Homework Assignment #2

Due in class, Friday, April 16  90 points total


2) An extremely important and useful property of the eigenfunctions of the hydrogen atom (and, in fact, of eigenfunctions of the Schrödinger Equation for any potential) is that they are “orthogonal.” That means that if we take any two different eigenfunctions $\psi_{n\ell m}(\mathbf{r})$ and $\psi_{n'\ell' m'}(\mathbf{r})$ with $n \neq n'$, $\ell \neq \ell'$, and $m \neq m'$, then

$$\int \int \int \psi^*_{n\ell m}(\mathbf{r}) \cdot \psi_{n'\ell' m'}(\mathbf{r}) dV = 0$$

In words, this integral over all space gives unity if all the quantum numbers are the same for the two wave functions (in which case it is just the normalization integral), but it gives zero if any one or more of the quantum numbers differs between the two wave functions.

   a) Demonstrate that $\psi_{2,1,1}$ is orthogonal to $\psi_{2,1,-1}$.
   b) Demonstrate that $\psi_{2,0,0}$ is orthogonal to $\psi_{2,1,0}$.
   c) Demonstrate that $\psi_{1,0,0}$ is orthogonal to $\psi_{2,0,0}$.

3) Consider a hydrogen atom in a superposition of two states of different $m_z$:

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{2}} \left[ \psi_{2,1,1}(\mathbf{r}) + \psi_{2,1,-1}(\mathbf{r}) \right]$$

   a) Show that this state is not an eigenfunction of the operator $L_z$ (that is $L_z \psi \neq \text{constant} \times \psi$).
   b) Calculate the expectation values $\langle L_z \rangle$ and $\langle L_z^2 \rangle$.
   c) Calculate the uncertainty in the $z$ component of the angular momentum:

   $$\Delta L_z = \sqrt{\langle L_z^2 \rangle - \langle L_z \rangle^2}$$

   d) Show by direct calculation that the probability density of this state is not independent of $\phi$.

   Explain how this result is related to the nonzero value obtained in part (c).

4) As a naïve classical model, pretend that an electron is a rigid sphere of radius $10^{-18}$ m with a uniform mass density. From the known mass and the known spin angular momentum calculate the rotational speed at the equator. Note that the radius given roughly corresponds to present experimental limits on the charge radius of the electron. In your calculation, please think back to Physics 5A (or equivalent) and use (or derive) the correct formula for the moment of inertia of a uniform sphere. What does your result tell you about the validity of this classical picture of the electron magnetic moment?

5) The separation between the upper and lower beams of atoms emerging from the magnet in a Stern-Gerlach experiment depends on the inhomogeneity of the magnetic field and on the length of the magnet.

   a) What value of $\partial B_z / \partial z$ is required to produce a separation of 1 mm between the upper and lower beams of hydrogen atoms emerging from a magnet 0.12 m long if the speed of the atoms is 600 m/s?
   b) If we use a beam of phosphorus atoms in a Stern-Gerlach experiment, we find four separate impact zones on the photographic plate. What is the total angular momentum of these atoms?

6) Problem 8.6 of Eisberg & Resnick. Splitting of hydrogen energy levels in a very strong field.


8) Problem 8.10 of Eisberg & Resnick. Angles between angular momentum vectors.


**Required Reading:**

- Read Chapter 8 of Eisberg & Resnick.

**Recommended:** Review in your introductory physics book the concept of a magnetic moment and the interaction between a magnetic moment and a magnetic field, or between two magnetic moments.