Anomalous magnetic moment of the muon in the two Higgs doublet model

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Abstract

We calculate the new physics effects on the anomalous magnetic moment of the muon in the framework of the two Higgs doublet model. We predict an upper bound for the lepton flavor violating coupling, which is responsible for the point like interaction between muon and tau, by using the uncertainty in the experimental result of muon anomalous magnetic moment. We see that the upper bound predicted is more stringent compared to the one which is obtained by using the experimental result of muon electric dipole moment.

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1 Introduction

The lepton flavor violating (LFV) interactions, non-zero electric dipole moments (EDM) and anomalous magnetic moments (AMM) of leptons are among the most promising candidates to search the physics beyond the standard model (SM). The AMM of muon have been studied in the literature extensively [1] and [2]. The experimental result of muon AMM by g-2 Collaboration [3]

\[ a_\mu = 116592023(151) \times 10^{-11}, \]

have opened a new window for testing the SM and the new physics effects beyond. At first, there were assumed a deviation of muon AMM over its SM prediction [4]

\[ \Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (4.26 \pm 1.65) \times 10^{-9}. \]

and various scenarios have been proposed to explain the nonvanishing value of deviation \( \Delta a_\mu \) [5] - [22]. The Supersymmetry (SUSY) contribution to \( a_\mu \) has been investigated in [2, 7, 12]. In [13] the new physics effect on \( a_\mu \) has been explained by introducing a new light gauge boson. The prediction of muon AMM has been done in the framework of leptoquark models in [14], the technicolor model with scalars and top color assisted technicolor model in [17], in the framework of the general two Higgs doublet model (2HDM) in [18] and also in [19]. The work [20] was devoted to the Higgs mediated lepton flavor violating interactions which contributed to \( a_\mu \).

In this study, only the scalar Higgs exchange was taken into account by assuming that the pseudoscalar Higgs particle was sufficiently heavier than the scalar one. Finally, in [22], scalar scenarios contributing to \( a_\mu \) with enhanced Yukawa coupling were proposed.

In [23] and the recent work [24], the hadronic contributions to the muon AMM have been calculated and the agreement with the experimental result has been obtained, in the framework of the SM.

In our work, we study the new physics effects on the AMM of muon in the model III version of 2HDM, including both scalar and pseudoscalar Higgs boson effects, based on the assumption that the numerical value should not exceed the present experimental uncertainty, \( \sim (1 - 2) \times 10^{-9} \). The new contribution to \( a_\mu \) exists at one-loop level with internal mediating neutral particles \( h^0 \) and \( A^0 \) in our case, since we do not take charged FC interaction in the leptonic sector due to the small couplings for \( \mu - \nu_l \) interactions. In the calculations, we take into account the internal \( \tau \) and \( \mu \) leptons and neglect the contribution coming from the internal \( e \)-lepton since the corresponding Yukawa coupling is taken to be smaller compared to
the others. Furthermore, we also neglect the internal $\mu$-lepton contribution by observing the weak dependence of $\Delta a_\mu$ on the $\mu$-$\mu$ coupling. We predict a stringent upper bound for the $\mu$-$\tau$ coupling and compare with the one, which is obtained by using the restriction coming from the EDM of $\mu$ lepton (see [25] for details).

The paper is organized as follows: In Section 2, we present the new physics effects on the AMM of muon in the framework of the general 2HDM. Section 3 is devoted to discussion and our conclusions.

2 Anomalous magnetic moment of muon in the model III version of two Higgs doublet model.

In the type III 2HDM, there exist flavor changing neutral currents (FCNC), mediated by the new Higgs bosons, at tree level. The most general Higgs-fermion interaction for the leptonic sector in this model reads as

$$\mathcal{L}_Y = \eta_{ij} E_{ij} \bar{l}_i L \phi_1 E_{jR} + \xi_{ij} E_{ij} \bar{l}_i L \phi_2 E_{jR} + h. c.$$  \hspace{1cm} (3)

where $i, j$ are family indices of leptons, $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $l_{iL}$ and $E_{jR}$ are lepton doublets and singlets respectively, $\phi_i$ for $i = 1, 2$, are the two scalar doublets

$$\phi_1 = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \frac{\sqrt{2}}{i} \chi^0 \right] ; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + i H_2 \end{pmatrix} ,$$ \hspace{1cm} (4)

with the vacuum expectation values

$$< \phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; < \phi_2 > = 0 .$$ \hspace{1cm} (5)

With the help of this parametrization and considering the gauge and $CP$ invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ as:

$$V(\phi_1, \phi_2) = c_1(\phi_1^+ \phi_1 - v^2/2)^2 + c_2(\phi_2^+ \phi_2)^2$$
$$+ c_3[(\phi_1^+ \phi_1 - v^2/2) + \phi_2^+ \phi_2]^2 + c_4[(\phi_1^+ \phi_1)(\phi_2^+ \phi_2) - (\phi_1^+ \phi_2)(\phi_2^+ \phi_1)]$$
$$+ c_5[Re(\phi_1^+ \phi_2)]^2 + c_6[Im(\phi_1^+ \phi_2)]^2 + c_7 .$$ \hspace{1cm} (6)

the SM particles and new particles beyond can be collected in the first and second doublets respectively. Here $H^0$ is the SM Higgs boson and $H_1$ ($H_2$) is the new neutral Higgs one. Since there is no mixing of neutral Higgs bosons at tree level for this choice of Higgs doublets, $H_1$ ($H_2$) is the usual scalar (pseudoscalar) $h^0$ ($A^0$).
In the Yukawa interaction eq. (3), the part which is responsible for the FCNC at tree level reads as

$$\mathcal{L}_{Y,FC} = \xi^E_{ij} \bar{l}_i L \phi \sigma_{\mu\nu} F_{\mu\nu} + h.c. .$$

(7)

Notice that, in the following we will replace \( \xi^E_{ij} \) by \( \xi^E_{N,ij} \) to emphasis that the couplings are related with the neutral interactions. The Yukawa matrices \( \xi^E_{N,ij} \) have in general complex entries and they are free parameters which should be fixed by using the various experimental results.

The effective interaction for the anomalous magnetic moment of the lepton is defined as

$$\mathcal{L}_{AMM} = a \frac{e}{4 m_l} \bar{l}_l \sigma_{\mu\nu} F_{\mu\nu} ,$$

(8)

where \( F_{\mu\nu} \) is the electromagnetic field tensor and "\( a \)" is AMM of the lepton "\( l \)". This interaction can be induced by neutral Higgs bosons \( h^0 \) and \( A^0 \) at loop level in the model III, beyond the SM. Notice that we do not take charged FC interaction in the leptonic sector due to the small couplings for \( \mu - \nu \) interactions.

In Fig. 1, we present the 1-loop diagrams due to neutral Higgs particles . Since, the self energy \( \Sigma (p) \) (diagrams a, b in Fig. 1) vanishes when \( l \)-lepton is on-shell, in the on-shell renormalization scheme, the vertex diagram c in Fig. 1 plays the main role in the calculation of the AMM of lepton \( l \). The most general Lorentz-invariant form of the coupling of a charged lepton to a photon of four-momentum \( q_\nu \) can be written as

$$\Gamma_\mu = G_1(q^2) \gamma_\mu + G_2(q^2) \sigma_{\mu\nu} q^\nu + G_3(q^2) \sigma_{\mu\nu} \gamma_5 q^\nu$$

(9)

where \( q_\nu \) is photon 4-vector and \( q^2 \) dependent form factors \( G_1(q^2), G_2(q^2) \) and \( G_3(q^2) \) are proportional to the charge, AMM and EDM of \( l \)-lepton respectively. Using the definition of AMM of the lepton \( l \) (eq. (8)), \( \Delta_{New} a_\mu \) is extracted as

$$\Delta_{New} a_\mu = a^{(1)}_\mu + \int_0^1 a^{(2)}_\mu (x) dx ,$$

(10)

where \( a^{(1)}_\mu (\int_0^1 a^{(2)}_\mu (x) dx) \) is the contribution coming from the internal \( \tau \) (\( \mu \)) lepton. The functions \( a^{(1)}_\mu \) and \( a^{(2)}_\mu \) are given by

$$a^{(1)}_\mu = \frac{G_F Q_\tau}{\sqrt{2} 64 \pi^2} \left\{ \left( \frac{1}{2} \right) (\xi^E_{N,\mu\tau}^*)^2 + (\xi^E_{N,\tau\mu})^2 \right\} \left( F_1(y_{h^0}) - F_1(y_{A^0}) \right)$$

$$+ \frac{1}{3} |\xi^E_{N,\tau\mu}|^2 \frac{m_\mu}{m_\tau} \left( G_1(y_{h^0}) + G_1(y_{A^0}) \right) \right\} ,$$

(11)
and

\[ a_{\mu}^{(2)}(x) = \frac{G_F}{\sqrt{2}} \frac{Q_\mu}{64 \pi^2} (x - 1)^2 \left\{ \left( \xi_{N,\mu}^E \right)^* \left( \xi_{N,\mu}^E \right) + \left( \xi_{N,\mu}^{E,\prime} \right)^* \left( \xi_{N,\mu}^{E,\prime} \right) + 2 \left| \xi_{N,\mu}^E \right|^2 x \right\} \]

where \( F_1(w) \) and \( G_1(w) \) are

\[ F_1(w) = \frac{w (3 - 4 w + w^2 + 2 \ln w)}{(-1 + w)^3}, \]
\[ G_1(w) = \frac{w (2 + 3 w - 6 w^2 + w^3 + 6 w \ln w)}{(-1 + w)^4} \]

(13)

Here \( y_H = \frac{m_\tau^2}{m_H^2} \) and \( r_H = \frac{m_\mu^2}{m_H^2} \), \( Q_\tau \) and \( Q_\mu \) are charges of \( \tau \) and \( \mu \) leptons respectively. In eqs. (11) and (12) \( \xi_{N,ij}^E \) is defined as \( \xi_{N,ij}^E \equiv \sqrt{\frac{4 G_F}{\sqrt{2}}} \xi_{N,ij}^E \). In eq. (10) we take into account internal \( \tau \) and \( \mu \)-lepton contributions since the Yukawa couplings \( \xi_{N,ij}^E \) (or \( j \)) are negligible (see Discussion part). Notice that, we make our calculations in arbitrary \( q^2 \) and take \( q^2 = 0 \) at the end.

In our analysis we take the couplings \( \bar{\xi}_{N,\tau \mu}^E \) and \( \bar{\xi}_{N,\mu \mu}^E \) complex in general and use the parametrization

\[ \bar{\xi}_{N,\mu \mu}^E = \left| \bar{\xi}_{N,\mu \mu}^E \right| e^{i \theta_{ll'}}. \]

(14)

The Yukawa factors in eqs. (11) and (12) can be written as

\[ \left( \left( \bar{\xi}_{N,\mu \mu}^E \right)^* + \left( \bar{\xi}_{N,\mu \mu}^{E,\prime} \right)^* \right)^2 = 2 \cos 2 \theta_{ll'} \left| \bar{\xi}_{N,\mu \mu}^E \right|^2 \]

(15)

where \( \mu, \tau \). Here \( \theta_{ll'} \) are CP violating parameters which play the main role in the existence of the lepton electric dipole moment.

3 Discussion

The new physics contribution to the AMM of the lepton is controlled by the Yukawa couplings \( \bar{\xi}_{N,ij}^E, i, j = e, \mu, \tau \) in the model III. These couplings can be complex in general and they are free parameters of the model under consideration. The relevant interaction (see eq. (8)) can be created by the mediation of the neutral Higgs bosons \( h^0 \) and \( A^0 \) beyond the SM, with internal leptons \( e, \mu, \tau \) (Fig. 1). However, in our predictions, we assume that the Yukawa couplings \( \bar{\xi}_{N,\tau e}^E \) and \( \bar{\xi}_{N,\mu \mu}^E \) are small compared to \( \bar{\xi}_{N,\tau \mu}^E \) since their strength is proportional to the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenrio [26]. Notice that,
we also assume $\xi_{E,ij}^E$ as symmetric with respect to the indices $i$ and $j$. Therefore, the number of free Yukawa couplings is reduced by two and one more coupling, namely $\xi_{E,\tau\mu}^E$, still exists as a free parameter. This parameter can be restricted by using the experimental result of $\mu$ EDM [27]

$$0.00 \times 10^{-19} e - cm < d_\mu < 10.34 \times 10^{-19} e - cm.$$  \hfill (16)

at 95% CL limit and the corresponding theoretical result for the EDM of muon in the model III (see [25] for details). Since non-zero EDM can be obtained in case of complex couplings, there exist a CP violating parameter $\theta_{\tau\mu}$ coming from the parametrization eq. (14). Using the experimental restriction in eq.(16), the upper limit of the coupling $\xi_{E,\tau\mu}^E$ is predicted at the order of the magnitude of $10^3 GeV$.

The other possibility to get a constraint for the upper limit of $\xi_{E,\tau\mu}^E$ is to use the experimental result of the muon AMM. In this work, we study the new physics effects on muon AMM and predict a more stringent bound for the coupling $\xi_{E,\tau\mu}^E$, with the assumption that the new physics effects lie in the experimental uncertainty of the muon AMM measurement. We also check the effect of the coupling $\xi_{N,\mu\mu}^E$ on AMM of muon and observe that AMM has a weak sensitivity on this coupling. This insensitivity is due to the suppression coming from factors $r_{h_0}$ and $r_{A_0}$ in the denominator of eq. (12). Therefore, we can take only $\xi_{E,\tau\mu}^E$ as a free parameter.

Fig. 2 shows $|\xi_{E,\tau\mu}^E|$ dependence of $\Delta_{new}a_\mu$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h_0} = 85 GeV$ and $m_{A_0} = 95 GeV$. Here, $\Delta_{new}a_\mu$ is at the order of the magnitude of $10^{-9}$, increases with increasing value of the coupling $|\xi_{E,\tau\mu}^E|$ and exceeds the experimental uncertainty, namely $10^{-9}$. This forces us to restrict the coupling $|\xi_{E,\tau\mu}^E|$ as $|\xi_{E,\tau\mu}^E| < 30 \pm 5 GeV$ for the intermediate values of $\sin\theta_{\tau\mu}$, $0.4 \leq \sin\theta_{\tau\mu} \leq 0.6$. This is extremely better upper limit compared the one obtained using the experimental result of $\mu$ EDM.

In Fig. 3, we show the $\sin\theta_{\tau\mu}$ dependence of $\Delta_{new}a_\mu$ for $\xi_{E,\tau\mu}^E = 30 GeV$, $m_{h_0} = 85 GeV$ and $m_{A_0} = 95 GeV$. Increasing values of $\sin\theta_{\tau\mu}$ causes $\Delta_{new}a_\mu$ to decrease and to lie in the experimental uncertainty.

In Fig. 4, we present $m_{h_0}$ dependence of $\Delta_{new}a_\mu$ for $\xi_{E,\tau\mu}^E = 30 GeV$, $\sin\theta_{\tau\mu} = 0.5$, and $m_{A_0} = 95 GeV$. This figure shows that the upper limit of $\Delta_{new}a_\mu$ decreases with the increasing values of $m_{h_0}$.

For completeness, we also show the $|\xi_{N,\mu\mu}^E|$ dependence of $\Delta_{new}a_\mu$ when the internal $\mu$-lepton contribution is taken into account. In this figure, it is observed that $\Delta_{new}a_\mu$ is weakly sensitive to $|\xi_{N,\mu\mu}^E|$ for $|\xi_{N,\mu\mu}^E| < 0.1 GeV$ and therefore the internal $\mu$-lepton contribution can be safely neglected, for these values.
In this work, we choose the type III 2HDM for the physics beyond the SM and assume that only FCNC interactions exist at tree level, with complex Yukawa couplings. We predict an upper limit for the coupling $|\xi_{N,\tau \mu}|$ for the intermediate values of imaginary part, by assuming that the new physics effects lie in the experimental uncertainty of muon AMM, namely $10^{-9}$, and see that this is an extremely better upper limit, $\sim 30 \text{GeV}$, compared the one obtained by using the experimental result of $\mu$ EDM, $\sim 10^3 \text{GeV}$. In the calculations, we studied the internal $\mu$ lepton contributions, however, we observe that gives a negligible contribution to AMM of muon. Furthermore, we neglect the $e$ lepton contribution. With the more reliable future measurements of muon AMM, it would be possible to check the new physics effects and restrict the corresponding new couplings, more accurately.

4 Acknowledgement

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References


Figure 1: One loop diagrams contributing to AMM of $l$-lepton due to the neutral Higgs bosons $h^0$ and $A^0$ in the 2HDM. Wavy (dashed) line represents the electromagnetic field ($h^0$ or $A^0$ fields).
Figure 2: $\Delta_{\text{New}} a_\mu$ as a function of $|\tilde{\xi}_{N,\tau\mu}|$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h^0} = 85 GeV$ and $m_{A^0} = 95 GeV$. 
Figure 3: $\Delta_{New} a_\mu$ as a function of $\sin \theta_{\tau\mu}$ for $|\tilde{E}_{N,\tau\mu}| = 30 \text{GeV}$, $m_{h^{0}} = 85 \text{GeV}$ and $m_{A^{0}} = 95 \text{GeV}$.

Figure 4: $\Delta_{New} a_\mu$ as a function of $m_{h^{0}}$ for $|\tilde{E}_{N,\tau\mu}| = 30 \text{GeV}$, $m_{A^{0}} = 95 \text{GeV}$ and $\sin \theta_{\tau\mu} = 0.5$. 

Figure 5: $\Delta_{N_{\mu\mu}}a_\mu$ as a function of $|\xi^{E}_{N,\mu\mu}|$ for $|\bar{\xi}^{E}_{N,\tau\mu}| = 30\, GeV$, $\sin\theta_{\tau\mu} = 0.5$, $\sin\theta_{\mu\mu} = 0.5$, $m_{\mu} = 85\, GeV$ and $m_{A^0} = 95\, GeV$. 