Homework Assignment #8
Due Monday, December 5 by 4:30 pm

Using the Born approximation is primarily an exercise in evaluating the appropriate integral. For these problems I will not accept solutions from computer programs such as Mathematica. Do the integrals by hand, at least to the point of reducing the integral to a standard form found in integral tables.

1. Using the Born approximation (Eqn. 11.79) calculate the differential cross section for scattering from the repulsive square-well potential:

\[ V(r) = \begin{cases} V_0 & r \leq a \\ 0 & r > a \end{cases} \]

Show that your result reduces to Eqn. 11.83 in the low-energy limit. Note: if you start from Eqn. 11.88 instead of Eqn. 11.79, then for this problem you must write out in detail the derivation of that equation starting from 11.79 (in the other problems below you may start from Eqn. 11.88 where appropriate).

2. Using the Born approximation, calculate the differential cross section and total cross section for scattering of a particle of mass \( m \) and energy \( E \) from an attractive Gaussian potential:

\[ V(r) = -V_0 e^{-r^2/a^2} \]

Do the integral by hand as follows: expand the sine function into a sum of two complex exponentials and complete the square in order to write the exponents in the integrands as perfect squares. Then the integrals are simple Gaussian definite integrals, as found in the inside back cover of the textbook. (If you don’t recall what “complete the square” means, then Google it or look it up somewhere, such as in Wikipedia.)

3. Problem 11.16 in Griffiths, finding the Green’s function for the 1-D Schrodinger equation. This requires a contour integration very similar to what is done for the 3-D case in Section 11.4.

4. Problems 11.17 and 11.18 in Griffiths, the 1-D Born approximation.

5. Suppose that two spin-½ particles are known to be in the singlet configuration \( |00\rangle \) (Eqn. 4.178). Let \( S_a^{(1)} \) be the component of the spin angular momentum of particle number 1 in the direction defined by the unit vector \( \hat{a} \). Similarly, let \( S_b^{(2)} \) be the component of the spin angular momentum of particle number 2 in the direction defined by the unit vector \( \hat{b} \). Derive the following quantum mechanical expectation value (i.e. Eqn. 12.4):

\[ \langle 00|S_a^{(1)}S_b^{(2)}|00\rangle = -\frac{\hbar^2}{4} \cos \theta \]

where \( \theta \) is the angle between \( \hat{a} \) and \( \hat{b} \). Hint: with no loss of generality you can take \( \hat{a} = \hat{z} \), so \( S_a^{(1)} = S_z^{(1)} \), and then take \( \hat{b} \) to be in the xz plane, so \( S_b^{(2)} = \cos \theta S_x^{(2)} + \sin \theta S_z^{(2)} \) (See Eqns. 4.145 and 4.147 for the relevant spin operators).

6. In Fig. 12.3, if the angle between \( \hat{a} \) and \( \hat{e} \) is \( \theta \), instead of 45°, for what range of \( \theta \) is the Bell inequality violated by quantum mechanics, and where is the maximum violation? You may do this numerically or graphically, if you prefer.