 3 \times 10^{11} \) GeV there is no Axion problem. But we have seen that in cases where \( \sqrt{F} \lesssim 3 \times 10^{6} \) GeV, one cannot solve the Saxion problem since it decays too late. It seems that for such low scales of supersymmetry breaking, then, the strong CP problem must be solved by some mechanism other than axions.

In cases where the saxion problem is solved, the axion problem is often solved as well. As before, we need to ask at what temperature the saxion comes to dominate the energy density of the universe. Consider the case \( f_A = 10^{15} \) GeV, \( \sqrt{F} = 10^{10} \) GeV. In this case, the saxion mass is of order \( \sqrt{\epsilon} \) 100 TeV, and the estimates are similar to those we encountered in the previous section. In particular, both the saxion and axion problems are solved. As we decrease \( \sqrt{F} \), we need also to decrease \( f_A \).
7 Alternative Solutions to the Strong CP Problem

We have seen that there are situations where the Axion solution to the Strong CP problem is likely to be unworkable, for any choice of decay constant. For example, for gauge mediation, with a SUSY breaking scale below $3 \times 10^6$ GeV, the Saxion problem is not easily solved even for a very small decay constant. Given that, on other grounds, such low scale supersymmetry breaking is a plausible picture of how nature works, it is interesting to examine carefully other solutions of the strong CP problem [1, 26]. We have already commented on the possibility of a massless $u$ quark. In this section we briefly comment on the Nelson-Barr mechanism [1]. The points we will make have largely appeared earlier in [25], and more recently in [26].

In [25], it was shown that implementing the Nelson-Barr mechanism is, in general, rather difficult in supersymmetric theories. The problem is that loop corrections involving squarks and gauginos give large corrections to $\theta$ unless there is a very high degree of degeneracy. In [26], it was noted that gauge mediation can provide the needed level of degeneracy. So gauge mediation, which is precisely the case where one might need an alternative to axions, is a situation in which there is an alternative solution to the strong CP problem.

8 Conclusions: Detectable Axion Dark Matter

We have seen that in supersymmetric theories, the problems of Saxion cosmology are much more serious than those associated with Axions. Mechanisms which solve the saxion problem usually modify the limits on the axion decay constant. One is thus left with several possibilities:

- There is no Axion. The strong CP problem is solved by a massless $u$ quark, or by a variation on the Nelson-Barr mechanism. We saw that this view is almost inevitable if one has gauge mediation, with a messenger scale below $3 \times 10^6$ GeV.

- The Axion solves the strong CP problem, but does not constitute the dark matter; it’s decay constant is, say, $10^{15}$ GeV, a scale which might plausibly emerge from string theory. Such a picture emerges naturally in the case of gravity mediation.

- The Axion solves the strong CP problem, constitutes the dark matter, but is not detectable in foreseeable experiments because of its large decay constant. Again, this can readily emerge from gravity mediation.
The Axion solves the strong CP problem, constitutes the dark matter, and its decay constant is such that it can be detected in future experiments.

While the discussion of this paper makes clear that the last possibility is far from inevitable, it also suggests that it might be possible. Indeed, our simple field theory model for axions suggests that this could come about naturally. If the symmetries of the theory are such that \( n = 1 \), then \( f_A \approx 10^{11} \text{ GeV} \). The axion, in such a model, will be sufficiently light provided, for example, one has a discrete symmetry which insures that the leading PQ violating correction to \( W \) goes as \( S^0 \) or larger.

Indeed, the usual axion limit poses the question: where does the scale \( 10^{11} \text{ GeV} \) come from? From the point of view of supersymmetry, the mechanism we described above was always a natural candidate. In contrast, in non-supersymmetric theories, this extra scale must simply be postulated, and raises all of the usual questions of hierarchy. Having added this scale to the theory, one has introduced a new fine tuning problem for the Higgs which is far worse than that connected with the strong CP problem.

Acknowledgements:

We thank Ed Witten for asking a set of questions which prompted this investigation, and Scott Thomas for discussion of a number of issues, particularly saxion decays to axions. This work is supported in part by the U.S. Department of Energy. MG would like to thank the Aspen Center of Physics where part of this work was completed.

References


