The physics of the International Linear Collider

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Frontier Physics at the Linear Collider
Lecture 1: Theory

Outline

• The Standard Model (SM) is not yet complete
  – The energy scale of electroweak symmetry breaking (EWSB) dynamics
  – Energy scales associated with neutrino masses and gravity
  – The SM as an effective theory

• Seeking the origin of EWSB dynamics
  – Implications of precision electroweak measurements
  – Weakly-coupled Higgs bosons
  – The decoupling limit
  – Evading the SM-like Higgs boson
Motivations for physics beyond the SM at the TeV-scale

- Hierarchy and naturalness
- Unification of gauge couplings
- Dark matter as a weakly-coupled thermal relic

Models of TeV-scale physics

- Low-energy supersymmetry
- Extra dimensions
- Alternative theories of EWSB
The Standard Model is not complete

One must determine the origin of the $W^\pm$ and $Z$ boson masses. In gauge theories, the gauge bosons are initially massless, but when quantum corrections are included, a mass may be generated. Gauge invariance implies:

$$i (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

The full gauge boson propagator is the sum of a geometric series

$$+ + + + + \ldots = \frac{-ig_{\mu\nu}}{q^2[1+\Pi(q^2)]}$$

neglecting terms $\propto q_\mu q_\nu$. The pole at $q^2 = 0$ is shifted to a non-zero value if:

$$\Pi(q^2) \underset{q^2 \to 0}{\approx} \frac{-g^2 v^2}{q^2}.$$ 

Then $q^2[1 + \Pi(q^2)] = q^2 - g^2 v^2$, corresponding to a gauge boson mass of $g v$. 
Interpretation of gauge boson mass generation:

In the sum over intermediate states, there is the propagation of a massless excitation, corresponding to a spin 0 state, called the Goldstone boson. This state could be either an elementary scalar boson or a bound state massless scalar.

**Experimental Observation:**

\[ m_W = 80.425 \pm 0.034 \text{ GeV} \]
\[ m_Z = 91.1875 \pm 0.0021 \text{ GeV} \]

The \( Z \) and \( W^\pm \) couple to neutral and charged weak currents

\[ \mathcal{L}_{\text{int}} = g_Z j^Z_{\mu} Z^\mu + g_W (j^W_{\mu} W^{+\mu} + \text{h.c.}) . \]

which are known to create neutral and charged pions from the vacuum. In the absence of quark masses, the pions are massless bound states of \( q\bar{q} \) [they are Goldstone bosons of chiral symmetry which is spontaneously broken by the strong interactions]. Thus, the diagram:

\[ \begin{align*}
\begin{array}{c}
\text{\( Z^0 \)}}
\end{array}
\end{align*} \]

yields \( \Pi(q^2) = -g_Z^2 f_\pi^2 / q^2 \), where \( f_\pi = 93 \text{ MeV} \) is the amplitude for creating a pion from the vacuum.
Thus, $m_Z = g_Z f_\pi$. Similarly $m_W = g_W f_\pi$. Thus,

$$\frac{m_W}{m_Z} = \frac{g_W}{g_Z} \equiv \cos \theta_W \simeq 0.88$$

which is remarkably close to the measured ratio. Unfortunately, since $g_Z \simeq 0.37$ we find $m_Z = 35$ MeV, which is too small by a factor of 2600. We need another source for the Goldstone bosons.

The quest for electroweak symmetry breaking is the search for the dynamics that generates the Goldstone bosons that are the source of mass for the $W$ and $Z$.

Possible choices for EWSB dynamics

- weakly-interacting self-coupled elementary (Higgs) scalar dynamics
- strong-interaction dynamics (mediated perhaps by new gauge forces)

Both mechanisms generate new phenomena with significant experimental consequences.
Significance of the TeV Scale—Part I

Let $\Lambda_{EW}$ be energy scale of EWSB dynamics. For example,

- Elementary Higgs scalar ($\Lambda_{EW} = m_H$).
- Strong EWSB dynamics (e.g., $\Lambda_{EW}^{-1}$ is the characteristic scale of bound states arising from new strong dynamics).

Consider $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ for $m_W^2 \ll s \ll \Lambda_{EW}^2$. The corresponding amplitude, to leading order in $g^2$, but to all orders in the couplings that control the EWSB dynamics, is equal to the amplitude for $G^+ G^- \rightarrow G^+ G^-$ (where $G^{\pm}$ are the charged Goldstone bosons). The latter is universal, independent of the EWSB dynamics. This is a rigorous low-energy theorem.

Applying unitarity constraints to this amplitude yields a critical energy $\sqrt{s_c}$, above which unitarity is violated. This unitarity violation must be repaired by EWSB dynamics and implies

$$\Lambda_{EW} \lesssim \mathcal{O}(\sqrt{s_c})$$.
For example, in the Standard Model, the $J = 0$ partial wave for $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ in the limit of $m_W^2 \ll s \ll m_H^2$ is equal to the corresponding amplitude for $G^+ G^- \rightarrow G^+ G^-$:

$$\mathcal{M}^{J=0} = \frac{G_F s}{16\pi\sqrt{2}}.$$ 

Partial wave unitarity implies that $|\mathcal{M}^J|^2 \leq |\text{Im} \ \mathcal{M}^J|$, which yields

$$(\text{Re} \ \mathcal{M}^J)^2 \leq |\text{Im} \ \mathcal{M}^J| \left(1 - |\text{Im} \ \mathcal{M}^J|\right) \leq \frac{1}{4}.$$ 

Setting $|\text{Re} \ \mathcal{M}^{J=0}| \leq \frac{1}{2}$ yields $\sqrt{s_c}$. The most restrictive bound arises from the isospin zero channel $\sqrt{\frac{1}{6}}(2W_L^+ W_L^- + Z_L Z_L)$:

$$s_c = \frac{4\pi\sqrt{2}}{G_F} = (1.2 \ \text{TeV})^2.$$ 

At or below $s = s_c$, one of two things must have happened:

(i) $\mathcal{M}^{J=0}$ is repaired by an elementary scalar Higgs, or
(ii) $\mathcal{M}^{J=0}$ is repaired by some other mechanism for EWSB.

Thus, the dynamics of electroweak symmetry breaking must be exposed at or below the 1 TeV energy scale.
What’s missing?

Not all observed microphysical phenomena has been included. Still missing:

- identification of the mechanism that produces the Goldstone bosons
- neutrino masses
- candidate for the dark matter
- gravity

In the standard parlance, the SM is defined such that the mass generation mechanism is due to the self-interactions of a single \( Y = 1 \) complex doublet of scalar fields (resulting in one neutral CP-even Higgs boson in the physical scalar sector). The other three items above necessarily lie outside the SM.

Thus, the Standard Model is at best an effective field theory, which will be superseded by a more fundamental theory at an energy scale \( (\Lambda) \) above which the SM breaks down.
The SM (with one Higgs doublet) contains information regarding its domain of validity. When radiative corrections are evaluated, one finds:

- The Higgs potential is unstable at large values of the Higgs field ($|\Phi| > \Lambda$) if the Higgs mass is too small.
- The value of the Higgs self-coupling runs off to infinity at an energy scale above $\Lambda$ if the Higgs mass is too large.

This is evidence that the SM must break down at energies above $\Lambda$.

![Graph showing Higgs mass bounds as a function of energy scale $\Lambda$](image)

Theoretical uncertainties on the lower [Altarelli and Isidori; Casas, Espinosa and Quirós] and upper [Hambye and Riesselmann]. Higgs mass bounds as a function of energy scale $\Lambda$ at which the Standard Model breaks down, assuming $m_t = 175$ GeV and $\alpha_s(m_Z) = 0.118$. The shaded areas above reflect the theoretical uncertainties in the calculations of the Higgs mass bounds.
Neutrino masses are perfectly well described by an effective SM. Simply, include the dimension-five interaction

\[ \mathcal{L}_{\text{int}} = \frac{\lambda}{\Lambda} (\overline{L} \Phi)(\Phi^\dagger L), \]

where \( L = (\nu_L, e_L) \) is the \( Y = -1 \) lepton doublet and \( \Phi = (\Phi^+, \Phi^0) \) is the \( Y = +1 \) complex Higgs doublet. When \( \Phi \) acquires a vacuum expectation value, \( \langle \Phi \rangle = v/\sqrt{2} \) [where \( v = 246 \) GeV], a neutrino mass is generated: \( m_\nu = \frac{1}{2} \lambda v^2/\Lambda \). If \( \lambda \sim 1 \) and \( m_\nu \sim 0.1 \) eV, one obtains \( \Lambda \gtrsim 10^{14} \) GeV. That is, the energy scale associated with neutrino mass generation lies way above the TeV scale.*

There is no candidate for dark matter within the Standard Model. Thus, dark matter must be associated with genuine new physics beyond the SM. More on the associated mass scale of this new physics later.

*To circumvent this conclusion, one needs to add additional new physics at the electroweak scale or below. Examples: (i) light right-handed neutrinos with tiny Yukawa couplings; (ii) R-parity-violating supersymmetry.
The Standard Model cannot be valid at energies above the Planck scale, \( M_{\text{PL}} \equiv (c\hbar/G_N)^{1/2} \simeq 10^{19} \text{ GeV} \), where gravity can no longer be ignored. The Planck scale arises as follows. The gravitational potential energy of a particle of mass \( M \), \( G_N M^2/r \) (where \( G_N \) is Newton’s gravitational constant), evaluated at a Compton wavelength, \( r = \hbar/Mc \), is of order the rest mass, \( Mc^2 \), when

\[
G_N M^2 \left( \frac{Mc}{\hbar} \right) \sim Mc^2,
\]

which implies that \( M^2 \sim c\hbar/G_N \). Equivalently, the Schwarzschild radius is of order the Compton wavelength. When this happens, the gravitational energy is large enough to induce pair production, which means that quantum gravitational effects can no longer be neglected. Thus, the Planck scale represents the energy scale at which gravity and all other forces of elementary particles must be incorporated into the same fundamental theory.
Constraints on the origin of EWSB Dynamics

The Standard Model has been tested with impressive accuracy by LEP, SLC, the Tevatron as well as data taken at lower energies (e.g., neutral currents in heavy atoms, Möller scattering, neutrino-nucleon scattering and \((g-2)_\mu\)). No significant deviations (say, above 3\(\sigma\)) have been observed. Many electroweak observables are sensitive to electroweak radiative corrections and provide one part per mille tests of the Standard Model. These data have been confronted with theoretical computations that incorporate virtual effects of the Higgs boson. For example, the \(W\) and \(Z\) masses are shifted slightly due to:

\[
W^\pm \quad \cdots \quad W^\pm \quad Z^0 \quad \cdots \quad Z^0
\]

The \(m_h\) dependence of the above radiative corrections is logarithmic. A global fit of many electroweak observables can test the overall consistency of the SM as well as providing bounds on the possible value of the Higgs mass.
<table>
<thead>
<tr>
<th>Measurement</th>
<th>Measurement Fit</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(5)}(m_Z)$</td>
<td>$0.02761 \pm 0.00036$</td>
<td>$0.02770$</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
<td>$91.1874$</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
<td>$2.4965$</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}}$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
<td>$41.481$</td>
</tr>
<tr>
<td>$R_l$</td>
<td>$20.767 \pm 0.025$</td>
<td>$20.739$</td>
</tr>
<tr>
<td>$A_{\ell}^{0,l}$</td>
<td>$0.01714 \pm 0.00095$</td>
<td>$0.01642$</td>
</tr>
<tr>
<td>$A_l(P_\tau)$</td>
<td>$0.1465 \pm 0.0032$</td>
<td>$0.1480$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$0.21630 \pm 0.00066$</td>
<td>$0.21562$</td>
</tr>
<tr>
<td>$A_{\ell}^{0,b}$</td>
<td>$0.0992 \pm 0.0016$</td>
<td>$0.1037$</td>
</tr>
<tr>
<td>$A_{\ell}^{0,c}$</td>
<td>$0.0707 \pm 0.0035$</td>
<td>$0.0742$</td>
</tr>
<tr>
<td>$A_{\ell}$</td>
<td>$0.923 \pm 0.020$</td>
<td>$0.935$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$0.670 \pm 0.027$</td>
<td>$0.668$</td>
</tr>
<tr>
<td>$A_l(\text{SLD})$</td>
<td>$0.1513 \pm 0.0021$</td>
<td>$0.1480$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}^{\text{lep}}(Q_{fb})$</td>
<td>$0.2324 \pm 0.0012$</td>
<td>$0.2314$</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>$80.425 \pm 0.034$</td>
<td>$80.390$</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>$2.133 \pm 0.069$</td>
<td>$2.093$</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>$178.0 \pm 4.3$</td>
<td>$178.4$</td>
</tr>
</tbody>
</table>
The global SM fit to high energy data from the Tevatron, SLC and LEP provide predictions for electroweak observables measured in low-energy data. In almost all cases there is consistency with one noticeable exception—the NuTeV low-energy neutrino scattering experiment. The discrepancy is best illustrated as follows:

**W-Boson Mass [GeV]**

- **TEVATRON**: $80.452 \pm 0.059$
- **LEP2**: $80.412 \pm 0.042$
- **Average**: $80.425 \pm 0.034$ ($\chi^2$/DoF: 0.3 / 1)
- **NuTeV**: $80.136 \pm 0.084$
- **LEP1/SLD**: $80.363 \pm 0.032$
- **LEP1/SLD/m_t**: $80.373 \pm 0.023
Constraints on $m_h$ from precision electroweak data

LEPEWWG SM fits to precision electroweak data $^\dagger$

$$m_h = 126^{+73}_{-48} \text{ GeV} \quad [m_h < 280 \text{ GeV at 95\% CL}]$$

(a) The 2005 blueband plot shows $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$ as a function of $m_h$. (b) Probability distribution function for $m_h$; each of the shaded regions corresponds to 50\% integrated probability [J. Erler, hep-ph/0310202].

$^\dagger$Global SM fits based on data from the $Z$-pole plus LEP/Tevatron data on $m_t$, $m_W$ and $\Gamma_W$ yield $\chi^2/\text{d.o.f.} = 18.28/13 \ [P = 14.7\%]$.  

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[Image of the blueband plot and probability distribution for $m_h$.]
Beyond the SM Higgs boson

The Higgs sector may be more complicated than that of the SM. There are a few constraints.

- The tree-level relation, \( m_W = m_Z \cos \theta_W \), is generic for models with singlet and doublet Higgs representations. In most other Higgs sectors, this mass relation is not respected (in contradiction to experimental measurements).

- In multi-Higgs models, phenomenologically unacceptable flavor-changing neutral currents mediated by Higgs bosons are present unless suppressed by additional symmetries and/or heavy mass effects.

- In the decoupling limit of a multi-Higgs models, there exists one neutral CP-even Higgs boson whose properties are indistinguishable from those of the SM Higgs boson; while all other physical Higgs scalars are significantly heavier. The constraints of precision electroweak observables apply to the lightest Higgs state.
**The Two-Higgs doublet model (2HDM)**

Minimizing the SM Higgs potential: \( V(\Phi) = -m^2 (\Phi^\dagger \Phi) + \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2 \), yields a physical Higgs mass: \( m_h^2 = \lambda v^2 \). The 2HDM potential is more complicated:

\[
V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
+ \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}.
\]

Nevertheless, if all \( \lambda_i \lesssim \mathcal{O}(1) \), the decoupling limit is achieved if the eigenvalues of the matrix \( m_{ij}^2 \) are opposite in sign and the positive eigenvalue is significantly larger than \( v^2 \) and the magnitude of the other eigenvalue. The effective low-energy theory is a one-doublet Higgs model.

The precision electroweak data favors a weakly-coupled Higgs sector, but cannot distinguish the SM from a Higgs sector close to the decoupling limit. A complete understanding of EWSB dynamics will likely require additional discoveries beyond a SM-like Higgs boson.
Can a Light Higgs Boson be avoided?

If new physics beyond the SM exists, it almost certainly couples to $W$ and $Z$ bosons. Then, there will be additional shifts in the $W$ and $Z$ mass due to the appearance of new particles in loops. In many cases, these effects can be parameterized in terms of two quantities, $S$ and $T$ [Peskin and Takeuchi].

It has been argued that to avoid the conclusion of a light Higgs boson, new physics beyond the SM must be accompanied by a variety of new phenomena at an energy scale between 100 GeV and 1 TeV that can be detected at future colliders (e.g., LHC and/or the ILC):

- either through direct observation of signatures of new physics
- or by improved precision measurements that can detect small deviations from SM predictions.
Peskin and Takeuchi actually defined three quantities $S$, $T$ and $U$ in terms of vector boson vacuum polarization amplitudes:

\[ i \Pi_{ij}^{\mu \nu} = i[g^{\mu \nu} A_{ij}(q^2) + q^\mu q^\nu B_{ij}(q^2)] \]

where $i, j = W, Z$ or $\gamma$. If one defines: $F_{ij}(q^2) \equiv [A_{ij}(q^2) - A_{ij}(0)]/q^2$, then:

\[ \alpha T = \frac{1}{m_W^2} [A_{11}(0) - A_{33}(0)] , \quad \alpha \equiv e^2/4\pi \]

\[ \alpha S = -4s_W \left[ F_{3\gamma}(m_Z^2) - s_W F_{33}(m_Z^2) \right] , \]

\[ \alpha U = 4s_W^2 \left[ F_{11}(m_W^2) - F_{33}(m_Z^2) \right] . \]

For convenience, we have defined $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$, $A_{11}(q^2) \equiv A_{WW}(q^2)$,

\[ A_{3\gamma}(q^2) \equiv s_W A_{\gamma\gamma}(q^2) + c_W A_{Z\gamma}(q^2) , \]

\[ A_{33}(q^2) \equiv c_W^2 A_{ZZ}(q^2) + 2c_W s_W A_{Z\gamma}(q^2) + s_W^2 A_{\gamma\gamma}(q^2) . \]

Note that gauge invariance implies that $A_{\gamma\gamma}(0) = 0$. $T$ is weak-isospin-violating and $S$ is weak-isospin-conserving. New physics contributions to $U$ tend to be very small and are often neglected.
Consider electroweak observables probed by energies no larger than the scale of electroweak symmetry breaking ($\nu = 246$ GeV). New physics effects are associated with a new energy scale $\Lambda$ that lies above the scale of electroweak symmetry breaking. Typically, effects governed by the new physics decouple when $E \ll \Lambda$ and so we would expect contributions to $S$ and $T$ from new physics to be quite small.

However, in some cases new physics effects do not decouple. A classic example is the case of a fourth generation of quarks with masses $m_T$ and $m_B$. For fun, let’s also assume that $m_h \gg m_Z$. Then, relative to the SM with $m_h \sim O(m_Z)$,

$$
\delta T = \frac{3}{16\pi m_W^2 \sin^2 \theta_W} \left[ m_T^2 + m_B^2 - \frac{2m_T^2 m_B^2}{m_T^2 - m_B^2} \log \left( \frac{m_T^2}{m_B^2} \right) - m_W^2 \tan^2 \theta_W \log \left( \frac{m_h^2}{m_Z^2} \right) \right],
$$

$$
\delta S = \frac{1}{2\pi} \left[ 1 - \frac{1}{3} \log \left( \frac{m_T^2}{m_B^2} \right) + \frac{1}{6} \log \left( \frac{m_h^2}{m_Z^2} \right) \right].
$$

There is no evidence for any deviation $\delta S$ and $\delta T$ relative to the SM with $m_t = 175$ GeV and $m_h = 150$ GeV. This puts strong constraints on any proposed “non-decoupling” new physics.
Although the data is suggestive of a weakly-coupled Higgs sector, one cannot definitively rule out another source of EWSB dynamics (although the measured $S$ and $T$ impose strong constraints on alternative approaches).
Motivations for physics beyond the SM at the TeV-scale

• Naturalness (in order to explain $m_W/M_{Pl}$)
  – Pro: successful explanation for the magnitude of $M_{proton}/M_{PL}$
  – Con: failure to understand the size of the cosmological constant

• Unification of gauge couplings
  – Pro: unsuccessful unification in the SM is repaired by introducing TeV-scale supersymmetry (which also provides a framework for explaining $m_W/M_{Pl}$)
  – Con: unification could be repaired by adding new (non-SUSY) phenomena at scales significantly above 1 TeV. Alternatively, gauge coupling unification could just be a coincidence (it’s just one data point)

• Dark matter
  – Pro: TeV-scale physics with a conserved multiplicative quantum number provides a candidate with the right annihilation cross section to yield a big bang relic with 25% critical density
  – Con: Models of dark matter models exist that have no connection to the TeV scale (e.g. “invisible” axions)
In 1939, Weisskopf computed the self-energy of a Dirac fermion and compared it to that of an elementary scalar. The fermion self-energy diverged logarithmically, while the scalar self-energy diverged quadratically. If the infinities are cut-off at a scale $\Lambda$, then Weisskopf argued that for the particle mass to be of order the self-energy,

- For the $e^-$, $\Lambda \sim m \exp(1/\alpha) \gg M_{\text{PL}}$ [$\alpha \equiv e^2/4\pi \simeq 1/137$];
- For an elementary boson, $\Lambda \sim m/g$, where $g$ is the coupling of the boson to gauge fields.

In modern times, this is called the hierarchy and naturalness problem. Namely, how can one understand the large hierarchy of energy scales from $v$ to $M_{\text{PL}}$ in the context of the SM? If the SM is superseded by a more fundamental theory at an energy scale $\Lambda$, one expects scalar squared-masses to exhibit (at one-loop order) quadratic sensitivity to $\Lambda$, in contrast to the logarithmic sensitivity of the fermions. That is, the natural value for the scalar squared-mass is roughly $(g^2/16\pi^2)\Lambda^2$. Thus,

$$\Lambda \simeq 4\pi m_h/g \sim O(1 \text{ TeV}).$$
On the Self-Energy and the Electromagnetic Field of the Electron

V. F. Weisskopf
University of Rochester, Rochester, New York
(Received April 12, 1939)

The charge distribution, the electromagnetic field and the self-energy of an electron are investigated. It is found that, as a result of Dirac's positron theory, the charge and the magnetic dipole of the electron are extended over a finite region; the contributions of the spin and of the fluctuations of the radiation field to the self-energy are analyzed, and the reasons that the self-energy is only logarithmically infinite in positron theory are given. It is proved that the latter result holds to every approximation in an expansion of the self-energy in powers of $\varphi/\hbar c$. The self-energy of charged particles obeying Bose statistics is found to be quadratically divergent. Some evidence is given that the "critical length" of positron theory is as small as $\hbar/(mc)\exp(-\hbar c/\varphi^2)$.

I. INTRODUCTION AND DISCUSSIONS OF RESULTS

The self-energy of the electron is its total energy in free space when isolated from other particles or light quanta. It is given by the expression

$$W = T + (1/8\pi) \int (H^2 + E^2) \, d\mathbf{r}. \tag{1}$$

Here $T$ is the kinetic energy of the electron; $H$ and $E$ are the magnetic and electric field strengths. In classical electrodynamics the self-energy of an electron of radius $a$ at rest and without spin is given by $W = mc^2 + e^2/a$ and consists solely of the energy of the rest mass and of its electrostatic field. This expression diverges linearly for an infinitely small radius. If the electron is in motion, other terms appear representing the energy produced by the magnetic field of the moving electron. These terms, of course, can be obtained by a Lorentz transformation of the former expression.

The quantum theory of the electron has put the problem of the self-energy in a critical state. There are three reasons for this:

(a) Quantum kinematics shows that the radius of the electron must be assumed to be zero. It is easily proved that the product of the charge densities at two different points, $\rho(r - \xi/2) \times \rho(r + \xi/2)$, is a delta-function $\delta^3(\xi)$. In other words: if one electron alone is present, the probability of finding a charge density simultaneously at two different points is zero for every finite distance between the points. Thus the energy of the electrostatic field is infinite as $W_{\text{st}} = \lim_{\epsilon \to 0} (e^2/\epsilon a)$.

(b) The quantum theory of the relativistic electron attributes a magnetic moment to the electron, so that an electron at rest is surrounded by a magnetic field. The energy

$$U_{\text{mag}} = (1/8\pi) \int H^2 \, d\mathbf{r}$$

of this field is computed in Section III and the result is

$$U_{\text{mag}} = e^2 \hbar^2 / (6\pi m^2 c^3 a^2).$$

This corresponds to the field energy of a magnetic dipole of the moment $e\hbar/2mc$ which is spread over a volume of the dimensions $a$. The spin, however, does not only produce a magnetic field, it also gives rise to an alternating electric field. The closer analysis of the Dirac wave equation has shown\(^1\) that the magnetic moment of the spin is produced by an irregular circular fluctuation movement (Zitterbewegung) of the electron which is superimposed to the translatory motion. The instantaneous value of the velocity is always found to be $c$. It must be expected that this motion will also create an alternating electric field. The existence of this field is demonstrated in Section III by the computation of the expression

$$U_{\text{el}} = (1/8\pi) \int E_s^2 \, d\mathbf{r}.$$  

There $E_s$ is the solenoidal part (div. $E_s = 0$) of the electric field strength created by the electron. The fact that the above expression does not vanish for an electron at rest proves the existence

\(^1\) E. Schroedinger, Berl. Ber. 1930, 418 (1930).
zero in the one-electron theory, is negative and quadratically divergent in the positron theory. This is because of the negative contribution of the magnetic field and the interference effect of the electric field of the vacuum electrons.

(c) The energy $W_{\text{fluct}}$ of forced vibrations under the influence of the zero-point fluctuations of the radiation field. The energies (b) and (c) compensate each other to a logarithmic term.

It is interesting to apply similar considerations to the scalar theory of particles obeying the Bose statistics, as has been developed by Pauli and the author. Here the probability of finding two equal particles closer than their wave-lengths is larger than at longer distances. The effect on the self-energy is therefore just the opposite. The influence of the particle on the vacuum causes a higher singularity in the charge distribution instead of the hole which balanced the original charge in the previous considerations. It is shown in Section V that this gives rise to a quadratically divergent energy of the Coulomb field of the particle. Thus the situation here is even worse than in the classical theory. The spin term obviously does not appear and the energy $W_{\text{fluct}}$ is exactly equal to its value for a Fermi particle.

A few remarks might be added about the possible significance of the logarithmic divergence of the self-energy for the theory of the electron. It is proved in Section VI that every term in the expansion of the self-energy in powers of $e^2/\hbar c$

$$W = \sum_n W^{(n)}$$

(3)

diverges logarithmically with infinitely small electron radius and is approximately given by

$$W^{(n)} \sim s_n a e^2 (e^2/\hbar c)^2 n [\log (h/\hbar m c a)]^n, \quad t \leq n.$$  

Here the $s_n$ are dimensionless constants which cannot easily be computed. It is therefore not sure, whether the series (3) converges even for finite $a$, but it is highly probable that it converges if $\delta = e^2/(\hbar c) \cdot \log (h/\hbar m c a) < 1$. One then get

$$W = m c^2 O(\delta)$$

where $O(\delta) = 1$ for a value of $\delta < 1$. We then can define an electron radius in the same way as the classical radius $e^2/m c^2$ is defined, by putting the self-energy equal to $m c^2$. One obtains then roughly a value $a \sim h/(m c) \cdot \exp (-\hbar c/\epsilon^2)$

which is about $10^{-38}$ times smaller than the classical electron radius. The "critical length" of the positron theory is thus infinitely smaller than usually assumed.

The situation is, however, entirely different for a particle with Bose statistics. Even the Coulombian part of the self-energy diverges to a first approximation as $W_{\text{st}} \sim e^4 h/(m c a^2)$ and requires a much larger critical length that is $a = (h c e^2)^{-1} \cdot h/(m c)$, to keep it of the order of magnitude of $m c^2$. This may indicate that a theory of particles obeying Bose statistics must involve new features at this critical length, or at energies corresponding to this length; whereas a theory of particles obeying the exclusion principle is probably consistent down to much smaller lengths or up to much higher energies.

II. The Charge Distribution of the Electron

The charge distribution in the neighborhood of an electron can be determined from the expression

$$G(\xi) = \int \rho(r - \xi/2) \rho(r + \xi/2) dr;$$

(4)

where $\rho(r)$ is the charge density at the point $r, G(\xi)$ is the probability of finding charge simultaneously at two points in a distance $\xi$. If applied to a situation in which one electron alone is present, direct information can be drawn from this expression concerning the charge distribution in the electron itself. The charge density is given by

$$\rho(r) = e |\psi^*(r)\psi(r)| - \sigma,$$

(5)

where $\psi(r)$, the wave function, is a spinor with four components $\psi_\mu, \mu = 1, 2, 3, 4$. We write

$$\{\psi^* \psi\} = \sum_{\mu=1}^4 \psi^*_\mu \psi_\mu$$

for the scalar product of two spinors. $\sigma$ is the charge density of the unperturbed electrons in the negative energy states which is to be subtracted in the positron theory. In the one-electron theory $\sigma$ is zero. The wave function $\psi$ can be expanded in wave functions $\varphi_\alpha$ of the

\footnote{W. Pauli and V. Weisskopf, Helv. Phys. Acta 7, 709 (1934).}
Is naturalness a valid principle for physics? Consider first its successes.

- **The proton mass**, $M_{\text{proton}} \ll M_{\text{PL}}$.

In QCD, $M_{\text{proton}} \propto \Lambda_{\text{QCD}}$. The scale $\Lambda_{\text{QCD}}$ is generated by dimensional transmutation. At one-loop,

$$
\Lambda_{\text{QCD}} = M_{\text{PL}} \exp \left( \frac{-2\pi}{b_0 \alpha_s(M_{\text{PL}})} \right),
$$

where $b_0 = 11 - \frac{2}{3}N_f$ in the SM. The value of $\alpha_s(M_{\text{PL}}) \sim 0.02$ is not particularly small. Thus, an exponential hierarchy is naturally achieved.

- **The electron mass is only logarithmically sensitive to ultraviolet physics.**

The electron self-energy in classical electromagnetism goes like $e^2/a \ (a \to 0)$, *i.e.*, it is linearly divergent. In quantum theory, fluctuations of the electromagnetic fields (in the “single electron theory”) generate a quadratic divergence. If these divergences are not canceled, one would expect that QED should break down at an energy of order $m_e/e$ far below the Planck scale (a severe hierarchy problem).

The linear and quadratic divergences will cancel exactly if one makes a bold hypothesis: the existence of the positron (with a mass equal to that of the electron but of opposite charge).
Weisskopf was the first to demonstrate this cancelation in 1934. . . well, actually he initially got it wrong, but thanks to Furry, the correct result was presented in an erratum.

A remarkable result:

The linear and quadratic divergences of a quantum theory of elementary fermions are precisely canceled if one doubles the particle spectrum—for every fermion, introduce an anti-fermion partner of the same mass and opposite charge.
The self-energy of the electron

V. WEISSKOPF


The self-energy of the electron is derived in a closer formal connection with classical radiation theory, and the self-energy of an electron is calculated when the negative energy states are occupied, corresponding to the conception of positive and negative electrons in the Dirac 'hole' theory. As expected, the self-energy also diverges in this theory, and specifically to the same extent as in ordinary single-electron theory.

1 Problem definition

The self-energy of the electron is the energy of the electromagnetic field which is generated by the electron in addition to the energy of the interaction of the electron with this field. Waller, 1 Oppenheimer, 2 and Rosenfeld 3 calculated the self-energy of the free electron by means of the Dirac relativistic wave equation of the electron and the Dirac theory of the interaction between matter and light. They here used an approximation method which represents the self-energy in powers of the charge e. They found that the first term, which is proportional to $e^2$, already becomes infinitely large. The essential reason for this is that the theory of the interaction of the electron with the electromagnetic field is built on the classical equations of motion of a point-shaped electron whose self-energy, as is well known, also becomes infinite in classical theory. 4

In the present note, the expressions for the self-energy shall be derived without direct application of quantum electrodynamics, but by means of the Heisenberg radiation theory, 5 which is linked much more closely to classical electrodynamics. The radiation field is calculated classically from the current and charge densities of the atom; however, the amplitudes of the electromagnetic potentials are regarded as non-commuting in the final result. Just as was shown in a corresponding paper by Casimir 6 concerning the natural linewidth, this method yields the same result as explicit quantum

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3 L. Rosenfeld, ZS. f. Phys. 70, 454, 1931.
4 Recently, G. Wentzel (ZS. f. Phys. 86, 479, 635, 1933) has shown that one can circumvent the divergence of the self-energy in classical electron theory by suitable limiting processes. The transfer of these methods to quantum theory has failed, however, since, according to Waller, the degree of infinity in quantum theory is higher than in classical theory. The hope expressed there that the degree of infinity will become smaller in the Dirac formalism of the 'hole' theory, does indeed hold for the electrostatic part but not for the electromagnetic part, so that the Wentzel method must fail here too.
6 H. Casimir, ZS. f. Phys. 81, 496, 1933.
Correction to the paper: The self-energy of the electron


On [p. 166] of the paper cited above, there is a computational error which has seriously garbled the results of the calculation for the electrodynamic self-energy of the electron according to the Dirac hole theory. I am greatly indebted to Mr Furry (University of California, Berkeley) for kindly pointing this out to me.

The degree of divergence of the self-energy in the hole theory is not, as asserted in [the preceding paper], just as great as in the Dirac one-electron theory, but the divergence is only logarithmic. The expression for the electrostatic and electrodynamic parts of the self-energy $E$ of an electron with momentum $p$ now correctly reads, in the notations used in [the preceding paper]:

$$E = E^5 + E^D,$$

$$E^5 = \frac{e^2}{h(m^2c^2 + p^2)^{1/2}} (2m^2c^2 + p^2) \int_{k_0}^{\infty} \frac{dk}{k} + \text{finite terms},$$

$$E^D = \frac{e^2}{h(m^2c^2 + p^2)^{1/2}} \left( m^2c^2 - \frac{4}{3} p^2 \right) \int_{k_0}^{\infty} \frac{dk}{k} + \text{finite terms}.$$

For comparison, we cite the expressions obtained on the basis of the single-electron theory:

$$E^5 = \frac{e^2}{h} \int_0^{\infty} \frac{dk}{k} + \text{finite terms},$$

$$E^D = \frac{e^2}{h} \left[ \frac{m^2c^2}{p(m^2c^2 + p^2)^{1/2}} \log \left( \frac{m^2c^2 + p^2)^{1/2} + p}{m^2c^2 + p^2)^{1/2} - p} \right] \int_0^{\infty} \frac{dk}{k}$$

$$+ \frac{2e^2}{h(m^2c^2 + p^2)^{1/2}} \int_0^{\infty} \frac{k \, dk}{k}.$$

The computational error arose in the transformation of the electrodynamic portion $E^D$ for the case of the hole theory:

$$E^D = J^k_+(\vec{p}) - J^k_-(-\vec{p}), \quad k = 1 \text{ or } 2,$$

where $J^k_\pm(\vec{p})$ is defined on [p. 166] whereas

$$J^k_\pm(\vec{p}) = -\frac{e^2}{2\pi h} \int \frac{dk}{k} \frac{PP_+ + \frac{1}{k^2} (\vec{k}\vec{p})^2 + (\vec{k}\vec{p}) + m^2c^2}{PP_+(P + P_+ + k)}$$

and is not equal to the quantity $J^k(\vec{p})$, from which it differs only by a sign. Likewise, one must set

$$E^D_{\text{vac}} = \sum_{k=1,2} \int J^k_\pm(\vec{p}) \, d\vec{p}$$

for the self-energy of the vacuum.

As a consequence of the new result, the question raised in note 4 of the paper requires a new examination, whether the Wentzel method,\textsuperscript{15} to avoid the infinite self-energy by suitable limiting processes, might not still lead to the objective in the hole theory.

\textsuperscript{15} G. Wentzel, ZS. f. Phys, 86, 479, 635, 1933.
Should the electroweak scale respect naturalness?

If we demand that the value of $m_h$ is natural (equivalently, the scale of EWSB is natural), i.e., without substantial fine-tuning, then $\Lambda$ cannot be significantly larger than 1 TeV.

Following Kolda and Murayama [JHEP 0007 (2000) 035], a reconsideration of the $\Lambda$ vs. Higgs mass plot with a focus on $\Lambda < 100$ TeV. Precision electroweak measurements restrict the parameter space to lie below the dashed line, based on a 95% CL fit that allows for nonzero values of $S$ and $T$ and the existence of higher dimensional operators suppressed by $v^2/\Lambda^2$. The unshaded area has less than one part in ten fine-tuning.
Since \( \Lambda \) represents the scale at which new physics beyond the SM must enter (which will provide a natural explanation for the scale of electroweak symmetry breaking), we should expect new physics at the TeV-scale that is intimately connected to EWSB dynamics.

However, naturalness has been a spectacular failure when applied to the cosmological constant problem. We have no compelling explanation for why \( \Lambda_{\text{cosmo}} \sim (0.003 \text{ eV})^4 \sim 10^{-120} M_{\text{Pl}}^4 \) is so tiny. The geometric mean of \( \Lambda_{\text{cosmo}}^{1/4} \) and \( M_{\text{Pl}} \) is a few TeV. Is this an accident? Perhaps naturalness is not a guiding principle for TeV-scale physics. Recent studies of string theory vacua have led to the idea of the landscape of possible universes. Our universe is one out of an almost uncountable number of possibilities. Naturalness may not be relevant in deciding which possibility corresponds to our universe.

Experimentation at the LHC (and beyond) will be decisive in discovering whether naturalness plays a role in determining the EWSB scale.

Without naturalness, the motivation for TeV-scale physics is much weaker.
Models of TeV-scale Physics:
1. TeV-Scale Supersymmetry

We solved one hierarchy problem by doubling the spectrum with the introduction of antiparticles. Let’s try it again by taking the Standard Model and double the particle spectrum. Introduce a new symmetry—supersymmetry—that relates fermions to bosons: for every fermion, there is a boson of equal mass and vice versa. Now, compute the self-energy of an elementary scalar. Supersymmetry relates it to the self-energy of a fermion, which is only logarithmically sensitive to the fundamental high energy scale. Conclusion: quadratic sensitivity is removed! The hierarchy problem is resolved.

However, no super-partner (degenerate in mass with the corresponding SM particle) has ever been seen. Supersymmetry, if it exists in nature, must be a broken symmetry. Previous arguments then imply that:

The scale of supersymmetry-breaking must be of order 1 TeV or less, if supersymmetry is associated with the scale of electroweak symmetry breaking.
Outstanding questions for TeV-scale supersymmetry

• Origin of supersymmetry (SUSY)-breaking.

This is the source of almost all the model dependence. Gravity-mediated, gauge-mediated, anomaly-mediated, gaugino-mediated and extra-dimensional SUSY-breaking is an incomplete list of proposed mechanisms. If SUSY is discovered, the main challenge for future experiments is to discover the underlying systematics of SUSY-breaking effects.

• Origin of R-parity conservation (if indeed it is conserved)

Unlike the SM, the supersymmetric extension of the SM does not guarantee the absence of dimension-four operators that violate baryon number (B) and lepton number (L). Thus, one typically imposes a $\mathbb{Z}_2$ discrete symmetry, $R = (-1)^{3(B-L)+2S}$ (where $S$ is the particle spin). An alternative is to impose a $\mathbb{Z}_3$ “baryon parity” that preserves B and violates L. This provides a new mechanism for neutrino masses, but one would have to explain the very small magnitudes of the R-parity-violating terms.

• Overcoming the flavor and CP-violation problems

Without making any special assumptions, supersymmetry-breaking effects will generically lead to FCNCs and CP-violating phases that are too large and not phenomenologically viable. This will impose strong constraints on any viable fundamental theory of SUSY-breaking.
Benefits of TeV-scale supersymmetry

- Quadratic sensitivity to $\Lambda$ is replaced by quadratic sensitivity to the SUSY-breaking scale.
- Provides a framework for the hierarchy of energy scales between the scale of electroweak symmetry breaking and $M_{PL}$ which characterizes the fundamental scale of gravity.
- Unification of the three gauge couplings at $\sim 10^{16}$ GeV.

A natural dark matter candidate—the lightest supersymmetric particle (LSP)—if R-parity is conserved.
- Radiative electroweak symmetry breaking triggered by a large Higgs-top quark Yukawa coupling.

This mechanism provides a direct link between SUSY-breaking and EWSB dynamics.

- The large value of $m_t$ can be understood as a result of a quasi-infrared fixed point of the Higgs-top quark Yukawa coupling.

- Origin of flavor and CP-violation may be associated with physics near $M_{\text{PL}}$; fermion masses arise from Higgs-fermion Yukawa couplings when the neutral Higgs fields acquire vacuum expectation values.
\[ \alpha_s(M_Z) = 0.118 \quad m_b(m_b) = 4.25 \quad M_{SUSY} = m_t^{pole} \]

\[ m_t \simeq \sqrt{\frac{1}{2}} \lambda_t(m_t) v \sin \beta \left[ 1 + \frac{4\alpha_s(m_t)}{3\pi} \right] \sim (185 \text{ GeV}) \lambda_t(m_t) \sin \beta \]
The Minimal Supersymmetric Standard Model (MSSM)

- Add a second complex Higgs doublet.
- Add corresponding super-partners and allow for all possible supersymmetric interactions (subject to the constraints of an approximately conserved B and L)
- Add supersymmetry-breaking (subject to experimental limits on super-partner masses and limits on FCNC’s and CP-violation.)

The MSSM spectrum and its parameters

- **Gauginos and Higgsinos** (spin-1/2 superpartners of the gauge and Higgs bosons)

States with the same color and electric charge can mix. The physical eigenstates are determined by three gaugino Majorana mass parameters ($M_1$, $M_2$ and $M_3$), a supersymmetric Higgs mass parameter $\mu$, and the ratio of the two neutral Higgs field vacuum expectation values, $\tan \beta$. The gluino $\tilde{g}$ is the SUSY-partner of the gluon. The charginos $\tilde{\chi}_i^\pm$ ($i = 1, 2$) are mixtures of charged gauginos and higgsinos. The neutralinos $\tilde{\chi}_j^0$ ($j = 1, 2, 3, 4$) are mixtures of neutral gauginos and higgsinos. In total, five real parameters and three CP-violating phases determine the gaugino/higgsino sector

\[\text{\textsuperscript{‡}required for anomaly cancelation by higgsino pairs of opposite hypercharge}\]
• **Squarks and sleptons** (Spin-0 superpartners of the quarks and leptons)
Each fermion $f$ has two associated scalar partners $\tilde{f}_L$ and $\tilde{f}_R$ (corresponding to the left-handed and right-handed fermion fields, respectively). In the case of one generation, the up-type and down-type squark and charged slepton physical eigenstates are determined by diagonalizing three $2 \times 2$ squared-mass matrices. (There is no $\tilde{\nu}_R$ in the MSSM without a SUSY generalization of the seesaw.) In addition to SUSY-breaking diagonal squared-masses, there is an off-diagonal squared-mass term:

$$M_{LR}^2 = \begin{cases} 
  m_b (A_b - \mu \tan \beta), & \text{for "down"-type } f, \\
  m_t (A_t - \mu \cot \beta), & \text{for "up"-type } f.
\end{cases}$$

In the three generation case, these squared-mass matrices can be more complicated. In total, 21 squark and slepton masses, 36 real mixing angles and 40 CP-violating phases are needed to fully fix the squark/slepton sector!

**The MSSM parameter count**
The SM depends on 19 parameters: (i) three gauge couplings; (ii) two Higgs potential parameters (or equivalently, the Higgs vev $v$ and mass $m_H$); (iii) six quark masses; (iv) three charged lepton masses; (v) three CKM mixing angles and one CP-violating phase; and (vi) $\theta_{QCD}$. The MSSM adds 105 new parameters, for a grand total of 124.
Higgs sector of the MSSM

Two complex doublets have eight degrees of freedom. Two neutral scalar components acquire vacuum expectation values: $v_1$ and $v_2$. The two scalar provide three Goldstone bosons that are absorbed by the $W^\pm$ and $Z$. The gauge boson masses fix the value of $v_1^2 + v_2^2 = 246$ GeV, while the ratio $\tan \beta \equiv v_2/v_1$ remains a free parameter. There are five remaining physical scalar degrees of freedom:

- $H^\pm$: a charged Higgs pair

- $h, H$: two CP-even Higgs scalars ($m_h \leq m_H$), and a CP-even Higgs mixing angle $\alpha$

- $A$: a CP-odd Higgs scalar

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be $m_A$ and $\tan \beta$. 
• When \( m_A \gg m_Z \), one finds that \( h \) exhibits couplings identical to that of the SM Higgs boson, while the other Higgs states \( H, A \) and \( H^\pm \) are all heavy and roughly degenerate in mass. This is called the decoupling limit.

• Due to supersymmetric relations among couplings, one finds that \( m_h \leq m_Z \) (a result already ruled out by LEP data). But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancelation, which would have been exact if supersymmetry were unbroken):

\[
m_h^2 \lesssim m_Z^2 + \frac{3g^2m_t^4}{8\pi^2m_W^2} \left[ \ln \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right],
\]

where \( X_t \equiv A_t - \mu \cot \beta \) governs top squark mixing and \( M_S^2 \) is the average top-squark squared-mass.
End result: $m_h \lesssim 130$ GeV [assuming that the top-squark mass is no heavier than about 2 TeV].

Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that $m_h$ is maximized (for a fixed $\tan \beta$). This occurs for $X_t/M_S \sim 2$. As $\tan \beta$ varies, $m_h$ reaches its maximal value, $(m_h)_{\text{max}} \simeq 130$ GeV, for $\tan \beta \gg 1$ and $m_A \gg m_Z$. 
Present status of the LEP Higgs Search\(^8\) [95% CL limits]

- Standard Model Higgs boson: \(m_h > 114.6\) GeV
- Charged Higgs boson: \(m_{H^\pm} > 78.6\) GeV
- MSSM Higgs: \(m_h > 92.9\) GeV; \(m_A > 93.4\) GeV

\(^8\)CP-conserving Higgs sector assumed. At one-loop CP-violating effects can enter, in which case a number of holes open up in previously excluded regions.
Supersymmetry is a theory that introduces new anti-commuting dimensions, thereby extending the space-time symmetry group. One may also contemplate the appearance of new commuting dimensions, that is the addition of new dimensions of physical space. To explain why this has not yet been observed, the new dimensions must either be compact with a size of order $\lesssim (1 \text{ TeV})^{-1}$ or warped. Our observed space is called a 3-brane, and one can consider models in which none, some, or all the particles of the SM are confined to the brane. Gravity must propagate in all the available space-dimensions (i.e, “the bulk”).

It is desirable to connect the size of the extra-dimensions with the scale of EWSB, with the possibility of providing a natural explanation for the TeV-scale.

Further theoretical details of these models and their phenomenology will be explored in depth by JoAnne Hewett in her lectures, so I will move on.
Models of TeV-scale Physics:

3. Alternative Approaches to EWSB

The Higgs sector of the SM is unnatural. Many attempts have been made to alter the dynamics of the EWSB sector to secure a natural explanation of the EWSB scale. Many of these approaches employed strong-coupling dynamics to generate the Goldstone bosons of electroweak physics in a natural way. Technicolor, walking technicolor, composite Higgs models, and top-quark condensate models are examples of this paradigm. Many of these attempts failed due to phenomenological constraints, while others were just too baroque. More recently, little Higgs models and a number of approaches inspired by extra-dimensional theories yielded new interesting possibilities. In judging any such models, it is critical that you:

- Take a close look at the predicted values of $S$ and $T$ (and constraints from precision electroweak physics).
- Examine closely the mechanism for generating fermion masses. (The simplicity of the SM approach is hard to beat!!)

Further theoretical details of these models and their phenomenology have already been discussed by Hitoshi Murayama in his lectures.
Lecture 2: Phenomenology of TeV-scale Physics at the ILC

Outline

• Anticipating the future of 21st century particle physics
  — electroweak symmetry breaking, TeV-scale physics and beyond

• The TeV-scale colliders
  — Large Hadron Collider (LHC)
    — International Linear Collider (ILC) from $\sqrt{s} = 500$ GeV to 1 TeV

• A path to the ILC
  — furthering the goals of the LHC

• Complementarity and synergy
  — New physics signals and precision measurements
• A few case studies
  – precision Higgs program
  – elucidation of TeV-scale supersymmetry
  – confusion scenarios
  – probing higher energies through virtual effects
  – connections with cosmology
1. Completing the Standard Model (SM)—elucidating the dynamics of electroweak symmetry breaking (EWSB)

- elementary Higgs bosons (weakly-coupled scalar dynamics)

- strongly-coupled EWSB dynamics (with or without Higgs-like scalars)

- strongly-coupled EWSB dynamics masquerading as weakly-coupled EWSB dynamics (with a scalar state resembling the SM Higgs boson) \textit{e.g.} little Higgs models

Precision electroweak physics provides strong hints for a SM-like Higgs boson. How devious is nature likely to be (are there new physics conspiracies?)
2. Is there new TeV-scale physics? If yes, how is the Standard Model superseded?

*Note:* the origin of flavor, CP violation and neutrino masses could in principle lie near the TeV-scale, but these are most likely are associated with much larger energy scales.

3. Scenarios for the future of particle physics

- After the SM Higgs boson, there is no TeV-scale physics. New physics is deferred to much higher energy scales.

- New TeV-scale physics reveals the existence of new 10 TeV-scale physics. Another layer of the onion is revealed...

- New TeV-scale physics provides a window to GUT/Planck-scale physics.
1. The Large Hadron Collider (LHC) is a $pp$ collider now under construction at CERN. Its center-of-mass energy is 14 TeV, with an anticipated luminosity of 10 fb$^{-1}$ per year in an initial period of running. This will be followed by higher luminosity running of 100 fb$^{-1}$ per year. The LHC will begin initial operations in 2007.

2. The International Linear Collider (ILC) is a high energy $e^+e^-$ collider presently under development. Its maximal center-of-mass-energy will initially be 500 GeV, with an upgrade path to 1 TeV. A luminosity of 100 fb$^{-1}$ per year are expected (with the potential for doubling the luminosity in later years). Over the next three years, the GDE (Global Design Effort) will organize the development of a conceptual design followed by a full technical design of the ILC. In parallel, detector designs for the ILC are also being formulated.
Large Hadron Collider at CERN

Circumference 26.7 km (16.6 miles)

Proton Beams

Experimental Hall (Collision point)

Tunnel cross section

Proton Injector

Booster rings

Experimental Hall (Collision point)

Detector for CMS experiment

Detector for ATLAS experiment (displaced for clarity)

Geneva
Generic Linear Collider

30-40 km

All collider elements are challenging

Albert De Roeck (CERN)
The International Linear Collider (ILC), with $\sqrt{s} = 500$ GeV (with an upgrade path to 1 TeV) is proposed as the facility that will complement the LHC in the twin goals of deciphering EWSB dynamics and revealing new TeV-scale physics. Realizing the ILC is a technical and political challenge, so it is critical to provide:

- a convincing case for the technological and financial viability of the machine; and
- a scientific case that is strong enough to be appreciated by other scientists and the political establishments.

Much work has gone in to building the scientific case for the ILC (some of which I will describe below). It is now imperative to develop the detailed conceptual and technical designs of the machine and the detectors. In this way, the international particle physics community will be ready to press for the final approval of the ILC (and begin its construction) around the time of the first major discoveries of the LHC. Of course, the nature of the LHC discoveries could have a considerable impact on the final form of the ILC proposal and its ultimate chances for successfully going forward.
The LHC and ILC provide complementary approaches to the TeV scale, in the same way that the CERN $Spp\bar{S}$/Tevatron and LEP/SLC provided complementary approaches to the 100 GeV scale. If the ILC is constructed to operate at some point during the LHC era, then there is potential for a synergetic interplay of the LHC and ILC physics programs:

- The combined interpretation of LHC and ILC data can yield a more unambiguous interpretation of the underlying physics than the results of both colliders taken separately.

- Combined analyses of data during concurrent LHC/ILC running implies that results obtained at one machine can influence the analysis techniques at the other machine, leading to optimized search strategies of new physics signals.
The LHC/ILC Study Group has documented numerous examples of the complementarity and potential synergy of the LHC and ILC [see G. Weiglein et al., hep-ph/0410364]. Two examples (of many) are:

- **The precision Higgs program at the ILC and the search for heavy Higgs scalars at the LHC**

Many models of weakly-coupled scalar EWSB dynamics predict the existence of a decoupling limit in which there exists one neutral Higgs boson whose properties are nearly indistinguishable from those of the SM Higgs boson. In multi-Higgs models, the approach to the decoupling limits scales as $m_Z^2/m_A^2$, where $m_A$ is a typical mass of the heavier Higgs states. Deviations of the couplings of the lightest Higgs boson from the corresponding SM predictions at the ILC can provide an estimate of the mass scale associated with the heavier Higgs states and provide crucial information for LHC searches for the heavier Higgs bosons.
• Precision ILC measurements of the light neutralino/chargino states in TeV-scale supersymmetry models can help LHC disentangle complex decay chains of heavier decaying supersymmetric particles.

Dots: LHC alone. Vertical bands: fixing the mass of $\tilde{\chi}_1^0$ to within $\pm 2\sigma$ with ILC input ($\sigma = 0.2\%$) [M. Chiorboli et al.].
SUSY ‘discovery’ should be simple at LHC

...for squark masses < ~2 TeV

require e.g. $E_T^{\text{miss}} > 300$ GeV and four jets for $m(\text{squark}) = 900$ GeV
But exclusive reconstruction is really difficult

typical SUSY diagram at LHC

3 isolated leptons
+ 2 b-jets
+ 4 jets
+ $E_{t}^{\text{miss}}$
will in this analysis look at the decay chain shown in Fig. 5.2

\[ \tilde{q}_L \rightarrow q \chi^0_2 \rightarrow q l^\pm \bar{\nu}_l \rightarrow q l^\pm \bar{\nu}_l \chi^0_1 \]  

(5.7)

where \( \tilde{q}_L \) can be \( \tilde{d}_L, \tilde{u}_L, \tilde{b}_2 \) or \( \tilde{b}_1 \). The first two have very similar masses, \( m_{\tilde{d}_L} = 543.0 \text{ GeV} \) and \( m_{\tilde{u}_L} = 537.2 \text{ GeV} \), and will in this analysis be grouped together and referred to as \( \tilde{q}_L \). For the fraction of the chain in Eq. (5.2) which starts with a sbottom, \( \tilde{b}_1 \) is responsible for 78\%, leaving us insensitive to the contribution from \( \tilde{b}_2 \). Decay chains involving the stop are not considered. The production cross section of the relevant squarks and their branching fractions to \( \chi^0_2 \) are

\[ \sigma(\tilde{q}_L) = 33 \text{ pb}, \quad BR(\tilde{q}_L \rightarrow q \chi^0_2) = 31.4\% \]
\[ \sigma(\tilde{b}_1) = 7.6 \text{ pb}, \quad BR(\tilde{b}_1 \rightarrow b \chi^0_2) = 35.5\% \]  

(5.8)

with many of the squarks coming from gluino decay. The stau is the lightest slepton, so it has the largest branching ratio. In this analysis we will however only use final states with electrons and muons.

We shall discuss the precision that can be achieved in the determination of this spectrum at the LHC, reconstructing it back to the squark mass, from measurements of various kinematical edges and thresholds of subsets of decay products. In particular, we shall determine how this precision can be improved with input from a Linear Collider [10].

**Kinematics** The invariant masses of various subsets of particles can be determined from kinematical edges and thresholds, as discussed in [11],

\[ (m^2_{qii})^\text{edge} = \frac{(m^2_{\chi^0_2} - m^2_{i_R})(m^2_{i_R} - m^2_{\chi^0_1})}{m^2_{i_R}} \]  

(5.9)

\[ (m^2_{qii})^\text{edge} = \frac{(m^2_{\tilde{q}_L} - m^2_{\chi^0_2})(m^2_{\chi^0_2} - m^2_{\chi^0_1})}{m^2_{\chi^0_2}} \]  

(5.10)

\[ (m^2_{qii})^\text{min} = \frac{(m^2_{\tilde{q}_L} - m^2_{\chi^0_2})(m^2_{i_R} - m^2_{\chi^0_1})}{m^2_{\chi^0_2}} \]  

(5.11)

\[ (m^2_{qii})^\text{max} = \frac{(m^2_{\tilde{q}_L} - m^2_{\chi^0_2})(m^2_{i_R} - m^2_{\chi^0_1})}{m^2_{i_R}} \]  

(5.12)

\[ (m^2_{qii})^\text{thres} = \left[ (m^2_{\tilde{q}_L} + m^2_{\chi^0_2})(m^2_{\chi^0_2} - m^2_{i_R})(m^2_{i_R} - m^2_{\chi^0_1}) \right. \]
\[ \left. - (m^2_{\tilde{q}_L} - m^2_{\chi^0_2}) \sqrt{(m^2_{\chi^0_2} + m^2_{i_R})^2 + m^2_{\chi^0_1} - 16m^4_{\chi^0_2}m^4_{i_R}} \right. \]
\[ \left. + 2m^2_{i_R}(m^2_{\tilde{q}_L} - m^2_{\chi^0_2})(m^2_{\chi^0_2} - m^2_{\chi^0_1}) \right] / (4m^2_{i_R}m^2_{\chi^0_2}) \]  

(5.13)

where “min” and “max” refer to minimising and maximising w.r.t. the choice of lepton. Furthermore “thres” refers to the threshold in the subset of the \( m_{qii} \) distribution for which the angle between the two lepton momenta (in the slepton rest frame) exceeds \( \pi/2 \), which corresponds to \( m_{qii}^\text{edge} / \sqrt{2} < m_{qii} < m_{qii}^\text{edge} \).
Monte Carlo simulations In order to assess quantitatively the precision that can be achieved, we have performed Monte Carlo simulations of SUSY production at SPS 1a, using the PYTHIA 6.2 program [12] and passing the particles through the ATLFAST detector simulation [13] before reconstructing invariant masses. We have used a sample corresponding to 100 fb$^{-1}$, one year at design luminosity. The results documented in this section have been found to be in agreement with results obtained using HERWIG [16].

The cuts used to isolate the chain were the following

- At least four jets, the hardest three satisfying:
  \[ p_{T,1} > 150 \text{ GeV}, \quad p_{T,2} > 100 \text{ GeV}, \quad p_{T,3} > 50 \text{ GeV}. \]
- \[ M_{\text{eff}} \equiv E_{T,\text{miss}} + p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4} > 600 \text{ GeV} \]
- \[ E_{T,\text{miss}} > \max(100 \text{ GeV}, 0.2 M_{\text{eff}}) \]
- Two isolated Opposite–Sign Same–Flavour (OS-SF) leptons (not $\tau$) satisfying \[ p_{T}(l) > 20 \text{ GeV} \] and \[ p_{T}(l) > 10 \text{ GeV}. \]

The basic signature of our decay chain are two OS-SF leptons. Two such leptons can also be produced in other processes. If the two leptons are independent of each other, one would expect equal amounts of OS-SF leptons and Opposite–Sign Opposite–Flavour (OS-OF) leptons. Their distributions should also be identical. This allows us to remove the background OS-SF contribution by subtracting the OS-OF events.

In addition to the two OS-SF leptons, our signal event will typically have considerable missing $E_T$ and two very hard jets, one from the decay of the squark in the chain we try to reconstruct, one from the decay of the squark in the other chain.

The only Standard Model process to have all the features of our signal event, is $t\bar{t}$ production where both $W$’s decay leptonically. However, with some help from the underlying event, pile-up and detector effects, other processes might also result in the signatures above. Together with $t\bar{t}$, we therefore considered the following PYTHIA processes: QCD, $Z/W+\text{jet}$, $ZZ/ZW/WW$.

The QCD processes are cut away by the requirement of two leptons and of considerable missing $E_T$. For the processes involving $Z$ and $W$ the requirement of high hadronic activity together with missing $E_T$, removes nearly all events. The only Standard Model background to survive the rather hard cuts listed above, is a small fraction of $t\bar{t}$ events. However, the rate of $W^+W^-$ (from the decay of $t$ and $\bar{t}$) going to $e^+\mu^-$ is identical to that going to $e^+e^-/\mu^+\mu^-$, so with the subtraction of OS-OF events the $t\bar{t}$ sample gives no net contribution to the mass distribution, only some minor contribution to the fluctuations.

In Fig. 5.3 the invariant mass of the two leptons for events passing the cuts is plotted. The Standard Model background is clearly negligible. The real background to our decay chain consists of other SUSY processes, and as is illustrated in Fig. 5.3 these are effectively removed by the OS-OF subtraction. This subtraction is also included for the invariant mass distribution in Fig. 5.4.

The edge value for $m_{ll}$, Fig. 5.3 is very accurately determined by fitting it to a triangular shape with Gaussian smearing. For the other distributions, Fig. 5.4, the end points are found with a naive linear fit. This method is known not to be optimal. In fact, by changing the binning or the range fitted, the fit values may change.
Figure 5.3: Effect of subtracting background leptons, for $\int L dt = 100$ fb$^{-1}$. Left: Solid: OS-SF, Dotted: OS-OF, Two upper curves: SUSY+SM, two lower curves: SM alone; Right: OS-SF–OS-OF. The triangular shape of the theoretical expectation is reproduced.

by typically a few GeV. For a more realistic situation one would need to investigate more thoroughly how the theoretical distributions are distorted by sparticle widths, by the detector resolution and the cuts applied. As already discussed in [11], it is not possible at the level of the present studies to perform a detailed estimate of the corresponding systematic errors, therefore we include only the statistical errors from the fitting procedure for each edge. In addition one has the systematic error on the energy scale. We use the ATLAS benchmark values, 0.1% for leptons and 1% for jets. The resulting values for the endpoints and the corresponding errors are given in table 5.5.

Table 5.5: Endpoint values found from fitting the edges in Fig. 5.3 and Fig. 5.4 for 100 fb$^{-1}$.

<table>
<thead>
<tr>
<th>Edge</th>
<th>Nominal Value</th>
<th>Fit Value</th>
<th>Syst. Error Energy Scale</th>
<th>Statistical Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(ll)^{edge}$</td>
<td>77.077</td>
<td>77.024</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>$m(qll)^{edge}$</td>
<td>431.1</td>
<td>431.3</td>
<td>4.3</td>
<td>2.4</td>
</tr>
<tr>
<td>$m(q)^{min}$</td>
<td>302.1</td>
<td>300.8</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$m(q)^{max}$</td>
<td>380.3</td>
<td>379.4</td>
<td>3.8</td>
<td>1.8</td>
</tr>
<tr>
<td>$m(ll)^{thres}$</td>
<td>203.0</td>
<td>204.6</td>
<td>2.0</td>
<td>2.8</td>
</tr>
<tr>
<td>$m(ll)^{thres}$</td>
<td>183.1</td>
<td>181.1</td>
<td>1.8</td>
<td>6.3</td>
</tr>
</tbody>
</table>

5.1.2.3 Gluino mass measurement

In the considered point the gluino decays through $\tilde{g} \rightarrow \tilde{q}q$, where $\tilde{q}$ is any squark flavour, except $\tilde{t}_2$, for which the decay $\tilde{g} \rightarrow \tilde{t}_2t$ is kinematically forbidden. Thus the reconstruction of the gluino can be attempted adding a quark to an identified
A few case studies

1. The precision Higgs program

- If nothing is discovered beyond the SM Higgs boson at the LHC, this may provide the only clue for the next energy scale of new physics.

- Close to the decoupling limit, precision Higgs measurements can provide evidence for Higgs physics beyond the SM.

- Provides strong tests of the physics of EWSB dynamics, with some sensitivity to loop effects.

- In TeV-scale supersymmetry, this can probe supersymmetry breaking parameters and new sources of CP violation.
Higgs production processes at hadron colliders

\[ gg \rightarrow h_{\text{SM}} \rightarrow \gamma \gamma, \]
\[ gg \rightarrow h_{\text{SM}} \rightarrow VV^{(*)}, \quad [V = W \text{ or } Z] \]
\[ qq \rightarrow qqV^{(*)}V^{(*)} \rightarrow qqh_{\text{SM}}, \quad h_{\text{SM}} \rightarrow \gamma \gamma, \tau^+\tau^-, VV^{(*)}, \]
\[ q\bar{q}^{(*)} \rightarrow V^{(*)} \rightarrow Vh_{\text{SM}}, \quad h_{\text{SM}} \rightarrow b\bar{b}, WW^{(*)}, \]
\[ gg, q\bar{q} \rightarrow t\bar{t}h_{\text{SM}}, \quad h_{\text{SM}} \rightarrow b\bar{b}, \gamma \gamma, WW^{(*)}. \]
Higgs production processes at the ILC

- Higgs-strahlung \((e^+e^- \rightarrow Z^* \rightarrow Zh_{SM})\)
- Vector boson fusion \((e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}h_{SM})\)
- Radiation off the top quark \((e^+e^- \rightarrow t\bar{t}h_{SM})\)

A program of precision measurements will begin at LHC and will reach maturity at the ILC.
Other precision measurements:

- Total width: use $\Gamma_{\text{tot}} = \Gamma_{h_{\text{SM}}WW}/\text{BR}(h_{\text{SM}} \to WW^*)$. $\delta\Gamma/\Gamma \simeq 6\%$ for $m_{h_{\text{SM}}} = 120$ GeV.
- Spin and CP quantum number
- Reconstructing the Higgs potential: e.g., for $m_{h_{\text{SM}}} = 120$ GeV, one can measure $\delta g_{hhh}/g_{hhh} \simeq 20\%$ using $e^+e^- \to Zh^* \to Zhh$ ($h = h_{\text{SM}}$).
Anticipated precision Higgs measurements at the ILC

\[ \sqrt{s} = 350—500 \text{ GeV and } \mathcal{L} = 500 \text{ fb}^{-1} \]

<table>
<thead>
<tr>
<th>Higgs coupling</th>
<th>( \delta \text{BR}/\text{BR} )</th>
<th>( \delta g/g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( hWW )</td>
<td>5.1%</td>
<td>1.2%</td>
</tr>
<tr>
<td>( hZZ )</td>
<td>—</td>
<td>1.2%</td>
</tr>
<tr>
<td>( hbb )</td>
<td>2.4%</td>
<td>2.1%</td>
</tr>
<tr>
<td>( hcc )</td>
<td>12.0%</td>
<td>—</td>
</tr>
<tr>
<td>( h\tau\tau )</td>
<td>5.0%</td>
<td>3.2%</td>
</tr>
<tr>
<td>( h\mu\mu^* )</td>
<td>( \sim 30% )</td>
<td>( \sim 15% )</td>
</tr>
<tr>
<td>( hgg )</td>
<td>8.2%</td>
<td>—</td>
</tr>
<tr>
<td>( h\gamma\gamma )</td>
<td>16%</td>
<td>—</td>
</tr>
</tbody>
</table>

*\( \sqrt{s} = 800 \text{ GeV assumed for the } \mu^+\mu^- \text{ channel} \)

\[ \sqrt{s} = 800—1000 \text{ GeV and } \mathcal{L} = 1000 \text{ fb}^{-1} \]

<table>
<thead>
<tr>
<th>Higgs coupling</th>
<th>( \delta \text{BR}/\text{BR} )</th>
<th>( \delta g/g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( hWW )</td>
<td>2.0%</td>
<td>—</td>
</tr>
<tr>
<td>( h\tau\tau )</td>
<td>6.0%</td>
<td>—</td>
</tr>
<tr>
<td>( hbb )</td>
<td>1.6%</td>
<td>—</td>
</tr>
<tr>
<td>( hcc )</td>
<td>8.3%</td>
<td>—</td>
</tr>
<tr>
<td>( h\tau\tau )</td>
<td>5.0%</td>
<td>—</td>
</tr>
<tr>
<td>( hgg )</td>
<td>2.3%</td>
<td>—</td>
</tr>
<tr>
<td>( h\gamma\gamma )</td>
<td>5.4%</td>
<td>—</td>
</tr>
<tr>
<td>( hh )</td>
<td>12%</td>
<td>—</td>
</tr>
<tr>
<td>total decay rate</td>
<td>—</td>
<td>3.4%</td>
</tr>
</tbody>
</table>

Expected fractional uncertainties for ILC measurements of Higgs branching ratios \([\text{BR}(h \rightarrow XX)]\) and couplings \([g_{hXX}]\) for various choices of final state \(XX\), assuming \(m_h = 120 \text{ GeV} \) [Battaglia, Boos, De Roeck, Desch, Kuhl, and others]. An upgraded ILC running at 1 TeV (with \(\mathcal{L} = 1000 \text{ fb}^{-1}\)) can provide further improvements via the processes \(e^+e^- \rightarrow \bar{\nu}_e\nu_e h\), \(e^+e^- \rightarrow \bar{\nu}_e\nu_e hh\) and \(e^+e^- \rightarrow t\bar{t}h\) [Barklow, Yamashita, Gay, Besson, Winter and others].
As an example, consider the MSSM Higgs sector. If we only keep the leading $\tan \beta$-enhanced radiative corrections, then for $m_A \gg m_Z$ (approaching the decoupling limit),

$$\frac{g_{hVV}^2}{g_{h_{SM}VV}^2} \approx 1 - \frac{c^2 m_Z^4 \sin^2 4\beta}{4m_A^4},$$

$$\frac{g_{htt}^2}{g_{h_{SM}tt}^2} \approx 1 + \frac{c m_Z^2 \sin 4\beta \cot \beta}{m_A^2},$$

$$\frac{g_{hbb}^2}{g_{h_{SM}bb}^2} \approx 1 - \frac{4c m_Z^2 \cos 2\beta}{m_A^2} \left[ \sin^2 \beta - \frac{\Delta_b}{1 + \Delta_b} \right],$$

where $c \equiv 1 + \mathcal{O}(g^2)$ and $\Delta_b \equiv \tan \beta \times \mathcal{O}(g^2)$ [$g$ is a generic gauge or Yukawa coupling].

The quantities $c$ and $\Delta_b$ depend on the MSSM spectrum. The approach to decoupling is fastest for the $h$ couplings to vector boson pairs and slowest for the couplings to down-type quarks.

Thus, deviations from the decoupling limit implicitly contain information about the EWSB sector and the associated TeV-scale dynamics.
Deviations of Higgs partial widths from their SM values in two different MSSM scenarios (Carena, Haber, Logan and Mrenna).
2. Confirming and elucidating TeV-scale supersymmetry

- If new physics signals are observed at the Tevatron and/or LHC, how can we be sure that it is supersymmetry?
  - Measure the spins of the new particles, and exhibit the superpartners of SM particles with spins differing by half a unit.
  - Verify that particle/sparticle interaction vertices are related to the corresponding SM vertices by the expected supersymmetric relations.

[Nojiri, Fujii and Tsukamoto]
– Confirm supersymmetric expectations for the Higgs sector [more model dependent]

• Do supersymmetric breaking parameters exhibit any definite organizing principle?
– Are there simplifications when low-energy parameters are extrapolated to the GUT/Planck scale?

RGE evolution of gaugino (left) and scalar quark and lepton (right) mass parameters from the electroweak scale to the GUT scale in an mSUGRA model with $m_0 = 200 \text{ GeV}$, $m_{1/2} = 190 \text{ GeV}$, $A_0 = 500 \text{ GeV}$, $\tan \beta = 30$ and $\mu < 0$. The bands indicate 95% CL contours. [Blair, Porod and Zerwas].
3. Confusion scenarios

An example: models of TeV-scale supersymmetry and universal extra dimensions (UED) with $R^{-1} \sim 1$ TeV both possess a spectrum of new particles (both colored and uncolored) that are accessible to the LHC.
Models of TeV-scale supersymmetry with R-parity, UED with KK-parity and little Higgs models with T-parity all possess a parity-odd lightest particle. These models therefore possess a dark matter candidate (LSP, LKP and LTP) and yield missing energy signals at colliders. A definitive interpretation may not be possible after an LHC discovery. Precision measurements at an $e^+e^-$ collider can provide the critical evidence to distinguish among different approaches [Battaglia, Datta, De Roeck, Kong and Matchev].
4. Probing higher energies through virtual effects

Precision measurements at the ILC (from Giga-Z to the highest center-of-mass energy) provide another means for distinguishing among different interpretations of new physics at the LHC.

Precision ILC measurements of $m_W$, $\sin^2 \theta_{\text{eff}}$, $m_\chi^0$, $\text{BR}(h \rightarrow b \bar{b})$ and $\text{BR}(h \rightarrow WW^*)$ can provide strong constraints and test the consistency of mSUGRA parameter assumptions [Ellis, Heinemeyer, Olive, Weiglein].
The direct detection of signals associated with strong EWSB dynamics lies beyond the kinematic reach of the ILC. Nevertheless, precision measurements of gauge boson pair production processes are sensitive to virtual effects that provide a significant window to new physics beyond 1 TeV.

ILC sensitivity at $\sqrt{s} = 500$ GeV and $L = 500$ fb$^{-1}$ to strong EWSB dynamics. Data from $e^+e^- \rightarrow W^+W^-$ is combined with results for $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$, $\nu\bar{\nu}ZZ$ to produce the statistical significances shown here [Barklow, hep-ph/0112286].
5. Connections with cosmology

The physics of the very early universe depends critically on our understanding of the fundamental laws of nature at the highest energy scales. For this reason alone, a thorough understanding of the physics of electroweak symmetry breaking and a comprehensive exploration of TeV-scale physics will have a profound impact on cosmology. Possible contributions of the ILC include [see J. Feng and M. Trodden]:

- A precision study of the particle that makes up the dark matter.
- Evidence for or against baryogenesis controlled by physics at the electroweak scale.
- New insights into the nature of the vacuum (through detailed studies of the Higgs boson), with implications for naturalness and vacuum energy.
- If supersymmetry and/or extra dimensions are confirmed, the implications for cosmology will be profound!
RELIC DENSITY DETERMINATIONS

Parts per mille agreement for $\Omega \chi \rightarrow$ discovery of dark matter