Some physical constants:
- Speed of light: \( c=3.00\times10^8 \) m/s.
- Stefan-Boltzmann constant: \( \sigma=5.67\times10^{-8} \) W/(m\(^2\)K\(^4\)).
- Planck's constant: \( h=6.63\times10^{-34} \) J s, \( h_0=1970 \) eV Å.
- Electron mass: \( m_e=9.11\times10^{-31} \) kg, \( m_e c^2=511000 \) eV.
- Electron charge: \( -e=-1.60\times10^{-19} \) C.

This exam includes 17 problems on 3 sheets and 6 pages. Please work problems 1 through 12 on this paper and 13 through 17 on separate paper.

1) (2 pts) The ground state \((n=1)\) of the hydrogen atom has one unit of orbital angular momentum (in units of \( \hbar \))
   
   a) True
   
   b) False
   
   \( \ell = m_\ell = 0 \) in the ground state

2) (2 pts) Light of frequency \( \nu \) shines on a metal surface and photoelectrons are emitted. Each electron emitted from the surface has a kinetic energy given by
   \[ E = h\nu - \phi \]
   where \( \phi \) is a constant for a given metal called the "work function."
   
   a) True
   
   b) False
   
   Most electrons will have less due to scattering on the way out.

3) (2 pts) In a Compton scattering experiment light of wavelength \( \lambda \) incident on a solid target emerges with a longer wavelength \( \lambda' \). The observed increase in wavelength is due to
   
   a) scattering from individual atomic nuclei.
   
   b) coherent scattering from planes of atoms.
   
   c) scattering from individual quasi-free electrons.
   
   d) inelastic scattering from an atom in which the atom is excited into one of its higher bound states.

4) (2 pts) An electron and a photon with the same energy have the same de Broglie wavelength.
   
   a) True
   
   b) False
   
   \( \frac{\hbar}{\lambda} = \frac{h}{E} = \frac{hc}{E} = \frac{h}{\sqrt{E^2+m_e^2c^2}} \)

5) (2 pts) Eigenfunctions of the Schrödinger equation are always either even or odd under space inversion, \( \bar{x} \rightarrow -\bar{x} \).
   
   a) True
   
   b) False
   
   True only if the potential is symmetric
   \[ (V(\bar{x}) = V(-\bar{x})) \]
7) (2 pts) There is a minimum frequency below which electromagnetic radiation incident on a metal will not cause any photoelectrons to be emitted, no matter how intense the radiation.
   a) True
   b) False

8) (3 pts) For the following 1-D potential, indicate for each energy range whether the energy eigenvalues are **continuous** or **quantized**. Assume that outside of the drawn region the potential is always equal to zero.

   ![Potential Diagram]

   a) \(-V_0 < E < 0\) **quantized**
   b) \(0 < E < V_0\) **continuous**
   c) \(E > V_0\) **continuous**

9) (2 pts) Suppose that a particle of mass \(m\) is in a harmonic oscillator potential with spring constant \(C\) in the \(n\)’th stationary state. Then the probability density \(|\Psi_n(x,t)|^2\) will oscillate back and forth as time progresses with angular frequency

   \[\omega = \frac{E_n}{\hbar} = (n + \frac{1}{2})\sqrt{C/m}\]

   a) True
   b) False

10) (2 pts) Suppose that a particle in a time-independent quantum system has a wave function \(\Psi(x,t)\) that, in the case of the harmonic oscillator potential, never goes to zero as \(x \to \pm \infty\).

    a) The wave function must be a solution to the Schrödinger equation.
    b) The wave function must be continuous and smooth.
    c) The wave function must be zero in the region where \(E < V_0\).

11) (2 pts) Which of the following sets of observable quantities (eigenvalues) fully distinguishes the eigenstates of a central potential \(V(r)\), such as the coulomb potential?

    a) The spherical coordinates of the particle: \(r, \theta, \varphi\).
    b) The three components of the angular momentum: \(L_x, L_y, L_z\).
    c) The energy \(E\), mean radius \(\langle r \rangle\), and the magnitude of the angular momentum \(|\vec{L}|\).
    d) The energy \(E\), magnitude of the angular momentum \(|\vec{L}|\), and \(L_z\).
12) (10 pts) Ultraviolet light shines on the metallic cathode in the tube shown here. Suppose that the wavelength of the light is \( \lambda = 2000 \, \text{Å} \) and the work function of the metal surface is \( \phi = 2 \, \text{eV} \).

a) What is the voltage \( V \) at which the current ceases to flow? Positive voltage is in the direction indicated in the figure.

The kinetic energy is \( K = h\nu - \phi \)

\[
K = \frac{2\pi \cdot 1.975 \, \text{eV} \cdot \text{Å}}{2000 \, \text{Å}} - 2\text{eV} = 4.17 \, \text{eV} \quad \text{maximum}
\]

A voltage of \(-4.17 \, \text{V}\) is needed to stop all of the electrons.

b) Make a sketch of how the current in the circuit depends on the voltage for two different intensities of light.
13) (6 pts) List all possible combinations of the quantum numbers $\ell$ and $m_\ell$ for the $n=3$ state of the hydrogen atom.

14) (10 pts) Electrons are incident upon the surface of a crystal in which the spacing of atomic planes is 1.5 Å. Given that the minimum angle $\theta$ for which a maximal amount of scattering is observed is $25^\circ$, what is the momentum of the electrons in the beam in units of keV/c? (You can ignore the work function of the crystal.)

15) (15 pts) Consider the following 1-D potential barrier. A monoenergetic beam of particles of mass $m$ and $E > V_0$ is incident from the left.

\[ V(x) \]
\[ E \]
\[ V_0 \]
\[ 0 \]
\[ x \]

a) Find the complete expression for the spatial part of the wave function of a beam particle in the region $x < 0$ in terms of $E$, $m$, $V_0$, and $\hbar$. There should be only a single arbitrary constant remaining (the overall normalization factor).

b) What is the probability for a particle to reflect from the step at $x=0$ if $E = 2V_0$?
13) (6 pts) List all possible combinations of the quantum numbers $\ell$ and $m$, for the n=3 state of the hydrogen atom.

14) (10 pts) Electrons are incident upon the surface of a crystal in which the spacing of atomic planes is 1.5 Å. Given that the minimum angle $\theta$ for which a maximal amount of scattering is observed is 25°, what is the momentum of the electrons in the beam in units of $\text{keV}/c$? (You can ignore the work function of the crystal.)

15) (15 pts) Consider the following 1-D potential barrier. A monoenergetic beam of particles of mass $m$ and $E > V_0$ is incident from the left.

(a) Find the complete expression for the spatial part of the wave function of a beam particle in the region $x < 0$ in terms of $E$, $m$, $V_0$, and $\hbar$. There should be only a single arbitrary constant remaining (the overall normalization factor).

(b) What is the probability for a particle to reflect from the step at $x=0$ if $E = 2V_0$?
\[ n = 2 \]

\[ \ell = \begin{array}{c|c}
0 & m_\ell = 0 \\
1 & m_\ell = -1, 0, +1 \\
2 & m_\ell = -2, -1, 0, +1, +2 \\
\end{array} \]
14. Bragg scattering

For constructive interference, \(2d \sin \theta = n \lambda\)

Minimum angle \(\theta\) for \(n = 1\)

\[\lambda = 2d \sin \theta = 2 \cdot 1.5 \cdot \sin 25^\circ = 1.27\]

\[p = \frac{\lambda}{2} = \frac{2 \pi hc}{\lambda} \frac{1}{c} = \frac{2 \pi \cdot 1970}{1.27} \frac{1}{c} = 9.75 \text{ keV}\]

15.

(a) \(\psi_i(x) = A e^{ik_x x} + B e^{-ik_x x} \quad k_x = \frac{\sqrt{2 \pi E}}{\hbar}\)

\(\psi_s(x) = C e^{ik_{s1} x} \quad k_{s1} = \frac{\sqrt{2 \pi (E-V_c)}}{\hbar}\)

\(\psi_i(0) = \psi_s(0) \left( A + B = C \right) i k_x\)

\(\psi_i'(0) = \psi_s'(0) \left( i k_x (A - B) = i k_x C \right)\)

\(i(k_{s1} - k_x) A + i(k_x + k_{s1}) B = 0\)

\[B = \frac{k_{s1} - k_x}{k_x + k_{s1}} A = \frac{\sqrt{E - V_c}}{\sqrt{E + V_c}} A\]

\(\psi_i(x) = A \left[ e^{ik_x x} + \left( \frac{\sqrt{E - V_c}}{\sqrt{E + V_c}} \right)^{ik_{s1} x} \right]\)

\[P = \left[ \frac{\sqrt{E - V_c}}{\sqrt{E + V_c}} \right]^2 = \left[ \frac{\sqrt{2} - \sqrt{2} - 1}{\sqrt{2} + \sqrt{2} - 1} \right]^2 = \left[ \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right]^2 = 0.029\]
(a) \[ E^\pi_1 + E^\pi_2 = m_K c^2 \]
\[ p^\pi_1 + p^\pi_2 = 0 \]
Since \( m_1 = m_2 \) this means that \( E^\pi_1 = E^\pi_2 \)

So, \[ 2E^\pi = m_K c^2 \]
\[ E^\pi = \frac{1}{2} \cdot 2.49 = 2.49 \text{ MeV} \]

\[ p^\pi = \sqrt{E^2 - m^2} = \sqrt{2.49^2 - 140^2} = 2.06 \text{ MeV/c} \]

(b) \[ \beta = 0.60 \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = 1.25 \]
\[ p^\prime = \gamma [ p/c + \beta E_1 ] \quad \text{Lorentz transformation} \]
\[ p^\prime = 1.25 [(2.06 + 0.60 \cdot 2.49)] = 4.44 \text{ MeV} \]
\[ p^\prime_1 = 1.25 [-2.06 + 0.60 \cdot 2.49] = -7.1 \text{ MeV} \]
\[ p^\prime_1 = 4.44 \text{ MeV/c in lab (in direction of the K_s)} \]
\[ p^\prime_2 = -7.1 \text{ MeV/c in lab} \]
17. \( \psi_1(x) = A \sin k_1 x \quad k_1 = \frac{\sqrt{2m(E-V_0)}}{\hbar} \)

\[ \Rightarrow 0 \quad \text{at} \quad x = 0 \]

\( \psi_2(x) = B \sin k_2 (x-a) \quad k_2 = \frac{\sqrt{2mV}}{\hbar} \)

\[ \Rightarrow 0 \quad \text{at} \quad x = a \]

\( \psi_1(x) = \psi_2(x) \quad A \sin k_1 x = B \sin k_2 (x-a) = -B \sin k_2 a \)

\( \psi_1'(x) = \psi_2'(x) \quad k_1 A \cos k_1 x = k_2 B \cos k_2 (x-a) = k_2 B \cos k_2 a \)

\( A \sin k_1 x = -B \sin k_2 x \quad k_1 A \cos k_1 x = k_2 B \cos k_2 x \)

Div. dc:

\[ \frac{1}{k_1 \tan k_1 \chi} = -\frac{1}{k_2 \tan k_2 \chi} \]

\[ \frac{1}{\sqrt{E-V_0}} \tan \frac{\sqrt{2m(E-V_0)}}{\hbar} \chi = -\frac{1}{\sqrt{E}} \tan \frac{\sqrt{2mE}}{\hbar} \chi \]