(b) We repeat the computation for the center of the sun, where
\[ T = 1.5 \times 10^7 \, \text{K} \quad \text{and} \quad N/V = 5 \times 10^{21} \, \text{m}^{-3} \]

\[
P_{\text{particles}} = \frac{N_k T}{V} = \frac{(5 \times 10^{21} \, \text{m}^{-3})(1.38 \times 10^{-23} \, \text{J/K})(1.5 \times 10^7 \, \text{K})}{1.036 \times 10^{14} \, \text{Nm}^{-2}}
\]

\[
P_{\text{particles}} = 1.28 \times 10^{13} \, \text{Nm}^{-2}
\]

and so,

\[
\frac{P_{\text{radiation}}}{P_{\text{particles}}} = 1.23 \times 10^{-5}
\]

Thus,

\[
P = \frac{4}{3} \left( 5.67 \times 10^{-8} \, \text{W m}^{-2} \text{K}^{-4} \right) (5800 \, \text{K})^4 \cdot 3 \times 10^4 \, \text{ms}^{-1}
\]

\[
= 2.85 \times 10^{11} \, \text{Nm}^{-2}
\]

(since 1 W = 1 J s^{-1} = 1 N m s^{-1}).

We conclude that

\[
P_{\text{radiation}} = 3.5 \times 10^{-5}
\]

\[
P_{\text{particles}}
\]
In this, the expression for $AE$, we can rewrite it as:

$$AE = \frac{c}{\sqrt{r^2 + (l - t)^2}}$$

This reduces to $AE = \frac{c}{r}$.

For a substitutive transformation, we have $AE = \frac{c}{r}$.

From the first law of thermodynamics,

$$\frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)} = \frac{\rho \cdot c}{n}$$

The energy relation for the system is:

$$\frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)} = \frac{\rho \cdot c}{n}$$

In this case, we can rewrite

$$\frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)} = \frac{\rho \cdot c}{n}$$

In part (a), we consider

$$\int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} p \, dV$$

In the case of $V_1$, the system is constant in its environment.

$$\frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)} = \frac{\rho \cdot c}{n}$$

For a substitutive transformation

$$\frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)} = \frac{\rho \cdot c}{n}$$

In part (b), we consider

$$p \cdot dV = \frac{\rho \cdot c}{(n + 1) \cdot (1 - \gamma)}$$

In this case, $p \cdot dV$ is constant in its environment.
3. Isothermal compression from $V_3$ to $V_4$ at temperature $T_c$

We again make use of part (a) and obtain

$$w_3 = \frac{4\alpha T_c^4}{3c} (V_4 - V_3)$$

4. Adiabatic compression from $V_4$ to $V_5$. The initial temperature is $T_c$ and the final temperature is $T_h$.

Again, as in step 2, $q = 0$, so

$$w_4 = -\Delta E = -\frac{4\alpha}{c} (T_h V_4 - T_c V_4)$$

We can evaluate $V_4$, since this is an adiabatic transition:

$$V_4 T_h^2 = V_3 T_c^2$$

Thus,

$$w_4 = -\frac{4\alpha}{c} V_4 T_h^2 (T_h - T_c)$$

The total work done by the gas of photons during the cycle is:

$$W = w_1 + w_2 + w_3 + w_4$$

First, note that

$$w_2 + w_4 = \frac{4\alpha}{c} V_2 T_h^3 (T_h - T_c) - \frac{4\alpha}{c} V_4 T_h^3 (T_h - T_c)$$

$$= \frac{4\alpha}{c} T_h^3 (V_2 - V_4) (T_h - T_c)$$

Next, we compute:

$$w_1 + w_3 = \frac{4\alpha T_h^4}{3c} (V_5 - V_4) + \frac{4\alpha T_c^4}{3c} (V_4 - V_3)$$

Using $V_2 = V_4 / T_h^3$ and $V_3 = V_2 / T_c^3$, we substitute for $V_3 + V_2$ in the above expression. The final result is

$$w_1 + w_3 = \frac{4\alpha T_h^4}{3c} (V_5 - V_4) + \frac{4\alpha T_c^4}{3c} T_h^3 T_c (V_4 - V_3)$$

$$= \frac{4\alpha}{3c} T_h^3 (T_h - T_c) (V_2 - V_4)$$

Therefore,

$$W = W_1 + W_2 + W_3 + W_4$$

or

$$W = \frac{16\alpha}{3c} T_h^4 (T_h - T_c) (V_2 - V_4)$$

Note that $T_h > T_c$ and $V_2 > V_4$, so that $W > 0$ as expected.

(c) The efficiency of the Carnot engine is defined by $\eta = W / Q_h$.

In part (a), we found that

$$Q_h = \frac{16\alpha}{3c} T_h^4 (V_2 - V_4)$$

Using the result for $W$ obtained above,

$$\eta = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$$

which is precisely the Carnot efficiency derived in RB Chapter 3.
According to the problem, 4.2 J of energy is required to raise the temperature of 1 kg of water by 1 K. Using the formula:

\[ E = \frac{Q}{m} \]

where \( E \) is the energy, \( Q \) is the heat, and \( m \) is the mass, we find:

\[ E = 4.2 \text{ J} \]

Thus, the energy required is:

\[ E = 4.2 \text{ J} \]

By the supply of water, it would have a length of 2.116 m.
(b) Suppose the earth radiates as a perfect black body at temperature $T_e$. Then, the earth radiates an energy per unit time of
\[
\sigma T_e^4 \left(4\pi R_e^2\right)
\]
If the earth is in a thermal steady state, this radiated flux must be exactly balanced by the incoming solar flux of radiation. In part (a), we computed the solar energy flux at the earth's orbital distance to be
\[
\sigma B T_0^4 \frac{R_0^2}{d^2}
\]
To get the energy per unit time absorbed by the earth, we must multiply by the effective surface area. This is a little tricky.

In the diagram at the right, I draw part of the earth's orbit and not to scale the sphere of the earth. The effective surface area is the projection of the sphere of the earth onto the large surface of radius $d$ which contains the earth's orbit. This effective area is just $\pi R_e^2$, the area of the circle the earth makes up the earth's equator.

(Imagine shining a flashlight on a sphere in front of the wall. The shadow on the wall will be a disk of radius $R$ and area $\pi R^2$, where $R$ is the radius of the sphere.)

Thus, the solar energy per unit time absorbed by the earth is
\[
\sigma B T_0^4 \frac{R_0^2}{d^2} \pi R_e^2
\]
Setting this equal to $\sigma T_e^4 \left(4\pi R_e^2\right)$, and solving for $T_e$. 

The energy flux of a perfect black body is equal to $\sigma T^4$.

Flux minus net unit area per unit time. Thus the units of $\sigma T^4$ is Wm$^{-2}$ or Jm$^{-2}$s$^{-1}$

The total power emitted by the sun is
\[
\left(\sigma B T_0^4\right) 4\pi R_0^2
\]
\[(T_0 = 5800 \text{ K})\]

where $R_0$ is the radius of the sun. This is because $4\pi R_0^2$ is the surface area of the sun, and radiation is emitted from the surface.

Let $d$ = distance from the sun to the earth. The radiation is emitted uniformly in all directions. Thus, the energy flux at the earth's orbital distance is equal to

\[
\frac{\text{total power emitted by sun}}{4\pi d^2}
\]

since $4\pi d^2$ is the total surface area at a distance $d$ from the sun receiving the sun's energy. That is,

\[
\text{energy flux at earth} = \frac{\sigma B T_0^4 R_0^2}{d^2}
\]

If $R_0 = 6.960 \times 10^8$ m (see p. 20 of RB)

\[d = 1.496 \times 10^{11} \text{ m}\]

we find:

\[
\text{energy flux at earth} = \frac{(5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4})(5800 \text{ K})^4 (6.960 \times 10^8 \text{ m})^2}{(1.496 \times 10^{11} \text{ m})^2}
\]

\[= 1389 \text{ Wm}^{-2}\]
(8) \[ E = \frac{8}{3h^2} \int_0^{4h/5} x^3 \left( e^{-x} \right) \, dx \]

In this case, we have:

\[ E = \frac{8}{3h^2} \left[ -\frac{x^3}{1} \right]_0^{4h/5} \]

\[ = \frac{8}{3h^2} \left( \frac{4h^3}{5} \right) \]

\[ = \frac{32}{15} \frac{h^2}{E} \]

where \( a = \frac{4h}{5} \). Again, we consider the high-temperature limit, \( T \gg T_e \).

Note: The equation at the top left corner of the page is missing, and it is not clear how to proceed from the given information. The rest of the page contains further mathematical calculations and explanations related to the problem.
In obtaining this result, I used the expansion
\[ \frac{1}{1+2z} = 1 - 2z + 2z^2 - \ldots \]
where \( z = \frac{x}{3} + \frac{x^2}{6} \). Note that I did not have to keep the term of \( O(x^3) \) obtained when squaring \( z \), since I am only interested in terms up to and including \( O(x^2) \).

Thus,
\[
\int_0^x \frac{x^2}{e^x - 1} \, dx = \int_0^x x^2 (1 - x + \frac{x^2}{2}) \, dx
\]
\[= \frac{x^3}{3} - \frac{x^6}{8} + \frac{x^9}{60} \]
\[= \frac{x^2}{3} \left( 1 - \frac{3x^3}{8} + \frac{x^6}{20} \right) \]

Therefore,
\[
E = \frac{3k^4 T^4 V}{2\pi^2 k_B^4} \frac{x^3}{3} \left( 1 - \frac{3x^3}{8} + \frac{x^6}{20} \right)
\]

Using the definition,
\[
x_0 = \frac{4k_B T}{\sqrt{\pi V}} \]
we can write:
\[
\frac{3N}{x_0^4} = \frac{3k_B^4 T^4 V}{2\pi^2 k_B^4}
\]

The expression for \( E \) simplifies to
\[
E = 3Nk_B T \left[ 1 - \frac{3}{8} \frac{T_0}{T} + \frac{1}{20} \frac{T_0^2}{T^2} \right]
\]
where we have put \( \Theta_0 = \Theta_0/T \).

The heat capacity is
\[
C_v = \frac{2E}{\Theta_0} = 3Nk_B \left[ 1 - \frac{1}{20} \frac{T_0^2}{T^2} \right]
\]

Note that the second term in the expression for \( E \) is independent of \( T \), so its derivative with respect to \( T \) vanishes. Thus, we need to compute \( E \) up to order \( T^2 \) with respect to the lowest order approximation in order to obtain the first non-trivial correction term in the expression for \( C_v \).

(b) For \( \Theta_0/T = 0.5 \)
\[
\frac{E - 3Nk_B T}{3Nk_B T} \approx -\frac{2T_0}{8} = -\frac{1}{4}\]

which is a correction of about \(-19\%\), and
\[
\frac{C_v - 3Nk_B}{3Nk_B} = -\frac{1}{20} \frac{T_0^2}{T^2} = -\frac{1}{80}
\]
which is a correction of \(-1.25\%\).
(a) The entropy of an ideal gas is given by

\[ S = \frac{n R}{2} \ln \left( \frac{V}{V_0} \right) + \frac{n R}{2} \ln \left( \frac{T}{T_0} \right) \]

Thus,

\[ S_0 - S = \frac{n R}{2} \ln \left( \frac{V}{V_0} \right) + \frac{n R}{2} \ln \left( \frac{T}{T_0} \right) \]

Given \( V_i = 5 \times 10^{-5} \) m\(^3\) and \( T_i = \frac{3}{2} T_0 \), with \( n = 10^{-3} \) m\(^3\) (moles)

\[ S = 3.8 \times 10^{-3} \text{ JK} \]

(b) According to Eq. 19 of PB on p. 62

\[ \frac{S}{T} = \int_0^t \frac{C(T)}{T} dt \]

According to Table 6 in the problem, \( T = 373 \)

\[ \therefore \frac{S}{T} = \frac{3 - S}{3 - S} \]

\[ \frac{S}{T} = \frac{1}{3} \]

\[ \frac{S}{T} = \frac{1}{T} \Theta_0 \]

\[ \frac{1}{T} = \frac{1}{3} \Theta_0 \]

\[ \frac{1}{T} = \frac{1}{3} \Theta_0 \]

\[ \frac{1}{T} = \frac{1}{3} \Theta_0 \]
Thus, we can use \( 18 \) or \( 6.57 \) for the heat capacity:

\[
C_v = \frac{12\pi^4 N\hbar}{5} \left( \frac{T}{\Theta} \right)^2
\]

For diamonds in helium, we are given the value of

\[
C_v(T=4.2K) = 10^{-6} J/K
\]

Thus,

\[
C_v(T) = 10^{-6} J/K \left( \frac{T}{4.2K} \right)^2
\]

So,

\[
S_T - S_i = 10^{-6} \int_{4.2K}^{T^2} T^2 dT
\]

\[
= \frac{1}{3} \frac{10^{-6}}{(4.2)^2} \left[ (77)^3 - (4.2)^3 \right]
\]

\[
= 2.05 \times 10^{-3} J/K
\]

(a) The energy loss per unit time is given by

\[
\frac{dE}{dt} = -\alpha \frac{T^4}{4\hbar^2 r_0^2}
\]

where \( \frac{T^4}{4\hbar^2 r_0^2} \) is the surface area of the sphere. By the chain rule,

\[
\frac{dE}{dt} = \frac{dE}{dT} \frac{dT}{dt} = C(T) \frac{dT}{dt}
\]

where \( C(T) \) is the heat capacity. Thus,

\[
\frac{dT}{dt} = \frac{1}{C(T)} \frac{dE}{dT}
\]

\[
= \frac{1}{C(T)} \frac{\alpha T^4}{4\hbar^2 r_0^2}
\]

Since \( \Theta = 275K \) for silver, in the range of temperatures between 250 K and 300 K, we will approximate

\[
C(T) = 3N\hbar
\]

(See e.g. Figures 6.8 and 6.9 on p.136 and 139 of RB)

So,

\[
\int_{T_i}^{T_f} \frac{dT}{T^4} = -\frac{1}{3N\hbar} \alpha \frac{9\hbar r_0^2}{4T_i} \int_{T_i}^{T_f} dt
\]

The cooling time required, \( \Delta t = t_f - t_i \) is then given by:

\[
\int_{T_i}^{T_f} \frac{dT}{T^4} = -\frac{1}{3N\hbar} \alpha \frac{9\hbar r_0^2}{4T_i} \Delta t
\]
\[ \frac{\sqrt{12.444444444444443}}{\sqrt{0.9658888888888888}} = \frac{1}{0.67} \]

This yields:

\[ x = \frac{74}{0.67} \approx 110.61 \]

\( C(T) = 12T \text{ m/s} \)

\[ s = \frac{1}{2} a t^2 = \frac{1}{2} \times 10 \times 10^2 = 500 \text{ m} \]

\[ \text{In 10 seconds we can go 500 m, thus we must use } \frac{60}{500} = \frac{3}{25} \text{ minutes.} \]

\[ (0.120 \times 180) = 21.6 \frac{m}{s} \]

\[ (2,550 \times 0.05) = 127.5 \]

\[ \text{Thus:} \quad Av = \frac{s}{t} = \frac{500}{10} = 50 \text{ m/s} \]